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THE LOST
YSTEM OF THE ANCIENTS
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VOL. I.

THE LOST
SOLAR SYSTEM OF THE ANCIENTS
DISCOVERED.

VOL. I.

LONDON:
Printed by SPOTTISWOODE & Co.,
New-street-Square.

THE LOST
SOLAR SYSTEM OF THE ANCIENTS
DISCOVERED.

BY JOHN WILSON.

IN TWO VOLUMES.

VOL. I.

LONDON:
LONGMAN, BROWN, GREEN, LONGMANS, & ROBERTS.
1856.

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THE

LOST SOLAR SYSTEM OF THE ANCIENTS DISCOVERED.

PART I.

GRAVITATION NEAR THE EARTH'S SURFACE. — CONSTRUCTION OF THE OBELISK. — VARIATION OF TIME, VELOCITY, AND DISTANCE REPRESENTED BY THE ORDINATES AND AXIS OF THE OBELISK. — THE OBELISCAL AND PARABOLIC AREAS COMPARED. — CONSTRUCTION AND SUMMATION OF OBELISCAL SERIES OF NUMBERS, SQUARES, AND CUBES. — SERIES OF OBELISKS AND PYRAMIDS COMPARED AND SUMMED. — SERIES TO THE SECOND, FOURTH, AND SIXTH POWERS. — SERIES OF CUBES CIRCUMSCRIBED BY SQUARES. — THE OBELISCAL STAR OR CROSS. — COMPLEMENTARY OBELISCAL SERIES. — PYLONIC CURVE GENERATED BY THE ORDINATE WHICH VARIES INVERSELY AS THE ORDINATE OF THE OBELISK. — THE HORN OF JUPITER AMMON FORMED BY THE SPIRAL OBELISK.

The Laws of Gravitation expounded by the Geometrical Properties of the Obelisk.

IT was found by Galileo that a heavy body, when allowed to fall freely from a state of rest towards the earth, described distances proportionate to the square of the times elapsed during the descent; or proportionate to the square of the velocities acquired at the end of the descent.

That is, at the end of the 1st second the body had described a distance of $16\frac{1}{12}$ feet English, which call 1 P.

At the end of the 2nd second, from the beginning of motion, the body had described a distance of 4 P.

At the end of the 3rd second, a distance of 9 P.

At the end of the 4th second, a distance of 16 P.

Thus the distances described at the end of

$$\begin{array}{lrrrrl}
 & 1, & 2, & 3, & 4 & \text{seconds are} \\
 & 1^2, & 2^2, & 3^2, & 4^2, & \text{or} \\
\text{1st series} & 1, & 4, & 9, & 16 & \text{P} \\
 & & 1, & 4, & 9 & \\
\cline{2-5}
\text{2nd series} & 1, & 3, & 5, & 7 & \text{difference} \\
 & & 1, & 3, & 5 & \\
\cline{2-5}
\text{3rd series} & 1, & 2, & 2, & 2 & \text{difference.}
\end{array}$$

Here 1, 4, 9, 16 P are the series of distances described in 1, 2, 3, 4, seconds.

1, 3, 5, 7, the series of distances described in each second.

1, 2, 2, 2, the series of incremental distances described in each second more than was described in the preceding second.

During the first second the distance described = 1 P. If the velocity had been uniform the distance would have been described in 1 second with the mean velocity = half the extreme velocities $=\frac{1}{2}(0+2)=1$ P. So that at the end of the 1st second the acquired velocity would = 2 P. The velocity acquired at the end of the 2nd second would = twice the mean velocity with which the whole distance 4 P was described in two seconds. The mean velocity will $=\frac{1}{2}(0+4)=2$ P; therefore the velocity at the end of the 2nd second will = 4 P; at the end of the 3rd second = 6 P; at the end of the 4th second = 8 P.

The velocity acquired at the end of the 1st second, if continued uniform during the 2nd second, would, of itself, have carried the body 2 P; but during the 2nd second the body received an additional accelerating velocity from gravity equal to that which caused it to describe 1 P in the 1st second. So that during the 2nd second the distance described will $=2+1=3=1+2$ P. In like manner, during the 3rd second, the distance described will $=4+1=5=3+2$ P. In the 4th second $6+1=7=5+2$ P, will be described.

The distances described in the successive seconds will be

1, 3, 5, 7 P.

The velocities at the beginning of the 1st, 2nd, 3rd, and 4th seconds will be

0, 2, 4, 6 P,

at the end 2, 4, 6, 8 P.

The mean of the extreme velocities in the successive seconds are

1, 3, 5, 7,

For

$$\tfrac{1}{2}(0+2)=1$$
$$\tfrac{1}{2}(2+4)=3$$
$$\tfrac{1}{2}(4+6)=5$$
$$\tfrac{1}{2}(6+8)=7.$$

Generally, the distance $(2n-1)$ P, described in the n^{th} second with an accelerated velocity, will be uniformly described with the mean of the velocities at the beginning and end of the n^{th} second ; which mean velocity will $= \tfrac{1}{2}(2n-2+2n)=(2n-1)$ P.

The whole distance described during n seconds will be proportionate to the square of the time, and $= n^2$ P.

At the end of the descent the acquired velocity will be proportionate to the whole time elapsed, and $= 2n$ P in a second.

During the descent equal increments of velocity 2 P are generated during each second.

Hence the effect produced by gravity may be regarded as constant for so small a distance as the body describes while falling freely near the earth's surface.

To construct the Obelisk.

When a body falls from a state of rest, near the earth's surface, by the action of gravity, the time elapsed and the velocity acquired at the end of the descent will vary as the square root of the distance described.

A body falling from rest will describe a straight line.

Let the point whence the body begins to fall be the apex of the obelisk, and the distance described be along the axis. (*fig.* 1.)

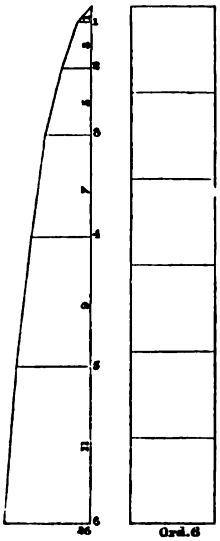


Fig. 1.

If at the end of the descent a straight line be drawn perpendicular to the axis, and made = the square root of the axis, this line will be an ordinate, and equal the square root of the axis.

Since the ordinate varies as $\overline{\text{axis}}^{\frac{1}{2}}$ and time varies as $\overline{\text{distance}}^{\frac{1}{2}}$, the ordinate will represent the variation of the time of descent, and the axis that of the distance described.

So that, when the body has descended 1 P along the axis, let an ordinate be drawn at the distance of unity from the apex and made = $\sqrt{1}$, or 1; this ordinate will represent 1 second, the time of describing 1 P along the axis. Again when the body has fallen from the apex to a distance of 4 P, there draw an ordinate = $\sqrt{4} = 2$, which will represent the time 2 seconds, during which the body fell from rest to a distance of 4 P. When the body has fallen from the apex to a distance of 9 P, there draw an ordinate = $\sqrt{9} = 3$, which will represent 3 seconds, the time of falling 9 P. Thus any number of ordinates may be drawn, and each made = the $\overline{\text{axis}}^{\frac{1}{2}}$.

When the extremities of these ordinates are joined by straight lines, the area included by these lines, the axis and the last ordinate will be an obeliscal area.

The ordinate of an obeliscal area will = in units the number of seconds elapsed during the descent from the apex to the ordinate; and the axis will = in units the number of P's described during the descent from the apex to the ordinate.

As the time and velocity both vary as the square root of the distance, and at the end of

1, 2, 3, 4 seconds
2, 4, 6, 8 P,

are the acquired velocities,

Then since ordinates made equal the square root of the axes represent the times, or number of seconds elapsed during the descent; it follows, that double ordinates, or ordinates twice the length of the corresponding time ordinates, will represent the velocity acquired in the descent from the apex to these ordinates.

As the n^{th} velocity ordinate will equal $2\,n$, or twice the corresponding time ordinate, so an additional ordinate like the time ordinate may be drawn on the other side of the axis; these together will represent the velocity ordinate. So that during n seconds the distance described will $= n^2$ P, and the velocity acquired at the end of the descent will $= 2\,n$ P in a second.

When the ordinates (*fig.* 6.) 1, 2, 3, 4, &c. are bisected and joined at the extremities by straight lines, an obeliscal area is formed equal to that of *fig.* 1.

An obeliscal sectional axis is the part of the axis intercepted by two consecutive ordinates, and are as 1, 3, 5, 7.

An obeliscal sectional area is the area included between two consecutive ordinates.

Sum of n sectional axes = whole axis.

$$\text{or } 1+3+5+7=n^2.$$

Sum of n ordinates $=1+2+3+4=\frac{1}{2}\overline{n+1}\,.\,n$

$$\text{Difference} = \tfrac{1}{2}\overline{n-1}\,.\,n.$$

Hence the difference between the sum of the sectional axes, or whole axis of the obelisk, and the sum of the corresponding ordinates will equal $\frac{1}{2}$ axis $-\frac{1}{2}$ ordinate $=\frac{1}{2}n^2-\frac{1}{2}n$.

Figs. 2. and 3. will represent

1st series,	1,	2,	3,	4	time ordinates.
2nd „	2,	4,	6,	8	velocity ordinates.
3rd „	1,	4,	9,	16	axes, or D.
4th „	1,	3,	5,	7	sectional axes, or *d*.
5th „	1,	2,	2,	2	sectional increments.

The 1st series represents the time ordinates. The 2nd series the velocity ordinates. The 3rd series their corresponding axes, or distances D, described from the apex to the time or velocity ordinates. The 4th series the sectional axes, or distances *d*, described during successive seconds. The 5th series is formed by taking from each term of the 4th series the term immediately preceding. Similarly, the 4th series is formed from the 3rd series; the 5th series form the increments of the 4th series; for 4 terms of the 5th series = the 4th term of the 4th series. So the terms of the 4th series form the increments of the 3rd series; since 4 terms of the 4th series = the 4th term of the 3rd series.

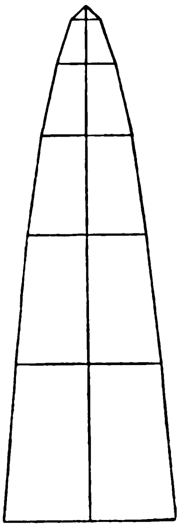


Fig. 2.

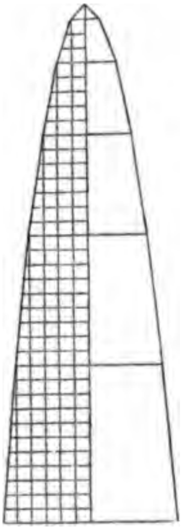


Fig. 3.

Generally n terms of the 5th series = the n^{th} term of the 4th series; or n terms of the 4th series = the n^{th} term of the 3rd series.

The sum of 4 terms of the 5th series, the increments of the 4th series, described with accelerating velocities, will = the sectional axis 7, described with an accelerating velocity during one second. Also the mean of the extreme velocities with which the sectional axis 7 would be described in the 4th second $=\frac{1}{2}(6+8)=7$. Also n terms of the 4th series, the sectional axes, or distances *d*, described in n successive seconds, will = the n^{th} term of the 3rd series, or whole axis, or distance D, described in n seconds.

The mean velocity with which the axis or whole distance D, (n^2P) would be uniformly described in n seconds

$$= \tfrac{1}{2} \text{ the extreme velocities} = \tfrac{1}{2}(o + 2 \,.\, n\text{P})$$
$$= n\text{P in a second.}$$

The mean velocity with which the sectional axis $\overline{2n-1}\,.\,\text{P}$, described in the n^{th} second would be uniformly described in one second

$$= \tfrac{1}{2} \text{ the extreme velocities}$$
$$= \tfrac{1}{2}(\overline{n-1} \times 2\text{P} + n \times 2\text{P}) = \overline{2n-1}\,.\,\text{P}.$$

Or, let the distance described $= 100\,.\,\text{P} =$ axis. The time ordinate will $= \sqrt{100} = 10$ seconds, and the velocity acquired at the end of 10 seconds, or of the descent, will = twice the time ordinate $= 2\sqrt{100} = 20\,.\,\text{P}$.

If this acquired velocity were continued uniform during another 10 seconds, the distance described would $= 10 \times 20\text{P} = 200\,.\,\text{P} =$ twice the distance described, when the body fell from rest till the acquired velocity equalled $20\,.\,\text{P}$ a second.

The velocities acquired and the distances described at the end of

	1, 2, 3, 4 seconds,
are	2, 4, 6, 8 P velocities,
and	1, 4, 9, 16 P distances.

The distance described in 4 seconds with an accelerating velocity will = the distance described uniformly in 4 seconds with the mean velocity

$$= 4 \times \tfrac{1}{2}(0 + 8) = 4 \times 4 = 16\,\text{P}.$$

As the body had no velocity at the beginning of the descent, the mean velocity will = half the last acquired velocity.

Hence with half the velocity acquired at the end of 4 seconds, if continued uniform during 4 seconds, the distance described would = the distance described in 4 seconds with an accelerating velocity.

Thus the axis of the obelisk represents the distance described. The single ordinate, made = the square root of the distance or axis, will represent the time elapsed during the descent, and the double ordinate will represent the velocity acquired at the end of the time, or descent.

The different distances intercepted by the ordinates, or the sectional axes, will represent the distances 1, 3, 5, 7 P, described during the 1st, 2nd, 3rd, 4th seconds. The distances 1, 3, 5, 7 also correspond with the mean velocities, or with the mean of the velocity ordinates at the beginning and end of each second.

The axis and ordinates are multiples of the same unity,—that of the obelisk,

Unity in the axis $= 1 \,.\, \mathrm{P}$
Unity in the velocity ordinates $= 1 \,.\, \mathrm{P}$

but unity in the time ordinates $= 1$ second.

The variation of velocity and distance described during each of six successive seconds will be seen below, where

s, denotes seconds;
v, velocity at the beginning of each second;
g, the additional effect of gravity during each second;
d, distance described in each second;
v′, velocity acquired at the end of each second;
D, the whole distance described at the end of the several seconds.

s.	*v.*	*g.*	*d.*	*v′.*	D.
1st.	0	+ 1 =	1	= 2	= 1
2nd.	2	+ 1 =	3	= 4	= 4
3rd.	4	+ 1 =	5	= 6	= 9
4th.	6	+ 1 =	7	= 8	=16
5th.	8	+ 1 =	9	=10	=25
6th.	10	+ 1 =	11	=12	=36
	30		36	42	

Half the sum of v + half the sum of $v' = \frac{1}{2}\overline{30+42} = 36$. Or the mean of the sum of the velocities at the beginning and end of each of the six seconds $= 36 =$ sum of the distances d, described during six seconds $=$ whole axis $=$ ordinate$^2 = 6^2$.

The 36 described with an uniform velocity during six seconds will $= \frac{36}{6} = 6$ during each second.

The mean of the velocities at the beginning and end of six seconds $= \frac{1}{2}\overline{0+2\times 6} = 6$.

Let $i.\ d$ denote the increment of d in a second, then during

s.	*i. d.*	*d.*	D.
1st.	=1	=1	= 1
2nd.	=2	=3	= 4
3rd.	=2	=5	= 9
4th.	=2	=7	=16
	7	16	

The sum of $i.\ d=7=d$, described in the fourth second.

The sum of $d=16=\text{D}$, described during the four seconds.

Let $i.\ v$ denote the increments of velocity at the beginning and end of each of the four seconds.

Then at the beginning of the

s.	*i. v.*	*v.*
1st.	=0	=0
2nd.	=2	=2
3rd.	=2	=4
4th.	=2	=6
	6	

At the end of the

s.	*i. v.*	*v.*
1st.	=2	=2
2nd.	=2	=4
3rd.	=2	=6
4th.	=2	=8
	8	

The sum of $i.\ v$ at the beginning of the fourth second $=6$; at the end $=8$.

Also the acquired velocities at the beginning and end of the fourth second are 6 and 8.

The mean $=\frac{1}{2}\overline{6+8}=7=$ the distance described in the fourth second.

The Obeliscal Area.

An obeliscal area $=\frac{1}{2}$ the area of *fig.* 3, or the whole of *fig.* 1. or 6., and is composed of sectional areas intercepted by the ordinates 1, 2, 3, 4, 5, 6, or defined by the sectional axes, 1, 3, 5, 7, 9, 11.

Fig. 3.	1st sectional area	$= \frac{1}{2}$ or $\frac{1}{2}$ of	1 or	1^2
	2nd ,,	$= 4\frac{1}{2}$	9	3^2
	3rd ,,	$=12\frac{1}{2}$	25	5^2
	4th ,,	$=24\frac{1}{2}$	49	7^2
	5th ,,	$=40\frac{1}{2}$	81	9^2
	6th ,,	$=60\frac{1}{2}$	121	11^2

The area from the apex to the 1st ordinate $=\frac{1}{2}$, and $\frac{2}{3}$ the circumscribing parallelogram = a parabolic area $=\frac{2}{3}$ axis × ordinate $=\frac{2}{3}\overline{1 \times 1}=\frac{2}{3}$.

Difference $= \frac{2}{3}-\frac{1}{2}=\frac{1}{6}$ unity.

Area from the apex to the 2nd ordinate $=\frac{1}{2}+4\frac{1}{2}=5$.

$\frac{2}{3}$ axis × ordinate $=\frac{2}{3}4 \times 2=\frac{2}{3}8=5\frac{1}{3}$.

Difference $=5\frac{1}{3}-5=\frac{1}{3}=\frac{2}{6}$.

Area from the apex to the 3rd ordinate $=5+12\frac{1}{2}=17\frac{1}{2}$.

$\frac{2}{3}$ axis × ordinate $=\frac{2}{3}9 \times 3=\frac{2}{3}27=18$.

Difference $=18-17\frac{1}{2}=\frac{1}{2}=\frac{3}{6}$.

Thus the curvilinear or parabolic areas will exceed the obeliscal areas contained by straight lines by $\frac{1}{6}$, $\frac{2}{6}$, $\frac{3}{6}$, $\frac{4}{6}$, $\frac{5}{6}$, $\frac{6}{6}$, corresponding to the ordinates 1, 2, 3, 4, 5, 6.

So that the difference between the curvilinear area and the area included by straight lines, or the parabolic and obeliscal areas at the 6th ordinate will be six times greater than the difference between these two areas at the 1st ordinate.

The difference between the two areas at the 1st and nth ordinate will be as $\frac{1}{6}1 : \frac{1}{6}n$.

Thus as n increases the two areas will continually approach to equality; since $\frac{2}{3}n^3-\frac{1}{6}n$ will continually approach to $\frac{2}{3}n^3$.

For parabolic area $=\frac{2}{3}$ axis × ordinate.

$$=\frac{2}{3}n^2 \times n=\frac{2}{3}n^3.$$

and obeliscal area $=\frac{2}{3}n^3-\frac{1}{6}n$.

Figs. 4, 5. The sum of the series

$$1^2+3^2+5^2+7^2+9^2+11^2=286=\frac{4}{3}n^3-\frac{1}{3}n,$$

Axis $=1+3+5+7+9+11=n^2=36$.

Ordinate $= 2n$.

$$\tfrac{2}{3}\text{ axis} \times \text{ordinate}$$
$$= \tfrac{2}{3}\, n^2 \times 2n$$
$$= \tfrac{4}{3} n^3 = \tfrac{4}{3}\, 6^3 = 288,$$

and $288 - 286 = 2 = \tfrac{1}{3} 6 = \tfrac{1}{3} n$.

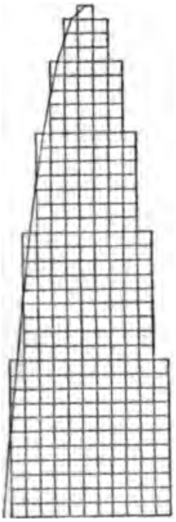


Fig. 4.

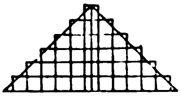


Fig. 5 a.

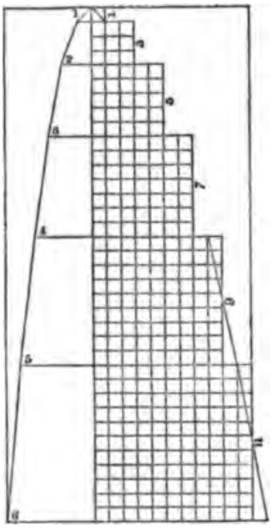


Fig. 5.

Hence the sum of the series $= \tfrac{4}{3} n^3 - \tfrac{1}{3} n$, and $\tfrac{1}{2}$ the sum of the series $= \tfrac{2}{3} n^3 - \tfrac{1}{6} n =$ the single obeliscal area.

Fig. 3. The sectional areas along the sectional axes, 1, 3, 5, 7, 9, 11, &c., and between the ordinates 0 and 1, 1 and 2, 2 and 3, &c., are

$$\begin{array}{rlrrrl}
1 \ldots\ldots & 0 \times 1 + \frac{1}{2} = & \frac{1}{2} = & 1 \times \frac{1}{2} = & \frac{1}{2} \text{ of } & 1^2 \\
3 \ldots\ldots & 2 \times 2 + \frac{1}{2} = & 4\frac{1}{2} = & 3 \times 1\frac{1}{2} = & ,, & 3^2 \\
5 \ldots\ldots & 4 \times 3 + \frac{1}{2} = & 12\frac{1}{2} = & 5 \times 2\frac{1}{2} = & ,, & 5^2 \\
7 \ldots\ldots & 6 \times 4 + \frac{1}{2} = & 24\frac{1}{2} = & 7 \times 3\frac{1}{2} = & ,, & 7^2 \\
9 \ldots\ldots & 8 \times 5 + \frac{1}{2} = & 40\frac{1}{2} = & 9 \times 4\frac{1}{2} = & ,, & 9^2 \\
11 \ldots\ldots & 10 \times 6 + \frac{1}{2} = & \underline{60\frac{1}{2}} = & 11 \times 5\frac{1}{2} = & ,, & 11^2 \\
 & & 143 & & &
\end{array}$$

circumscribing parallelogram = axis × ordinate
$=36\times 6=216.$

Parabolic area $=\frac{2}{3}\,216=144.$
$144-143=1$
Obeliscal area $=\frac{2}{3}n^3-\frac{1}{6}n=\frac{2}{3}6^3-\frac{1}{6}6=143.$

Though the actual difference between every two corresponding obeliscal and parabolic sectional areas equals $\frac{1}{6}$ unity; yet the relative difference between two such areas will be greater nearer the apex, and less as the ordinates recede from the apex.

Generally the corresponding areas of the n^{th} section will be as $\frac{1}{2}\,.\,\overline{2n-1}^2 : \frac{1}{2}\,\overline{2n-1}^2+\frac{1}{6}n$

When $n=6$, the areas will be as $60\frac{1}{2} : 61\frac{1}{2}$.

When $n=12$, the areas will be as $264\frac{1}{2} : 266\frac{1}{2}$.

The sum of the two ordinates = the axis of an obeliscal sectional area. As the successive sectional axes, or distance between the two ordinates, are continually increasing by 2, while the difference between the two ordinates,— unity, — remains the same, it follows that the opposite sides of the single obelisk (*fig.*6.), will continually approach to parallelism, but which they can never attain; for how great soever the sectional axes, or the sum of the two ordinates may be, still their difference will equal unity, so the sides of a sectional obeliscal area can never become parallel to the axis.

The two sides of an obeliscal sectional area are always equal, and the two ordinates are always parallel. If the two ordinates were also equal, then the four sides would form a rectangular parallelogram, the opposite sides of which would be parallel to each other, as are the ordinates.

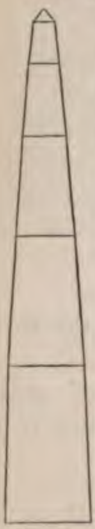


Fig. 6.

An ordinate equal the mean ordinate of any obeliscal sectional area will always correspond to an axis equal to the distance from the apex to the point of bisection of that sectional axis, less $\frac{1}{4}$ unity, a constant quantity.

For the sectional axis intercepted by the $\overline{n-1}$ and n^{th} ordinates $=\overline{2n-1}$, the half of which $=\overline{n-\frac{1}{2}}=$ the mean of the two ordinates $\overline{n-1}$ and n.

So the whole axis from the apex to the point of bisection of the sectional axis will

$$=n^2-(n-\tfrac{1}{2})=n^2-n+\tfrac{1}{2}.$$

But the axis corresponding to the ordinate $\overline{n-\frac{1}{2}}$ will $=\overline{n-\frac{1}{2}}^{\,2}$ $=n^2-n+\frac{1}{4}$, which is less than $n^2-n+\frac{1}{2}$ by $\frac{1}{4}$. Hence the mean ordinate of the 1st sectional area, which $=\frac{1}{2}$, will be at the distance from the apex $=\frac{1}{2}-\frac{1}{4}=\frac{1}{4}$ unity; so that an ordinate drawn at $\frac{1}{4}$ from the apex, and made $=\frac{1}{2}$ unity, will be an ordinate to the parabola.

The parabolic area of the 1st section will be to the corresponding obeliscal area $:: \frac{2}{3} : \frac{1}{2} :: 4 : 3$.

The ordinates of the parabolic and obeliscal area are equal at the beginning and end of each section, but the intermediate ordinates of the parabola are greater than the corresponding intermediate ordinates of the obeliscal area. This difference of the ordinates makes a sectional area of the parabola exceed the corresponding sectional obeliscal area by $\frac{1}{6}$ unity.

If the double ordinates, like the velocity ordinates, were made ordinates of an obeliscal area; then the successive sectional areas would equal 1^2, 3^2, 5^2, 7^2 (*Figs.* 3, 4, 5), or equal twice the single obeliscal series of sectional areas of *Figs.* 1. or 6. Then each parabolic sectional area will exceed the corresponding obeliscal sectional by $\frac{1}{3}$ of 1.

The Construction and Summation of Obeliscal Series.

The sum of the series $1+2+3+4$, &c. $=\frac{1}{2}\,\overline{n+1}\,.\,n$.

Fig. 7—2. The number of squares of unity $=1+2+3+4+5+6$

$$\begin{aligned}&=\tfrac{1}{2}\text{ the area of the triangle }+\tfrac{1}{2}\,6\\&=\tfrac{1}{2}\,6\times 6+\tfrac{1}{2}\,6\\&=\tfrac{1}{2}\,n\times n+\tfrac{1}{2}\,n\\&=\tfrac{1}{2}\,\overline{n+1}\,.\,n.\end{aligned}$$

Fig. 7. The sum of the series $1^2+2^2+3^2+4^2$, &c. $=$

$\frac{2}{3}$ axis × ordinate = $\frac{2}{3}$ the circumscribing parallelogram, or $\frac{1}{3}\,\overline{n+1}\,.\,n\,.\,\overline{n+\frac{1}{2}}$.

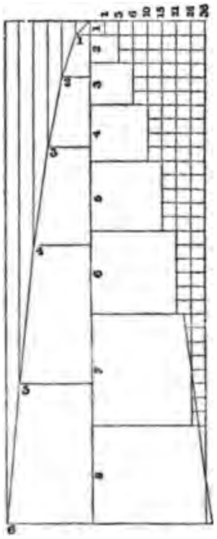


Fig. 7.

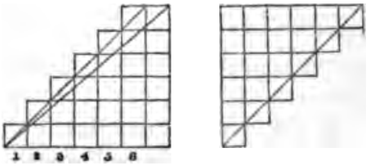


Fig. 7—2.

For axis = sum of the series 1 + 2 + 3 + 4, &c. = $\frac{1}{2}\,\overline{n+1}\,.\,n$; but here the ordinate = $\frac{1}{2}$ of unity more than the number of terms, or side of the last square. Or ordinate = $\overline{n+\frac{1}{2}}$

Sum of the series = $\frac{2}{3}$ axis × ordinate

$$= \tfrac{2}{3} \text{ of } \tfrac{1}{2}\,\overline{n+1}\,.\,n\,.\,\overline{n+\tfrac{1}{2}}$$
$$= \tfrac{1}{3}\,\overline{n+1}\,.\,n\,.\,\overline{n+\tfrac{1}{2}}$$

The circumscribing parallelogram will = $\frac{1}{2}\,\overline{n+1} \times n \times \overline{n+\frac{1}{2}}$ = axis × ordinate.

Also by construction the sum of the areas limited by the ordinates will equal the sum of the corresponding squares = $\frac{1}{3}\,\overline{n+1}\,.\,n\,.\,\overline{n+\frac{1}{2}}$.

For the straight line joining the two ordinates $8\frac{1}{2}$ and $7\frac{1}{2}$ cuts off a triangle from the square of 8 = the triangle added to the same square; consequently the area contained by this straight line, the sectional axis 8, and the two ordinates will = the square of 8.

These series of areas would form an obeliscal area = the sum of the corresponding squares.

Fig. 7. The ordinate of the series of squares $= n+\frac{1}{2}$, the square of which $= \overline{n+1} \,.\, n+\frac{1}{4}$ = twice the axis of the squares $+ \frac{1}{4}$.

In order to construct a parabolic area, the axis should vary as the square of the ordinate. If $n+\frac{1}{2}$, the ordinate of the series of squares, be made the ordinate of a parabolic area, the corresponding axis should $= \frac{1}{2}(\overline{n+1} \,.\, n+\frac{1}{4}) = \frac{1}{2}\,\overline{n+1} \,.\, n+\frac{1}{8}$ or $= \frac{1}{2}$ ordinate^2 of the parabolic area = the axis of the squares $+ \frac{1}{8}$.

Hence the parabolic area will have an axis greater than the series of squares by $\frac{1}{8}$ unity; or equal $\frac{1}{2}$ ordinate^2 $= \frac{1}{2}\,\overline{n+\frac{1}{2}}^{\,2}$.

This parabolic area will $= \frac{2}{3}$ axis × ordinate

$$= \tfrac{2}{3} \text{ of } \tfrac{1}{2} \text{ ordinate}^2 \times \text{ordinate}$$
$$= \tfrac{1}{3}\,\overline{\text{ordinate}}^{\,3} = \tfrac{1}{3}\,\overline{n+\tfrac{1}{2}}^{\,3}$$

The apex of the parabola will be in the produced axis of the squares at the distance of $\frac{1}{8}$ above the first square. The n^{th} ordinate of the squares, which $= n+\frac{1}{2}$, will be common to both areas; but the parabolic area being curvilinear, the ordinate will continually vary as $\overline{\text{axis}}^{\frac{1}{2}}$ from the apex to the n^{th} ordinate, which parabolic area so generated will be to the corresponding series of n squares,

as $\frac{1}{3}\,\overline{n+\frac{1}{2}}^{\,3} : \frac{1}{3}\,\overline{n+1} \,.\, n \,.\, \overline{n+\frac{1}{2}}$
or $\frac{1}{3}\,(n^3+1\frac{1}{2}\,n^2+\frac{3}{4}\,n+\frac{1}{8}) : \frac{1}{3}(n^3+1\frac{1}{2}\,n^2+\frac{1}{2}\,n)$

$$\text{Difference} = \tfrac{1}{3}\,(\tfrac{1}{4}\,n+\tfrac{1}{8})$$
$$= \tfrac{1}{12}\,n+\tfrac{1}{24} \,.\, 1$$

Fig. 7. The difference between the two areas at the 8th ordinate, which are as 204·708 : 204,

$$\text{will} = \cdot 708, \text{ or } = \tfrac{17}{24} = \tfrac{8}{12}+\tfrac{1}{24}$$
$$\text{and } \tfrac{1}{12}n+\tfrac{1}{24} = \tfrac{8}{12}+\tfrac{1}{24}.$$

When $n=24$ the two areas are as 4902·041 : 4900.

Difference $=2{\cdot}041 = 2\frac{1}{24}$
and $\frac{1}{12}n+\frac{1}{24}=2\frac{1}{24}$.

$\frac{1}{12}n+\frac{1}{24}=\frac{1}{12}$ of n squares of unity $+\frac{1}{24}$ of 1 square of unity.

Fig. 7. The parabolic area corresponding to the series of squares has the apex $\frac{1}{8}$ 1, above the single obeliscal or parabolic area on the other side of the axis. In order to compare the two parabolic areas having a common axis, let the two apices coincide. The parabolic area corresponding to the obeliscal area, will be to the parabolic area corresponding to the series of squares,

$$\begin{array}{lcl} \text{as } \frac{2}{3}\text{ axis} \times \text{ordinate } 6 & : & \frac{2}{3}\text{ axis} \times \text{ ordinate } \surd 72, \\ \frac{2}{3}\text{ axis} \times \overline{\text{axis}}^{\frac{1}{2}} & : & \frac{2}{3}\text{ axis} \times 2\,\overline{\text{axis}}^{\frac{1}{2}} \\ 1 & : & 2^{\frac{1}{2}} \end{array}$$

or as side to diagonal of a square. Hence the first double parabolic area will be to the parabolic area of the squares, as

$$2 : 2^{\frac{1}{2}}$$
$$2^{\frac{1}{2}} : 1$$

or as diagonal to side of a square.

The difference between the 1st parabolic area and the 1st square, or the difference between the two areas to the 1st ordinate, will $= \frac{1}{12}n + \frac{1}{24}$

$$= \frac{1}{12} + \frac{1}{24} = \frac{1}{8}.$$

The difference between the two areas to the 2nd ordinate will

$$= \frac{1}{12}n + \frac{1}{24} = \frac{2}{12} + \frac{1}{24}$$

from which take $\frac{1}{12} + \frac{1}{24}$, the 1st difference, and $\frac{1}{12}$ will $=$ the difference to be added to the 2nd square to equal the corresponding parabolic sectional area.

So the difference between every two corresponding sectional areas in succession will $= \frac{1}{12}$ unity.

As n increases, the area of the series of squares $\frac{1}{3}\overline{n+1}\,.\,n\,.\,\overline{n+\frac{1}{2}}$, will continually approach to equality with $\frac{1}{3}\overline{n+\frac{1}{2}}^{3}$, the corresponding parabolic area; though their difference $\frac{1}{12}n + \frac{1}{24}$ will continually increase.

Also, whatever be the increase of n, the last, or n^{th} square will be less by $\frac{1}{12}$ unity than the corresponding parabolic sectional area.

The parabolic area may also be represented in terms of the axis. For parabolic area = $\frac{2}{3}$ axis × ordinate,

$$= \tfrac{2}{3}\text{ axis} \times \overline{2\text{ axis}}^{\frac{1}{2}},$$
$$= \tfrac{1}{3}\, 2\text{ axis} \times \overline{2\text{ axis}}^{\frac{1}{2}}$$
$$= \tfrac{1}{3}\, \overline{2\text{ axis}}^{\frac{3}{2}}.$$

The series of squares may be so arranged that the axis shall divide the series into two equal parts.

Fig. 8. The sum of the series $2^2 + 4^2 + 6^2$, &c., will = $\frac{2}{3}\overline{n+1} \cdot n \cdot \overline{2n-1}$, or = $\frac{4}{3}\overline{n+1} \cdot n \cdot \overline{n+\frac{1}{2}}$.

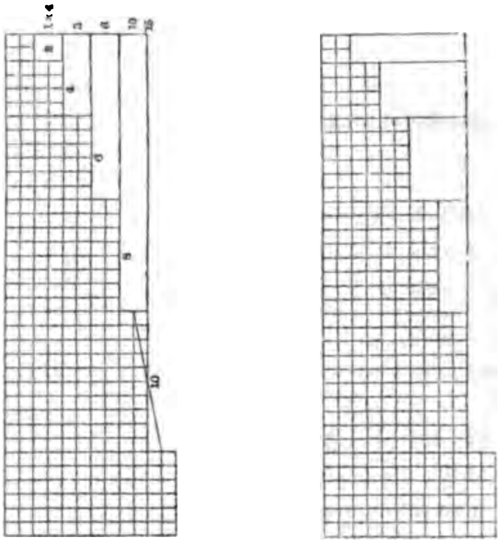


Fig. 8.

Since the sum of $1+2+3$, &c. = $\frac{1}{2}\overline{n+1} \cdot n$,

∴ the sum of $2+4+6$, &c. = $\overline{n+1} \cdot n$,

= the axis of the series $2^2+4^2+6^2$, &c., and the ordinate will $= 2n+1$, or $2 \cdot \overline{n+\frac{1}{2}}$

$\frac{2}{3}$ axis × ordinate, or $\frac{2}{3}\overline{n+1} \cdot n \cdot 2\,\overline{n+\frac{1}{2}}$,

or $\frac{4}{3}\overline{n+1} \cdot n \cdot \overline{n+\frac{1}{2}}$,

will = the sum of the series $2^2+4^2+6^2$.

Thus the axis and ordinate of n terms of the series $2^2 + 4^2 + 6^2$ will be double the axis and ordinate of n terms of the series $1^2 + 2^2 + 3^2$, and their areas will be as their rectangles, or as $4 : 1$.

The parabolic area corresponding to the series of squares $2^2 + 4^2 + 6^2$ will have an axis = the axis of the squares + $\frac{1}{4}$, in order that the parabolic axis may vary as $\overline{\text{ordinate}}^2$ of the squares, or vary as $(2 \,.\, \overline{n + \frac{1}{2}})^2$

$$\begin{aligned}
\text{axis of squares} &= \overline{n+1} \,.\, n \\
\text{ordinate} &= 2\, \overline{n + \tfrac{1}{2}} \\
\overline{\text{ordinate}}^2 &= 4\,(n + \tfrac{1}{2})^2 \\
&= 4\,(\overline{n+1} \,.\, n + \tfrac{1}{4}) \\
\tfrac{1}{4}\ \text{ordinate}^2 &= \overline{n+1} \,.\, n + \tfrac{1}{4} \\
&= \text{axis of squares} + \tfrac{1}{4} \\
&= \text{axis of parabolic area.}
\end{aligned}$$

Hence parabolic area will $= \frac{2}{3}$ axis × ordinate

$$\begin{aligned}
&= \tfrac{2}{3} \text{ of } \tfrac{1}{4}\ \text{ordinate}^2 \times \text{ordinate} \\
&= \tfrac{1}{6}\ \text{ordinate}^3 = \tfrac{1}{6}(2 \,.\, \overline{n + \tfrac{1}{2}})^3
\end{aligned}$$

or $\frac{2}{3}$ axis × ordinate

$$\begin{aligned}
&= \tfrac{2}{3}\ \text{axis} \times \overline{4\ \text{axis}}^{\frac{1}{2}} \\
&= \tfrac{2}{3}\ \text{axis} \times 2 \,.\, \overline{\text{axis}}^{\frac{1}{2}} \\
&= \tfrac{4}{3}\ \overline{\text{axis}}^{\frac{3}{2}} .
\end{aligned}$$

The parabolic area will be to the corresponding series of n squares

$$\begin{aligned}
&\text{as } \tfrac{1}{6}\,(2 \,.\, \overline{n + \tfrac{1}{2})}^3 : \tfrac{4}{3}\, \overline{n+1} \,.\, n \,.\, \overline{n + \tfrac{1}{2}} \\
&\text{as } \tfrac{8}{6} \text{ or } \tfrac{4}{3} \,.\, \overline{n + \tfrac{1}{2}}^3 : \tfrac{4}{3}\, \overline{n+1} \,.\, n \,.\, \overline{n + \tfrac{1}{2}} \\
&\qquad \text{The difference} = \tfrac{4}{3}\,(\tfrac{1}{4} n + \tfrac{1}{8}) \\
&\qquad\qquad = \tfrac{4}{12} n + \tfrac{4}{24} \\
&\qquad\qquad = \tfrac{1}{3} n + \tfrac{1}{6}.
\end{aligned}$$

Fig. 4, 5. The sum of the series $1^2 + 3^2 + 5^2 + 7^2$, will $= \frac{4}{3} n^3 - \frac{1}{3} n$.

It has been shown that the single obeliscal area $= \frac{1}{2} (1^2 + 3^2 + 5^2 + 7^2) = \frac{2}{3} n^3 - \frac{1}{6} n.$, (*fig.* 3.); consequently the double obeliscal area, or the sum of $1^2 + 3^2 + 5^2 + 7^2$, will $= \frac{4}{3} n^3 - \frac{1}{3} n$.

The single parabolic area $= \frac{2}{3} n^3$

$\therefore$ the double parabolic area will $= \frac{4}{3} n^3$.

The single parabolic area exceeds the single obeliscal area by $\frac{1}{6}$ unity in each corresponding sectional area.

$\therefore$ the double parabolic area will exceed the double obeliscal area $\frac{1}{3}$ unity in each sectional area.

$$\begin{array}{ll}
\text{1st } S. & 1+4+9+16+25+36=91 \\
\text{2nd } S. & \phantom{1+{}}4\phantom{{}+9}+16\phantom{{}+25}+36=56 \\
\hline
\text{3rd } S. & 1\phantom{{}+4}+9\phantom{{}+16}+25\phantom{{}+36}=35
\end{array}$$

Sum of the 1st series to n terms

$$\begin{aligned}
&= \tfrac{1}{3}\,\overline{n+1}\,.\,n\,.\,\overline{n+\tfrac{1}{2}} \\
&= 91 \text{ when } n = 6.
\end{aligned}$$

Sum of $\frac{1}{2}n$ terms of the 2nd series $=$ 4 times the sum of $\frac{1}{2}n$ terms of the 1st series;

$$\begin{array}{l}
\text{as } 1+4+9=14 \\
\text{and } 4+16+36=56=4\times 14;
\end{array}$$

or n terms of the 2nd series $=$ 4 times n terms of the 1st series, and n terms of the 3rd series $= \frac{4}{3}n^3-\frac{1}{3}n$.

Hence sum of

$$\begin{aligned}
n \text{ terms of 1st series} &= \tfrac{1}{3}\,\overline{n+1}\,.\,n\,.\,\overline{n+\tfrac{1}{2}} \\
n \text{ terms of 2nd series} &= \tfrac{4}{3}\,\overline{n+1}\,.\,n\,.\,\overline{n+\tfrac{1}{2}} \\
n \text{ terms of 3rd series} &= \tfrac{4}{3}n^3-\tfrac{1}{3}n.
\end{aligned}$$

$$\begin{array}{lcl}
\text{1st series} &=& 1^2+2^2+3^2+4^2+5^2+6^2 = 91 \\
\text{2nd series} &=& 2^2+4^2+6^2+8^2+10^2+12^2 = 364 \\
\text{3rd series} &=& 1^2+3^2+5^2+7^2+9^2+11^2 = 286
\end{array}$$

$$\text{when } n = 6.$$

The difference between the 2nd and 3rd series will equal

$$3+7+11+15+19+23=78,$$

$$\text{or } 2n^2+n=S.$$

$$\begin{array}{ll}
\text{1st } S. = & 1^2+2^2+3^2+4^2+5^2+6^2=91 \\
\text{2nd } S. = & \phantom{1^2+{}}2^2\phantom{{}+3^2}+4^2\phantom{{}+5^2}+6^2=56 \\
\hline
\text{3rd } S. = & 1^2\phantom{{}+2^2}+3^2\phantom{{}+4^2}+5^2\phantom{{}+6^2}=35
\end{array}$$

$$\begin{array}{lcl}
S.\ \text{1st} &=& \tfrac{1}{3}\,.\,\overline{n+1}\,.\,n\,.\,\overline{n+\tfrac{1}{2}}=91 \text{ when } n = 6 \\
S.\ \text{2nd} &=& \tfrac{4}{3}\,\overline{n+1}\,.\,n\,.\,\overline{n+\tfrac{1}{2}}=56 \text{ when } n = 3 \\
S.\ \text{3rd} &=& \text{difference} = \overline{35} \\
&=& \tfrac{4}{3}n^3-\tfrac{1}{3}n=35 \text{ when } n = 3.
\end{array}$$

Sum of $1^2+2^2+3^2 = 14$

and $4 \times 14=56 =$ sum of 2nd series.

Hence the sum of the 2nd series = 4 times the sum of $\frac{1}{2}n$ terms of the 1st series $= 4 \times 14=56$.

The difference between the two series = the sum of $\frac{1}{2}n$ terms of the 3rd series.

To sum the series $1+3+5$, &c.

$$\begin{array}{r} \text{sum of } 1+2+3+4+5+6=21 \\ 2\ +\ 4\ +\ 6=12 \\ \hline 1\ +\ 3\ +\ 5\ \ =\ 9 \end{array}$$

$1+2+3=6$, and $2 \times 6=12 =$ sum of second series, which subtract from the first series $= 21-12 = 9 =$ sum of 3rd series.

Or, *S.* of $\frac{1}{2}n$ terms of the 1st series × by 2 = *S.* of $\frac{1}{2}n$ terms of the 2nd series, which subtracted from n terms of the 1st series = *S.* of $\frac{1}{2}n$ terms of the 3rd series.

The Formation of Increasing Series from a Series in which all the Terms are equal, excepting the first.

By reversing the order of the three series, the least will be placed the first, from which the other two increasing series will be formed thus:—

	1,	2,	2,	2,	2,	2	Sum $=2n-1$.
and forms	1,	3,	5,	7,	9,	11	$=n^2$
and forms	1,	4,	9,	16,	25,	36	$=\frac{1}{3}\overline{n+1}\,.\,n\,.\,\overline{n+\frac{1}{2}}$
and forms	1,	5,	14,	30,	55,	91	

The first series represents the incremental distances described in each second more than was described in the preceding second.

The second series represents the distances described in each of the n seconds. So that the distance described in the n^{th} second will = the sum of the incremental distances described during n seconds.

The third series represents the whole distances described during the several descents from the apex to the different ordinates; as the whole distance described during n seconds from the apex to the n^{th} ordinate will = the sum of the distances described during each of the n seconds.

The formation of these series may be further illustrated by the triangle, *fig.* 7–2., where the first horizontal line

$$\begin{aligned}
&=1+2+2+2+2+2\\
&=6 \text{ times } 2 \text{ less } 1=2\times 6-1=11\\
&=n \text{ times } 2 \text{ less } 1=2n-1.
\end{aligned}$$

Again, $2n-1$ forms the columnar series 1, 3, 5, 7, 9, 11, the sum of which series = the area of the triangle when each square = 2, and each $\frac{1}{2}$ square = 1.

$$\begin{array}{lrl}
\text{1st series,} & 1+2+2+2+2+2 & \text{Sum}=2n-1.\\
 & 1+2+2+2+2 & \\
 & 1+2+2+2 & \\
 & 1+2+2 & \\
 & 1+2 & \\
 & 1 & \\ \cline{2-2}
\text{2nd, formed from } 2n-1= & 1+3+5+7+9+11. & \text{Sum}=n^2.
\end{array}$$

The area of the triangle $=1+2+3+4+5+6$ squares, each = 2 in area, less 6 half squares,

$$\begin{aligned}
&=\tfrac{1}{2}\,\overline{n+1}\,.\,n \text{ less } \tfrac{1}{2}n\\
&=\tfrac{1}{2}\,7\times 6-\tfrac{1}{2}n\\
&=\quad 21\ -3=18 \text{ squares.}
\end{aligned}$$

Or the triangle will contain 36 half squares = sum of
$1+3+5+7+9+11=n^2$

$$\begin{array}{lrl}
\text{Next,} & 1+3+\ 5+\ 7+\ 9+11 & \\
 & 1+\ 3+\ 5+\ 7+\ 9 & \\
 & 1+\ 3+\ 5+\ 7 & \\
 & 1+\ 3+\ 5 & \\
 & 1+\ 3 & \\
 & 1 & \\ \cline{2-2}
 & 1+4+\ 9+16+25+36 & \text{Sum}=\tfrac{1}{3}\,\overline{n+1}\,.\,n\,.\,\overline{n+\tfrac{1}{2}}.\\
 & 1+\ 4+\ 9+16+25 & \\
 & 1+\ 4+\ 9+16 & \\
 & 1+\ 4+\ 9 & \\
 & 1+\ 4 & \\
 & 1 & \\ \cline{2-2}
 & 1+5+14+30+55+91, & \text{which is formed from}\\
 & & \tfrac{1}{3}\overline{n+1}\,.\,n\,.\,\overline{n+\tfrac{1}{2}}.
\end{array}$$

The series $1+4+9$, or $1^2+2^2+3^2$ is represented by the complementary area of the obeliscal series, *fig.* 7.*a.*

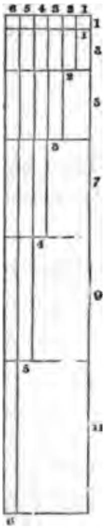


Fig. 7.*a.*

These series and others may be formed from the column of units, and line of twos, by adding together two numbers in a diagonal line to form a third; the third with its diagonal number will form a fourth, and so the numbers may be increased to any extent.

As $2+1=3$ $\quad 2+3=5$
$3+1=4$ $\quad 5+4=9$
$4+1=5$ $\quad 9+5=14$

1					
1,	2,	2,	2,	2,	2
1,	3,	5,	7,	9,	11
1,	4,	9,	16,	25,	36
1,	5,	14,	30,	55,	91
1,	6,	20,	50,	105,	196
1,	7,	27,	77,	182,	378
1,	8,	35,	112,	294,	672
1,	9,	44,	156,	450,	1122.

The sum of any line of numbers = the number below the last term = the sum of the preceding column:

as the line $= 1+5+14 = 20$,
and the column $= 2+3+4+5+6 = 20$.

The two series cross each other at 5, the last term but one in both series.

The last terms of the two series together $= 14+6 = 20$, the sum of either series.

Again, 36, the axis corresponding to the 6th ordinate of the obeliscal area, or distance described in 6 seconds, = the sum of 6 sectional areas, or = the line of series $1+3+5+7+9+11=36$.

25 is the distance described in 5 seconds,
9 " " in the 5th,
and $9+2=11$ " " " 6th.

The distance described in 6 seconds $=25+11=36=$ the column $2+9+25$.

Thus the line of series = the column of series = the sum of the last terms of both series = the sum of either series.

Fig. 7. The sum of the series of cubes of 1, 2, 3, 4, 5, 6, 7, 8 $=(\frac{1}{2}\overline{n+1}\,.\,n)^2=\overline{\text{axis}}^2$.

For $1+2+3+4+5+6+7+8=\frac{1}{2}\overline{n+1}\,.\,n=$ axis $=$ 36,

and $1^3+2^3+3^3+4^3+5^3+6^3+7^3+8^3=(\frac{1}{2}\overline{n+1}\,.\,n)^2=\overline{\text{axis}}^2=1296$,

as before call the ordinate $n+\frac{1}{2}$,
then the last or 8th ordinate $=8{\cdot}5$,

axis $\times\overline{\text{ordinate}}^2=36\times\overline{8{\cdot}5}^2$,

$\frac{1}{2}$ axis $\times\overline{\text{ordinate}}^2=\frac{1}{2}36\times72{\cdot}25=1300{\cdot}5$,
but the series of cubes $=1296$
difference $=4{\cdot}5$
$=\frac{36}{8}$

The first ordinate will $=1{\cdot}5$,
then $\frac{1}{2}$ axis $\times\overline{\text{ordinate}}^2=\frac{1}{2}1\times\overline{1{\cdot}5}^2=1{\cdot}125$,
1st cube $=1$
difference $=\ {\cdot}125=\frac{1}{8}$.

The 2nd ordinate $=2{\cdot}5$,
$\frac{1}{2}$ axis $\times\overline{\text{ordinate}}^2=\frac{1}{2}3\times\overline{2{\cdot}5}^2=9{\cdot}375$,
The 2 cubes $=1^3$ and $2^3=9$
difference $=\ {\cdot}375=\frac{3}{8}$.

So $\frac{1}{2}$ axis $\times\overline{\text{ordinate}}^2$ exceeds the 1st cube by $\frac{1}{8}$ cube of 1,
the 1st and 2nd cubes, or 1^3+2^3 " $\frac{3}{8}$,
$1^3+2^3+3^3$ " $\frac{6}{8}$,
$1^3+2^3+3^3+4^3$ " $\frac{10}{8}$,
1^3 .... $+8^3$ " $\frac{36}{8}$.

So the sectional solids having $\overline{\text{ordinate}}^2=(n+\frac{1}{2})^2$ exceed

1^3, 2^3, 3^3, 4^3, 5^3, 6^3, 7^3, 8^3
by $\frac{1}{8}$, $\frac{3}{8}$, $\frac{6}{8}$, $\frac{10}{8}$, $\frac{15}{8}$, $\frac{21}{8}$, $\frac{28}{8}$, $\frac{36}{8}$
less $\frac{1}{8}$, $\frac{3}{8}$, $\frac{6}{8}$, $\frac{10}{8}$, $\frac{15}{8}$, $\frac{21}{8}$, $\frac{28}{8}$
or $\frac{1}{8}$, $\frac{2}{8}$, $\frac{3}{8}$, $\frac{4}{8}$, $\frac{5}{8}$, $\frac{6}{8}$, $\frac{7}{8}$, $\frac{8}{8}$,

the sum of which $=\frac{36}{8}$, or $4\frac{1}{2}$ cubes of unity for the series of 8 cubes $=\frac{1}{8}$ axis.

Or sum of the 8 sectional solids $= \frac{1}{2}$ axis × ordinate^2.
" " $= \frac{1}{2}$ axis × $8{\cdot}5^2$.
" " $= \frac{1}{2}$ axis × $72{\cdot}25$.
" " $= (\frac{1}{2}$ axis × 2 axis$) + \frac{1}{8}$ axis.
" " $=$ axis$^2 + \frac{1}{8}$ axis.

Instead of taking the $\overline{\text{ordinate}}^2 = (n+\frac{1}{2})^2$,

$$\begin{aligned}
\text{let the } \overline{\text{ordinate}}^{\,2} &= (n+\tfrac{1}{2})^2 - \tfrac{1}{4} \\
&= n^2 + n + \tfrac{1}{4} - \tfrac{1}{4} \\
&= n^2 + n = \overline{n+1} \,.\, n \\
\text{axis} \times \overline{\text{ordinate}}^{\,2} &= \tfrac{1}{2}\, \overline{n+1} \,.\, n \times \overline{n+1} \,.\, n \\
&= \tfrac{1}{2}\, (n+1 \,.\, n)^2 \\
\tfrac{1}{2}\, \text{axis} \times \overline{\text{ordinate}}^{\,2} &= \tfrac{1}{4}\, \overline{(n+1} \,.\, n)^2 \\
\text{content} &= (\tfrac{1}{2} \overline{n+1} \,.\, n)^2 = \overline{\text{axis}}^{\,2}
\end{aligned}$$

when $n = 8 = (\frac{1}{2}\, 9 \times 8)^2 = 36^2 = 1296$ = the content of the 8 cubes.

Thus the series of n cubes of 1, 2, 3 &c., will $= (\frac{1}{2}\, \overline{n+1} \,.\, n)^2$ $= \overline{\text{axis}}^{\,2}$ = as many cubes of 1 as the $\overline{\text{axis}}^{\,2}$ contains squares of 1.

Since the ordinate^2 ∝ axis, or ordinate ∝ axis$^{\frac{1}{2}}$, the solid will be of the parabolic form, and the content = the sum of the series of cubes, both having equal axes.

Fig. 8. Sum $2^3 + 4^3 + 6^3 + 8^3 + 10^3 = 1800$

$$\begin{aligned}
\text{axis} &= 2 \times \tfrac{1}{2}\, \overline{n+1} \,.\, n = 2 \times \text{axis } 1+2+3 \\
&= 2+4+6 \text{ \&c. } = \overline{n+1} \,.\, n.
\end{aligned}$$

Let ordinate $= 2n+1$

$$\text{axis} \times \text{ordinate}^2 = \overline{n+1} \,.\, n \,.\, (2n+1)^2.$$

Here the sectional solids having ordinate$^2 = (2n+1)^2$ will exceed

$$\begin{array}{rrrrrr}
 & 2^3, & 4^3, & 6^3, & 8^3, & 10^3 \\
\text{by} & 1, & 3, & 6, & 10, & 15 \\
\text{less} & & 1, & 3, & 6, & 10 \\
\hline
\text{or} & 1, & 2, & 3, & 4, & 5,
\end{array}$$

the sum $= \frac{1}{2}\overline{n+1} \,.\, n = 15$, or 15 cubes of 1 for the series of 5 cubes, or 36 cubes of 1 for the series of 8 cubes, which $= \frac{1}{2}$ axis.

$$\begin{aligned}
\text{Let the ordinate}^2 &= (2n+1)^2-1\\
&= 4n^2+4n+1-1\\
&= 4(n^2+n)\\
&= 4(\overline{n+1}\,.\,n)\\
\text{axis} \times \text{ordinate}^2 &= \overline{n+1}\,.\,n\,.\,4(\overline{n+1}\,.\,n)\\
&= 4(\overline{n+1}\,.\,n)^2\\
\tfrac{1}{2}\,\text{axis} \times \text{ordinate}^2 &= 2(\overline{n+1}\,.\,n)^2\\
\text{content} &= 2\ \text{axis}^2 = 2\times(6\times5)^2 = 1800,\\
\text{when } n &= 5.
\end{aligned}$$

Here the ordinate $\propto \overline{\text{axis}}^{\frac{1}{2}}$, so the solid will be parabolic, and the content = the series of cubes, both having equal axes.

The content of n terms of $2^3+4^3+6^3=8$ times that of n terms of $1^3+2^3+3^3$. Thus the series of n cubes of 2, 4, 6 will $=2(\overline{n+1}\,.\,n)^2=2\times\overline{\text{axis}}^2$.

Figs. 4, 5. Sum $1^3+3^3+5^3+7^3+9^3+11^3=2556$

$\text{axis} = 1+3+5 \text{ \&c.} = n^2.$

Let ordinate $=2n$, then ordinate will $\propto \overline{\text{axis}}^{\frac{1}{2}}$, and a parabolic solid will be generated by the ordinate^2, or $\overline{2n}^2$.

$$\begin{aligned}
\tfrac{1}{2}\,\text{axis}\times\overline{\text{ordinate}}^2 &= \tfrac{1}{2}n^2\times\overline{2n}^2\\
\text{when } n=1 &= \tfrac{1}{2}1^2\times\overline{2\times1}^2=2,\ \text{difference}=2-1=1\\
n=6 &= \tfrac{1}{2}6^2\times\overline{2\times6}^2=2592\\
6\ \text{cubes} &= \underline{2556}\\
\text{difference} &= \quad 36
\end{aligned}$$

When $n=1$ difference $=\ \ 2-\ \ 1=1$
$n=2$ „ $=\ 32-\ 28=4$
$n=3$ „ $=162-153=9$
$\vdots \qquad\qquad \vdots \quad \vdots$
$n=6$ „ $=2592-2556=36.$

The parabolic sectional solids when ordinate$^2=\overline{2n}^2$ will exceed

	1^3,	3^3,	5^3,	7^3,	9^3,	11^3
by	1,	4,	9,	16,	25,	36
less		1,	4,	9,	16,	25
or	1,	3,	5,	7,	9,	11

Sum $=n^2=6^2=36$, or 36 cubes of 1 for the series of 6 cubes $=$ axis.

Let ordinate $^2=(2n)^2-2$

$$\begin{aligned}\tfrac{1}{2}\text{ axis}\times\text{ ordinate}^2&=\tfrac{1}{2}n^2\times(\overline{2n}^2-2)=2n^4-n^2\\ \text{when } n=1&=\tfrac{1}{2}1^2\times(2^2-2)=1\\ n=2&=\tfrac{1}{2}2^2\times(4^2-2)=28\\ \vdots\quad\vdots&\quad\vdots\\ n=6&=\tfrac{1}{2}6^2\times(12^2-2)=2556,\end{aligned}$$

or sum of 6 terms of $1^3+3^3+5^3$ &c. 2556.

Thus the parabolic solid $=$ sum of the series of cubes $+$ axis, both having a common axis,

$$\begin{aligned}&=\tfrac{1}{2}n^2\times(\overline{2n}^2-2)+n^2\\ &=2n^4-n^2+n^2=2n^4=2\text{ axis}^2;\end{aligned}$$

therefore sum of $1^3+3^3+5^3$ &c. $=2$ axis $^2-$ axis $=2n^4-n^2$.

The axis of the series of 8 cubes of 1, 2, 3, 4, 5, 6, 7, 8 (*fig.* 7.)$=\tfrac{1}{2}\overline{n+1}.n=36$, and the content of the series $=$axis^2.

The axis of the series of 6 cubes of 1, 3, 5, 7, 9, 11, (*figs.* 4, 5.)$=n^2=36$, and the corresponding parabolic solid, having the same axis, $=2$ axis ^{2}.

$\therefore$ the series of 8 cubes of *fig.* 7. $=\tfrac{1}{2}$ the content of the parabolic solid corresponding to the series of 6 cubes of *figs.* 4, 5.

$$\begin{array}{lllllll} & 1^3, & 2^3, & 3^3, & 4^3, & 5^3, & 6^3\\ = & 1, & 8, & 27, & 64, & 125, & 216\\ & 1, & 9, & 36, & 100, & 225, & 441=1^3,\ 1^3+2^3,\ 1^3+2^3+3^3\\ = & 1^2, & 3^2, & 6^2, & 10^2, & 15^2, & 21^2.\end{array}$$

The series 1, 3, 6 is formed of 1, $1+2$, $1+2+3$. The n^{th} term of $1+2+3$ &c. $=\tfrac{1}{2}\overline{n+1}\,.\,n=\tfrac{1}{2}(n^2+n)$. Thus the sum of n terms of the cubes of 1, 2, 3, &c., $=$ the square of the sum of n terms of $1+2+3$, &c., $=(\tfrac{1}{2}\overline{n+1}\,.\,n)^2=(\tfrac{1}{2}\times\overline{n^2+n})^2$.

$$\begin{aligned}\text{Let } n=6,\ s&=(\tfrac{1}{2}\times\overline{n^2+n})^2=(\tfrac{1}{2}\times 6^2+6)^2=21^2\\ &=441\text{ cubes of unity}\end{aligned}$$

$=$ a stratum of the depth of unity and area $=21^2=441$.

If the squares 1, 2, 3, 4, 5, 6 represent the 6 cubes, then a square stratum having the side $=$ the sum of the ordinates

$1+2+3+4+5+6=21$, will = the sum of 6 cubes $=(\frac{1}{2}$ of $\overline{6^2+6})^2=21^2=441$ cubes of unity.

If the sum of the cubes were $1^3+2^3+3^3=36$, then a square stratum having the side = 6 would contain 36 cubes of unity, the sum of the cubes of 1, 2, 3; or the square of the sum of the sides of the cubes of 1, 2, 3 will = the sum of the cubes of 1, 2, 3. For $1^3+2^3+3^3=36=6\times 6$ cubes of unity; if 6 cubes of unity be placed along the side of the squares of 6, then the square would contain 6 times 6 units, or 6 columns of 6 units each.

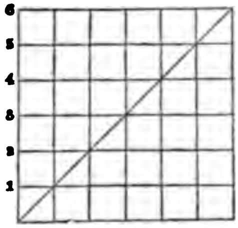


Fig. 7. *b.*

Or the number of cubes of unity in $1^3+2^3+3^3$ = the number of square units in $(1+2+3)$ squared $= 6\times 6=6$ times 6 columns of single squares = one column of 36 squares = the length of 36 linear units.

Thus 36 cubits of unity placed side by side in a straight line will extend to the same distance as 36 squares of unity placed in a straight line, equal to a straight line of 36 linear units.

Thus the length of a side of a cube of unity = the length of a side of a square of unity = the length of a linear unit.

So that, in measuring distances, a cube of unity, a square of unity, and a linear unit are all equal in length.

Fig. 7. Sum the obeliscal series of

$$1^3+2^3+3^3+4^3+5^3+6^3+7^3+8^3,$$

$$\text{axis} = 1+2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots + 8$$

$$=\tfrac{1}{2}\overline{n+1}\,.\,n=\tfrac{1}{2}\overline{n^2+n}=\tfrac{1}{2}\,\overline{64+8}=36,$$

$\overline{\text{ordinate}}^2$ at the end of the 8th cube,

$$=\overline{8{\cdot}5}^2-{\cdot}25$$

$$=72{\cdot}25-{\cdot}25=72$$

$$\tfrac{1}{2}\,\text{axis}\times\overline{\text{ordinate}}^2=\tfrac{1}{2}\,36\times 72=1296.$$

But the sum of the series of 8 cubes = the square of the sum of the sides of the 8 cubes

$=(\frac{1}{2}\overline{n^2+n})^2$

$=(\frac{1}{2}\overline{8^2+8})^2=36^2=1296$

which $=\overline{\text{axis}}^2$ of the obeliscal series

$=\frac{1}{2}\text{axis}\times\overline{\text{ordinate}}^2$.

In this obeliscal series of cubes the axis $=\frac{1}{2}\overline{\text{ordinate}}^2$, or ordinate$^2=2$ axis.

The $\overline{\text{ordinate}}^2$ at the end of every cube of the series will = the square of (the side of the cube + ·5) less ·25; which will = 2 axis.

Or the axis being known, the $\overline{\text{ordinate}}^2$ will = 2 axis.

Let a straight line = 6 linear units. Then 6 × 1 will form a rectangled parallelogram = 6 square units, and 6 rectangled parallelograms will form a square

= 6 × 6 = 36 square units, the square of 6.

The square of 6 × by 1 will form a square stratum = 6 × 6 = 36 cubes of unity, and 6 square strata will = 6 × 36 = 216 cubes of unity

= 6 × 6 × 6 = the cube of 6.

Otherwise. Let a straight line = a linear unit = 1.
Then 6 × 1 = 6, a line of 6 units in length.
Next 1 × 1 = square of 1,
6 × square of 1 = rectangled parallelogram of 6 squares of unity,
6 × rectangled parallelogram = 6 × 6 = 36 square units,
= the square of 6.

Thirdly. 1 × 1 × 1 = cube of 1,
6 × cube of 1 = parallelopipedon of 6 cubes of unity,
6 × parallelopipedon = 6 × 6 = square stratum of 36 cubes of unity,
6 × stratum = 6 × 6 × 6 = 216 cubes of unity,
= the cube of 6.

Thus 1 = a linear unit,
6 × 1 = a line of 6 units in length,
1 × 1 = square of 1,
6 × 6 = square of 6,
1 × 1 × 1 = cube of 1,
6 × 6 × 6 = cube of 6.

Solid Obelisks and Pyramids.

To compare a Series of Obelisks with a corresponding Series of Pyramids.

Figs. 9, 10. The first section of the obelisk is a pyramid, the 1st in the series of pyramids, and therefore the apices of both will coincide.

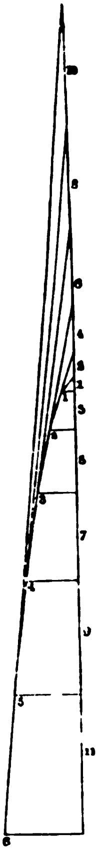


Fig. 9.

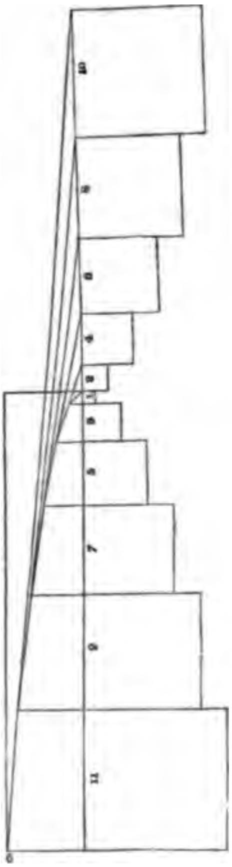


Fig. 10.

All the other sections of the solid obelisk are frustums or frusta of pyramids having their several apices beyond the apex of the obelisk, in the produced axis. Their several bases will = the square ordinates of the obelisk.

The 2nd pyramid has the side of the base = 2, the side of the 2nd square ordinate of the obelisk.

The height or axis of the pyramid is bisected by the 1st ordinate, which = 1 = $\frac{1}{2}$ the second ordinate 2; and 3 is the distance between, or the sectional axis of the 1st and 2nd ordinates,

$$\begin{aligned} \therefore \text{ axis of pyramid will} &= 2 \times 3 = 6, \\ &= \text{ordinate} \times \text{sectional axis}, \\ &= n \text{ times the sectional axis}, \\ &= n \times \overline{2n-1}, \end{aligned}$$

or $= 2n^2 - n =$ twice the axis less the ordinate; or $= 2\,\overline{\text{ordinate}}^2$ less ordinate of obelisk.

Axis of the 2nd pyramid beyond the apex of the obelisk $= 6 - 4 = 2 = n^2 - n = \overline{\text{ordinate}}^2$ less ordinate = axis of obelisk less ordinate.

So the axis of the 1st pyramid will $= 2n^2 - 1 = 2 - 1 = 1$, when $n = 1$.

The axis of the 1st pyramid beyond the apex of the obelisk will $= n^2 - n = 1^2 - 1 = 0$, when $n = 1$.

Hence the axis of the pyramid, having the n^{th} $\overline{\text{ordinate}}^2$ obelisk for the base will $= n . \overline{2n-1} = n$ times the sectional axis of the obelisk, or $= 2n^2 - n =$ twice the axis less the ordinate.

The distance of the apex of the pyramid from the apex of the obelisk will $= n^2 - n =$ axis of obelisk less ordinate.

The whole axis of pyramid = axis of obelisk + produced axis; and produced axis = axis obelisk $- \frac{1}{n}$ axis obelisk,

1st axis = 1, produced axis = $1 - \frac{1}{1} 1 = 0$,
2nd „ = 4, „ = $4 - \frac{1}{2} 4 = 2$,
3rd „ = 9, „ = $9 - \frac{1}{3} 9 = 6$,
4th „ = 16, „ = $16 - \frac{1}{4} 16 = 12$,
5th „ = 25, „ = $25 - \frac{1}{5} 25 = 20$,
6th „ = 36, „ = $36 - \frac{1}{6} 36 = 30$.

Thus series of whole axes will be

1st $= 1 + 0 = 1$, or 1, 6, 15, 28, 45, 66,
2nd $= 4 + 2 = 6$, D. 1, 5, 9, 13, 17, 21,
3rd $= 9 + 6 = 15$, D. 1, 4, 4, 4, 4, 4,
4th $= 16 + 12 = 28$,
5th $= 25 + 20 = 45$,
6th $= 36 + 30 = 66$.

Produced axes will be

0, 2, 6, 12, 20, 30,
D. 0, 2, 4, 6, 8, 10,
D. 0, 2, 2, 2, 2, 2.

Axis of pyramid $= 2$ axis obelisk $- \frac{1}{n}$ axis obelisk,
$= 2$ axis obelisk — ordinate,
$= 2$ axis obelisk — axis$^{\frac{1}{2}}$,
$=$ sectional axis obelisk × ordinate,
$= 2$ ordinate^2 obelisk — ordinate.

The several distances of the apices of 6 pyramids from the apex of the obelisk will be 0, 2, 6, 12, 20, 30.

If from the end of the 6th ordinate a straight line be drawn to a distance from the apex of the obelisk along the produced axis $= n^2 - n = 6^2 - 6 = 30$, that line will represent the side of a triangle or pyramid; and the frustum of that pyramid, between the ordinates 5 and 6, will be the 6th sectional solid of the obelisk.

The axis of a pyramid $= n^2 + n^2 - n = 2n^2 - n$
content $= \frac{1}{3}$ axis × ordinate
$= \frac{1}{3}(2n^2 - n) \times n^2$.

From which take the section having the area of its base $= \overline{n-1}^2$. The axis of this pyramid

$$= \overline{2n-1} \,.\, n - \overline{2n-1}$$
$$= 2n^2 - n - 2n + 1$$
$$= 2n^2 - 3n + 1$$

content $= \frac{1}{3}(2n^2 - 3n + 1) \,.\, \overline{n-1}^2,$

hence the frustum will equal

$$\tfrac{1}{3}(2n^2-n)\,.\,n^2-\tfrac{1}{3}(2n^2-3n+1)\,.\,\overline{n-1}^2$$

or $\tfrac{1}{3}(2n^4-n^3)-\tfrac{1}{3}(2n^4-7n^3+9n-5n-1)$

which $=\tfrac{1}{3}(6n^3-9n^2+5n-1)$, when $n=6$

$=\tfrac{1}{3}(6\times 216-9\times 36+30-1)=333\tfrac{2}{3}$

similarly the 5th frustum $=183$
4th „ $= 86\tfrac{1}{3}$
3rd „ $= 31\tfrac{2}{3}$
2nd „ $= 7$
1st „ $= \tfrac{1}{3}$

content of the obelisk $=642$

The cubes of the sectional axes

	1	3	5	7	9	11	
are	1	27	125	343	729	1331	$=2556$
$\tfrac{1}{4}=$	$\tfrac{1}{4}$	$6\tfrac{3}{4}$	$31\tfrac{1}{4}$	$85\tfrac{3}{4}$	$182\tfrac{1}{4}$	$332\tfrac{3}{4}=$	639
obeliscal series $=$	$\tfrac{1}{3}$	7	$31\tfrac{2}{3}$	$86\tfrac{1}{3}$	183	$333\tfrac{2}{3}=$	642
difference	$=\tfrac{1}{12}$	$\tfrac{3}{12}$	$\tfrac{5}{12}$	$\tfrac{7}{12}$	$\tfrac{9}{12}$	$\tfrac{11}{12}=\tfrac{n^2}{12}=$	3

Thus the solid obelisk will exceed $\tfrac{1}{4}$ the series of cubes, or of 1^3, 3^3, 5^3, 7^3, 9^3, 11^3, by $\tfrac{1}{12}$ the cube of 1 for every unit of the axis; or by 1 cube for every 12 units of the axis, or by as many cubes of 1 as would extend $\tfrac{1}{12}$ the axis; or by a stratum of cubes of 1 that would cover $\tfrac{1}{12}$ the square ordinate.

Sum of the cubes of 1, 3, 5 $=2\,\overline{\text{axis}}^2-$ axis, when $n=6$, $=2\times 36^2-36=2556$

$$\tfrac{1}{4}\text{ sum of the cubes } = 639$$

$$\begin{aligned}\tfrac{1}{4}\text{ sum of cubes } &=\tfrac{1}{4}\left(2\,\overline{\text{axis}}^2-\text{axis}\right)\\ &=\tfrac{1}{2}\,\overline{\text{axis}}^2-\tfrac{1}{4}\text{ axis}\\ &=\tfrac{1}{2}\,36^2-9\\ &=648-9=639.\end{aligned}$$

$$\begin{aligned}\text{Content obelisk } &=\tfrac{1}{4}\text{ sum of the cubes }+\tfrac{1}{12}\text{ axis}\\ &=\tfrac{1}{2}\,\overline{\text{axis}}^2-\tfrac{1}{4}\text{ axis }+\tfrac{1}{12}\text{ axis}\\ &=\tfrac{1}{2}\,\overline{\text{axis}}^2-\tfrac{1}{6}\text{ axis}\\ &=\tfrac{1}{2}\,36^2-6=648-6=642.\end{aligned}$$

$$\begin{aligned}\text{Content parabolic obelisk} &=\tfrac{1}{2}\text{ axis }\times\overline{\text{ordinate}}^2\\ &=\tfrac{1}{2}\,\overline{\text{axis}}^2\\ &=\tfrac{1}{2}\,36^2=648.\end{aligned}$$

If the quadruple parabolic obelisk generated by the double, or velocity ordinate squared $= \overline{2n}^2$, descending along the axis and varying as axis $= \frac{1}{2}$ circumscribing parallelopiped.

Sum of the cubes $= 2\,\overline{\text{axis}}^2 - \text{axis}$,
$= 2 \times 36^2 - 36 = 2556$.

Content of quadruple obelisk $=$ sum of the cubes $+ \frac{1}{3}$ axis,
$= 2\,\overline{\text{axis}}^2 - \text{axis} + \frac{1}{3}\,\text{axis}$,
$= 2\,\overline{\text{axis}}^2 - \frac{2}{3}\,\text{axis}$,
$= 2 \times 36^2 - 24$,
$= 2592 - 24 = 2568$.

Content of quadruple parabolic obelisk $= \frac{1}{2}$ circumscribing parallelopiped,
$= \frac{1}{2}\,\text{axis} \times \text{ordinate}$,
$= \frac{1}{2}\,\text{axis} \times \overline{2n}^2$,
$= \frac{1}{2}n^2 \times 4n^2$,
$= 2n^2 \times n^2$,
$= 2\,\overline{\text{axis}}^2$,
$= 2 \times 36^2 = 2592$.

The content of the obelisk exceeds $\frac{1}{4}$ the sum of the series of $1^3 + 3^3 + 5^3$, &c., by $\frac{1}{12}$ the axis; or by $\frac{1}{12}$ of a cube for every unit of axis of the obelisk.

The content of the obelisk is less than the content of the parabolic solid by $\frac{1}{6}$ the axis, or $\frac{1}{6}$ of a cube for every unit of the axis.

Hence the content of obelisk will lie between $\frac{1}{4}$ the sum of the cubes of 1, 3, 5, &c., and parabolic content.

But the sum of the series of cubes of 1, 2, 3, &c., $= \overline{\text{axis}}^2$, and parabolic content $= \frac{1}{2}\,\text{axis}^2$; therefore content of obelisk will lie between $\frac{1}{4}$ the sum of the cubes of 1, 3, 5, &c., and $\frac{1}{2}$ the sum of the cubes of 1, 2, 3, &c., the axes being equal.

Again, the sum of the cubes of 2, 4, 6, &c., $= 2\,\overline{\text{axis}}^2$; therefore content of obelisk will lie between $\frac{1}{4}$ the sum of the cubes of 1, 3, 5, &c., and $\frac{1}{4}$ the sum of the cubes of 2, 4, 6, &c.

Or the content of the quadruple obelisk, generated by the (double ordinate)^2, will lie between the sum of the cubes of 1, 3, 5, &c., and the sum of the cubes of 2, 4, 6, &c.

If, on the produced axis of the obelisk (*fig.* 10.), squares be drawn having their sides equal 2, 4, 6, &c., the differences between the values of n^2-n, where $n=1$, 2, 3, &c., these squares will represent the cubes of 2, 4, 6, &c.; and the squares having their sides = the sectional axes 1, 3, 5, &c., will represent the cubes of 1, 3, 5, &c. Thus the content of the single obelisk will lie between $\frac{1}{4}$ the sum of the 1st and $\frac{1}{4}$ the sum of the 2nd series of cubes, if the axes were equal.

$$1+3+5+7+9+11=n^2=6^2=36,$$

and $$0+2+4+6+8+10=n^2-n=30;$$

therefore $$1+5+9+13+17+21=2n^2-n=66.$$

Here $n^2=$ axis of obelisk,

$n^2-n=$ axis produced,

$2n^2-n=$ axis of pyramid.

Area of the triangle corresponding to the pyramid having base $= n$, and axis $= 2n^2-n=\frac{1}{2}(2n^2-n)\,.\,n=n^3-\frac{1}{2}n^2$.

$\frac{2}{3}$ area of triangle $= \frac{2}{3}$ of $\frac{1}{2}(2n^2-n)\,.\,n$

$= \frac{2}{3}n^3-\frac{1}{3}n^2$.

Area of obelisk $= \frac{2}{3}n^3-\frac{1}{6}n$,

and $\frac{2}{3}$ the circumscribing parallelogram $=\frac{2}{3}n^3$.

Hence the area of obelisk, which $=\frac{2}{3}n^3-\frac{1}{6}n$, will lie between $\frac{2}{3}n^3$, which $= \frac{2}{3}$ the circumscribing parallelogram, or = the parabolic area, and $\frac{2}{3}n^3-\frac{1}{3}d^2$, which $= \frac{2}{3}$ the triangular area formed by the vertical section of the pyramid.

When the series $0+2+4+6$, &c., begins with 0 (and 0 is reckoned a term), the sum of n terms $= n^2-n$.

When the series begins with 2 (and n is reckoned from 2), the series $2+4+6+8+10=n^2+n=30$, when $n=5$, which is the same as 6 terms of $0+2+4+6+8+10$, where $n^2-n=30$.

Since series $1+3+5+7=n^2$
and series $2+4+6+8=n^2+n$.
Therefore series $3+7+11+15=2n^2+n$,
and series $1+5+9+13=2n^2-n$.
Therefore series $4+12+20+28=4n^2$,
$= 4 \times \overline{\text{axis}}^2$, or $= \overline{2\text{ axis}}^2$ of obelisk.

Ordinate and sectional axis obelisk = axis of pyramid.

1st. $1 \times 1 = 1$
2nd. $2 \times 3 = 6$
3rd. $3 \times 5 = 15$
4th. $4 \times 7 = 28$
5th. $5 \times 9 = 45$
6th. $6 \times 11 = 66$.

1st. 1, 6. 15, 28, 45, 66 axes of pyramids.
2nd. 1, 5, 9, 13, 17, 21 difference of axes.
3rd. 1, 4, 4, 4, 4, 4 difference between the last distances.

Hence the sum of the series of the axes of pyramids will, if formed by squares of unity = the sum of the areas formed by each sectional axis and its ordinates, which area will exceed the area of the obelisk by half the number of squares of unity of the series 1, 3, 5, 7. 9, 11, or $\frac{1}{2}$ the squares of unity along the whole axis of obelisk or $\frac{1}{2}\,\overline{\text{ordinate}}^2$

For $1+6+15+28+45+66=161$
and area of obelisk $=\frac{2}{3}n^3-\frac{1}{6}n$.
when $n=6=144-1=143$
area of obelisk $+\frac{1}{2}\,\overline{\text{ordinate}}^2 = 143+18=161$.

Thus each number of the series of axes is formed by its sectional axis × ordinate, and the sum of this series = area of obelisk $+\frac{1}{2}\,\overline{\text{ordinate}}^2$.

$$\text{Or sum of series} = \text{area obelisk} + \tfrac{1}{2}\,\overline{\text{ordinate}}^2$$
$$= \tfrac{2}{3}n^3 - \tfrac{1}{6}n + \tfrac{1}{2}n^2$$
$$= \tfrac{2}{3}n^3 + \tfrac{1}{2}n^2 - \tfrac{1}{6}n.$$

The sum of any number of terms in the second series will = the number itself in the first series immediately above the last of these terms, which sum will also = the sectional axis

× ordinate. Thus the sum of 6 terms of the second series, or $1+5+9+13+17+21=66$, the number above 21, or will
$=$the 6th ordinate × its sectional axis
$=6\times 11=66$, or, generally,
$=n\times\overline{2\,n-1}$.

The sum of the 3rd series will
$=1+4+4+4+4+4=21$
the 6th term of the second series, or, generally,

$$=1+\overline{n-1}\,.\,4$$
$$\text{or}=4\,n-3$$

Each of the sectional axes of the obelisk 1, 3, 5, 7, &c. equals the sum of the two ordinates, or the difference of their squares;

$$\text{for } n+\overline{n-1}=2\,n-1$$
$$\text{and } n^2-\overline{n-1}^2=n^2-(n^2-2\,n+1)$$
$$=2n-1$$

Subtracting the less from the next greater axis of the series of pyramids gives the series 1, 5, 9, 13 for the differences between the axes of the pyramids.

To sum of $1+3+5+7+9+11$, &c. $=n^2$
add $0+2+4+6+8+10$, &c. $=\overline{n-1}\,.\,n$
then *S.* of $1+5+9+13+17+21$, &c. will
$=n^2+\overline{n-1}\,.\,n=\overline{2n-1}\,.\,n$
$=$axis of the n^{th} pyramid.

Or by making the 3d the 1st series and the 1st the 3rd, it will be seen that the sum of n terms of the 1st series will form each of the n terms of the 2nd series, and the sum of the 2nd series will form each of the n terms of the 3rd series, and the sum of the 3rd series will form each of the n terms of a 4th series.

	1,	4,	4,	4,	4,	4,	Sum $=4n-3$
and forms	1,	5,	9,	13,	17,	21,	$=\overline{2n-1}\,.\,n$
and forms	1,	6,	15,	28,	45,	66,	$=\frac{2}{3}n^3+\frac{1}{2}n^2-\frac{1}{6}n$.
and forms	1,	7,	22,	50,	95,	161,	

Thus the axis of obelisk + the rectangle by the two ordinates of the last section will = the axis of pyramid having the same base as the obelisk.

The axis of pyramid $= n^2$ below, and $n^2 - n$ above, the apex of the obelisk, or whole axis of pyramid $= \overline{2n-1} \, . \, n$. , and $2n-1$, forms the series 1, 3, 5, 7, the sectional axes, or series of the differences between the series of the whole axes of obelisk. If to this series there be added the series 0, 2, 4, 6, 8, &c., formed from $2n-2$, the distances between the several apices of pyramids, the sum of which series $= 0+2+4+6$, &c. $= \overline{n-1} \, . \, n =$ the rectangle by the two ordinates of the last section of obelisk = the portion of the axis of each of these several pyramids beyond the apex of the obelisk. Then will $2n-1+2n-2=4n-3$, the difference between the entire axes of pyramids, form the series 1, 5, 9, 13, &c.; the sum of which $=1+5+9+13$, &c. $= n^2 + \overline{n-1} \, . \, n = \overline{2n-1} \, . \, n =$ the whole axis of the n^{th} pyramid.

The series of the axes of pyramids will be 1, 6, 15, 28, &c., each term being formed by $\overline{2n-1} \, . \, n$, or by sectional axis × ordinate of obelisk, which equals the whole axis of a pyramid $= n \times n^{\text{th}}$ sectional axis.

Again, $n^2 - n$, the distance of the apex of the pyramid from the apex of the obelisk, forms the series 0, 2, 6, 12, &c., the sum of which series $= 0 + 2 + 6 + 12 + 20 + 30$, &c. $= \frac{1}{3}n^3 - \frac{1}{3}n$.

n^2 forms the series of the whole axes of obelisks, 1, 4, 9, 16, &c. which = the series of the parts of the axes of pyramids below the apex of obelisk; the sum of which series $= 1+4+9+16$, &c. $= \frac{1}{3}(\overline{n+1} \, . \, n \, . \, \overline{n+\tfrac{1}{2}}) = \frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n$.

Then $n^2 - n + n^2 = \overline{2n-1} \, . \, n$ will form the series 1, 6, 15, 18, &c.; the sum of which $= 1+6+15+18$, &c. $= \frac{2}{3}n^3 + \frac{1}{2}n^2 - \frac{1}{6}n$, the sum of the series of entire axes of pyramids.

The formation and sum of each of these three series will be

1. *S.* of $n^2 - n \quad = 0+2+ \ 6+12$ &c. $= \frac{1}{3}n^3 - \frac{1}{3}n$
2. *S.* of $n^2 \quad = \underline{1+4+ \ 9+16 \text{ \&c.}} = \frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n$
3. *S.* of $\overline{2n-1} \, . \, n = 1+6+15+28$ &c. $= \frac{2}{3}n^3 + \frac{1}{2}n^2 - \frac{1}{6}n$

1. Sum of axes of pyramids beyond the apex of obelisk

2. Sum of axes of pyramids below the apex of obelisk, which is also the sum of the axes of obelisk.
3. Sum of the entire axes of pyramids.

Since the axes of the pyramids are as $\overline{2n-1}\,.\,n$, the areas of the triangular vertical sections of the pyramids will be as $\frac{1}{2}$ axis × ordinate, or $\frac{1}{2}\,(\overline{2n-1}\,.\,n^2)$, or $n^3-\frac{1}{2}n^2$.

The areas of the triangles will be expressed by the difference between the series of n^3 and $\frac{1}{2}n^2$.

$$\begin{aligned} S.\ n^3 &= 1+8+27+64+125+216=441 \\ S.\ \tfrac{1}{2}n^2 &= \underline{\tfrac{1}{2}+2+4\tfrac{1}{2}+8+12\tfrac{1}{2}+18=45\tfrac{1}{2}} \\ S.\text{ of difference} &= \tfrac{1}{2}+6+22\tfrac{1}{2}+56+112\tfrac{1}{2}+198=395\tfrac{1}{2} \end{aligned}$$

The sum of the cubes of $1+2+3=(\frac{1}{2}\,\overline{n+1}\,.\,n)^2$, and the sum of their squares $=\frac{1}{3}\,\overline{n+1}\,.\,n\,.\,\overline{n+\frac{1}{2}}$.

So half the sum $=\frac{1}{6}\overline{n+1}\,.\,n\,.\,\overline{n+\frac{1}{2}}$.

Hence the sum of their difference, or the sum of the triangular areas will

$$\begin{aligned} &=(\tfrac{1}{2}\,\overline{n+1}\,.\,n)^2-\tfrac{1}{6}\,\overline{n+1}\,.\,n\,.\,\overline{n+\tfrac{1}{2}} \\ \text{if } n=6\qquad &=\tfrac{1}{2}\,\overline{21}^2-\tfrac{1}{6}\,273 \\ &=441-45\tfrac{1}{2}=395\tfrac{1}{2}. \end{aligned}$$

To Sum the Series of Pyramids.

Content of pyramid $=\frac{1}{3}$ axis × ordinate.

Here ordinate $=n^2$, and axis $=\overline{2n=1}\,.\,n$.

$$\begin{aligned} \text{Pyramid}\quad &=\tfrac{1}{3}(\overline{2n-1}\,.\,n\,.\,n^2). \\ &=\tfrac{1}{3}(2n^4-n^3). \\ &=\tfrac{2}{3}\,n^4-\tfrac{1}{3}\,n^3. \end{aligned}$$

Sum of series $\frac{2}{3}\,n^4$

$$\begin{aligned} &=\tfrac{2}{3}(1^4+2^4+3^4\ \&c.) \\ &=\tfrac{2}{3}\text{ of }\tfrac{1}{6}\,(\overline{n+1\,.\,n}^{\,2}\,.\,\overline{n+\tfrac{1}{2}}-\tfrac{1}{3}\,\overline{n+1}\,.\,n\,.\,\overline{n+\tfrac{1}{2}}). \end{aligned}$$

Sum of series $\frac{1}{3}\,n^3=\frac{1}{3}\,(1^3+2^3+3^3\ \&c.)$

$$=\tfrac{1}{3}\,(\tfrac{1}{2}\overline{n+1}\,.\,n)^2.$$

When $n=6$, Sum of series $\frac{2}{3}\,n^4=1516\frac{2}{3}$

$$\begin{aligned} \text{,,}\qquad \tfrac{1}{3}\,n^3 &= \underline{\ 147\ } \end{aligned}$$

Sum of their difference $=1369\frac{2}{3}$,

= content of the series of pyramids.

These series, when $n=6$, will be

$$\begin{array}{l}
\tfrac{2}{3}n^4=\tfrac{2}{3}+10\tfrac{2}{3}+54+170\tfrac{2}{3}+416\tfrac{2}{3}+864=1516\tfrac{2}{3},\\
\tfrac{1}{3}n^3=\tfrac{1}{3}+\ \ 2\tfrac{2}{3}+\ \ 9+\ \ 21\tfrac{1}{3}+\ \ 41\tfrac{2}{3}+\ \ 72=\ \ 147,\\
\text{dif.}=\tfrac{1}{3}+\ \ 8\ \ +45+149\tfrac{1}{3}+375\ \ +792=1369\tfrac{2}{3},
\end{array}$$

or content of series of pyramids $=1369\tfrac{2}{3}$;
or

$$\begin{array}{l}
\tfrac{2}{3}n^4=\tfrac{2}{3}(1^4+2^4+3^4+4^4+5^4+6^4)=1516\tfrac{2}{3},\\
\tfrac{1}{3}n^3=\tfrac{1}{3}(1^3+2^3+3^3+4^3+5^3+6^3)=\ 147,\\
\text{dif.}=1516\tfrac{2}{3}-147=1369\tfrac{2}{3}.
\end{array}$$

The series $\tfrac{1}{3}n^3$ × by $2n$ will form the series of $\tfrac{2}{3}n^4$;

$$\begin{array}{llllll}
\tfrac{1}{3}n^3=\tfrac{1}{3}, & 2\tfrac{2}{3}, & 9, & 21\tfrac{1}{3}, & 41\tfrac{2}{3}, & 72,\\
2n\ =2, & 4, & 6, & 8, & 10, & 12,\\
\tfrac{2}{3}n^4=\tfrac{2}{3}, & 10\tfrac{2}{3}, & 54, & 170\tfrac{2}{3}, & 416\tfrac{2}{3}, & 864.
\end{array}$$

Hence the n^{th} term in the series $\tfrac{2}{3}n^4$ will = the n^{th} term in the series $\tfrac{1}{3}n^3$ × by $2n$; as the 6th in

$$\tfrac{2}{3}n^4=864=72\times\text{by }6\times2.$$

Also the series $\tfrac{1}{3}n^3$ multiplied by $2n-1$ will form the series of $\tfrac{2}{3}n^4=\tfrac{1}{3}n^3$

$$\begin{array}{llllll}
\tfrac{1}{3}n^3\qquad=\tfrac{1}{3}, & 2\tfrac{2}{3}, & 9, & 21\tfrac{1}{3}, & 41\tfrac{2}{3}, & 72,\\
2n-1\quad=1, & 3, & 5, & 7, & 9, & 11,\\
\tfrac{2}{3}n^4-\tfrac{1}{3}n^3=\tfrac{1}{3}, & 8, & 45, & 149\tfrac{1}{3}, & 375, & 792.
\end{array}$$

Thus the series $\tfrac{1}{3}n^3$ is a pyramidal series, each term being = to a pyramid, $\tfrac{1}{3}n^3$, which, multiplied by twice the ordinate, or $2n$, will form the first series $\tfrac{2}{3}n^4$.

The third series, the difference between the series $\tfrac{2}{3}n^4$ and $\tfrac{1}{3}n^3$, will be formed by multiplying the pyramidal series $\tfrac{1}{3}n^3$ successively by 1, 3, 5, 7, or the corresponding sectional axes $2n-1$.

Thus each pyramid, the frustum of which forms a section of the obelisk, will $=\tfrac{1}{3}n^3$, or $\tfrac{1}{3}$ ord³ obelisk multiplied by the sectional axis of that ordinate, or $=\tfrac{1}{3}n^3\times\overline{2n-1}=\tfrac{2}{3}n^4-\tfrac{1}{3}n^3$.

Hence the pyramid having its axis $=\overline{2n-1}\ .\ n$, and base $=n^2$, the base of the obelisk, may be compared with the corresponding obelisk having its axis $=n^2$.

$$\begin{array}{rlcl} & \text{Content pyramid} & : & \text{content obelisk.} \\ :: & \frac{2}{3}n^4-\frac{1}{3}n^3 & : & \frac{1}{2}n^4-\frac{1}{6}n^2 \\ :: & \frac{2}{3}n^2-\frac{1}{3}n & : & \frac{1}{2}n^2-\frac{1}{6} \\ :: & \frac{12}{3}n^2-\frac{6}{3}n & : & \frac{6}{2}n^2-\frac{6}{6} \\ :: & 4n^2-2n & : & 3n^2-1. \end{array}$$

The sum of the series of cubes of 1, 2, 3, 4 $= \frac{1}{2}(\overline{n+1}\,.\,n)^2$ $=\text{axis}^2$.

For

$$S.\ 1+2+3+4+5+6=\tfrac{1}{2}\overline{n+1}\,.\,n=21=\text{axis}.$$

$$S.\ 1^3+2^3+3^3+4^3+5^3+6^3=441=\overline{21}^2=\overline{\text{axis}}^2,$$

or sum of the cubes $= (\frac{1}{2}\overline{n+1}\,.\,n)^2=\overline{\text{axis}}^2$.

The sum of the series of cubes of 2, 4, 6, 8 $= 2(\frac{1}{2}\overline{n+1}\,.\,n)^2$ $=2\,\overline{\text{axis}}^2$.

For the axis of n terms of this series will = twice the axis of n terms of the series 1, 2, 3, 4,

$$=2\,(\tfrac{1}{2}\overline{n+1}\,.\,n)=\overline{n+1}\,.\,n,$$

and each term in the 1st series = 8 times the corresponding term in the last series.

Therefore $S.$ of $2^3+4^3+6^3+8^3$ will = 8 times the $S.$ of $1^3+2^3+3^3+4^3=8\,(\frac{1}{2}\overline{n+1}\,.\,n)^2$

$$=2\,(\overline{n+1}\,.\,n)^2$$

$$=2\,\overline{\text{axis}}^2.$$

The sum of the series of cubes of 1, 3, 5, 7,

$$=2n^4-n^2=2\,\overline{\text{axis}}^2-\text{axis}.$$

$$\begin{array}{lcccccc} \text{From} & 1^3, & 2^3, & 3^3, & 4^3, & 5^3, & 6^3, \\ \text{take} & & 2^3, & & 4^3, & & 6^3, \\ \hline \text{difference} & 1^3, & & 3^3, & & 5^3, & \end{array}$$

Let $n=$ the number of terms in each of the two last series, then $2n$ will equal the number of terms in the 1st series.

$$\begin{array}{ll} S.\ \text{of 1st, which} & =(\frac{1}{2}\overline{n+1}\,.\,n)^2, \\ \text{will now} & =(\frac{1}{2}\overline{2n+1}\,.\,2n)^2 \\ & =(\overline{n+\frac{1}{2}}\,.\,2n)^2 \\ & =(2n^2+n)^2. \end{array}$$

$$S.\ \text{of 2nd series} =2(\overline{n+1}\,.\,n)^2=2\,(n^2+n)^2.$$

$$\begin{aligned} S.\ \text{of 3rd series} &= \text{difference} \\ &= (2n^2+n)^2 - 2(n^2+n)^2 \\ &= 2n^4 - n^2 = 2\,\overline{\text{axis}}^2 - \text{axis}. \end{aligned}$$

For axis $= 1+3+5 = n^2$; $2\,\overline{\text{axis}}^2 - \text{axis} = 2$ strata, each stratum having an area $= \overline{\text{axis}}^2$, and a depth of unity, less a line of cubes of unity = the length of the axis.

In the 3rd series of $1^3, 3^3, 5^3$, $n=3$, axis $= 1+3+5 = 9 = n$.

$$\begin{aligned} \text{Sum of } 1^3+3^3+5^3 &= 2\,\overline{\text{axis}}^2 - \text{axis} \\ &= 2n^4 - n^2 \\ &= 2 \times 3^4 - 3^2 \\ &= 2 \times 81 - 9 \\ &= 153. \end{aligned}$$

Or in the series $1^3, 3^3, 5^3$,

$n=3$, axis $= 1+3+5 = n^2 = 9$.

$$\begin{aligned} \text{Sum} &= 2\,\overline{\text{axis}}^2 - \text{axis} \\ &= 2 \times 9^2 - 9 \\ &= 153. \end{aligned}$$

Otherwise,

$$\begin{array}{rl} & 1^3+2^3+3^3+4^3+5^3+6^3=441 \\ & \quad\ 2^3 \quad +4^3 \quad +6^3=288 \\ \hline & 1^3 \quad +3^3 \quad +5^3 \quad =153. \end{array}$$

Sum of the 2nd series = the sum of $8 \times \frac{1}{2}\,n$ terms of the 1st series. The difference of the two series = the sum of $\frac{1}{2}\,n$ terms of the 3rd series.

As sum of $1^3+2^3+3^4=36$, and $8 \times 36 = 288$ = sum of 2nd series, which, subtracted from the sum of the 1st series 441, leaves 153, the sum of the 3rd series,

Sum of series of $1^3+2^3+3^3+4^3 = \overline{\text{axis}}^2$.

Sum of series of $2^3+4^3+6^3+8^3 = 2\,\overline{\text{axis}}^2$.

These axes become equal at the 20th term of the 1st series, and the 14th term of the second series.

$$\begin{aligned} \text{Sum of 1st series} &= (\tfrac{1}{2}\,\overline{n+1}\,.\,n)^2 = \overline{\text{axis}}^2 \\ &= (\tfrac{1}{2}\,21 \times 20)^2 \\ &= \overline{210}^2 = 44100. \end{aligned}$$

$$\begin{aligned}\text{Sum of 2nd series} &= 2(\overline{n+1}\,.\,n)^2 = 2\,\overline{\text{axis}}^2\\ &= 2(15\times 14)^2\\ &= 2\times\overline{210}^2 = 88200.\end{aligned}$$

Or when the two series have a common axis, their contents will be as 1 : 2.

When both series have the same number of terms, their contents will be as 1 : 8.

Let each of the series

$$\begin{array}{llllll}1, & 2, & 3, & 4, & 5, & 6,\\ 2, & 4, & 6, & 8, & 10, & 12,\\ 1, & 3, & 5, & 7, & 9, & 11,\end{array}$$

form an axis, *figs.* 7, 8. 5., then the series of squares described on the side of the axis will represent both squares and cubes, or areas and solids of an obeliscal form.

$$\begin{array}{lll}\text{The axis of the} & \text{1st series} & = \tfrac{1}{2}\overline{n+1}\,.\,n,\\ & \text{2nd} \;\;\text{,,} & = \overline{n+1}\,.\,n,\\ & \text{3rd} \;\;\text{,,} & = n^2,\\ \text{The ordinate of the} & \text{1st} \;\;\text{,,} & = n+\tfrac{1}{2},\\ & \text{2nd} \;\;\text{,,} & = 2(n+\tfrac{1}{2}),\\ & \text{3rd} \;\;\text{,,} & = 2n.\end{array}$$

$$\begin{array}{ll}\text{Or sum of} & 1+2+3=\text{axis}=\tfrac{1}{2}\overline{n+1}\,.\,n,\\ & 2+4+6=\text{axis}=\overline{n+1}\,.\,n,\\ & 1+3+5=\text{axis}=n^2.\end{array}$$

The sum of their squares, or

$1^2+2^2+3^2=\text{area}=\tfrac{1}{3}\overline{n+1}\,.\,n\,.\,\overline{n+\tfrac{1}{2}}=\tfrac{2}{3}\,\text{axis}\times\text{ordinate},$

$2^2+4^2+6^2=\text{area}=\tfrac{4}{3}\overline{n+1}\,.\,n\,.\,\overline{n+\tfrac{1}{2}}=\tfrac{2}{3}\,\text{axis}\times\text{ordinate},$

$1^2+3^2+5^2=\text{area}=\tfrac{4}{3}n^3-\tfrac{1}{3}n=\tfrac{4}{3}\overline{\text{axis}}^{\frac{3}{2}}-\tfrac{1}{6}\,\text{ordinate}.$

The sum of their cubes, or

$1^3+2^3+3^3=\text{solid}=(\tfrac{1}{2}\overline{n+1}\,.\,n)^2=\overline{\text{axis}}^2,$

$2^3+4^3+6^3=\text{solid}=2(\overline{n+1}\,.\,n)^2=2\,\overline{\text{axis}}^2,$

$1^3+3^3+5^3=\text{solid}=(2n^2-1)^2\,.\,n^2=2\,\overline{\text{axis}}^2-\text{axis}.$

Also the axis of the obeliscal area $=n^2=\overline{\text{ordinate}}^2$,

$$\begin{aligned}\text{area} &= \tfrac{2}{3}n^3-\tfrac{1}{6}n,\\ &= \tfrac{2}{3}\overline{\text{axis}}^{\frac{3}{2}}-\tfrac{1}{6}\,\text{ordinate},\end{aligned}$$

$$\begin{aligned} \text{solid} &= \tfrac{1}{2}n^4 - \tfrac{1}{6}n^2, \\ &= \tfrac{1}{2}\overline{\text{axis}}^2 - \tfrac{1}{6}\text{axis}; \end{aligned}$$

and $\frac{1}{4}$ the sum of $1^3 + 3^3 + 5^3$

$$\begin{aligned} &= \tfrac{1}{4}(2\,\overline{\text{axis}}^2 - \text{axis}), \\ &= \tfrac{1}{2}\overline{\text{axis}}^2 - \tfrac{1}{4}\text{axis}; \\ \text{solid obelisk} &= \tfrac{1}{2}\overline{\text{axis}}^2 - \tfrac{1}{6}\text{axis}. \end{aligned}$$

$\therefore$ the solid obelisk is greater than $\frac{1}{4}$ the sum of $1^3 + 3^3 + 5^3$ by $\frac{1}{4}\text{axis} - \frac{1}{6}\text{axis}$,

$$\text{or } \tfrac{1}{12}\text{axis}, \text{ or } \tfrac{1}{12}n^2.$$

The obeliscal area $= \frac{2}{3}n^3 - \frac{1}{6}n = \frac{1}{2}(\frac{4}{3}n^3 - \frac{1}{3}n) = \frac{1}{2}$ the sum of $1^2 + 3^2 + 5^2$.

The axis $1 + 3 + 5 = n^2$ is common to both the obeliscal solid and to this obeliscal series of cubes.

The corresponding parabolic area $= \frac{2}{3}n^3$,

$$\text{solid} = \tfrac{1}{2}\overline{\text{axis}}^2.$$

Let each of the squares in the series $1^2 + 3^2 + 5^2 + 7^2 + 9^2 + 11^2$ represent a cube of unity.

Fig. 5 *a.* Then these square strata, each having a depth of unity, will form a terraced pyramid, the content of which will $= \frac{4}{3}n^3 - \frac{1}{3}n$ in cubes of 1.

The content of the rectilineal-sided pyramid having a height $= n$, and side of base $= 2n$, will $= \frac{4}{3}n^3$, which will exceed the content of the stratified pyramid by $\frac{1}{3}n$ cubes of 1.

Next compare their sectional triangular areas, made by dividing each pyramid vertically into two equal parts.

$$\begin{aligned} &\text{Height of the triangle} = n = 6. \\ &\text{Side of the base} = 2n = 12. \\ &\therefore \text{Triangular area,} = \tfrac{1}{2}2n \,.\, n = \tfrac{1}{2}12 \times 6. \\ &\phantom{\therefore \text{Triangular area,}} = n^2 = 6^2 = 36. \end{aligned}$$

$$\text{Stratified area} = 1 + 3 + 5 + 7 + 9 + 11 = n^2 = 6^2 = 36.$$

Thus the triangular and stratified areas are equal. But the stratified pyramid is less than the triangular pyramid by $\frac{1}{3}n$.

Since the double obeliscal area, *fig.* 5. $= 1^2 + 3^2 + 5^2 + 7^2 +$

9^2+11^2, it follows that if each of these squares were converted into a stratum having a depth of 1, together they would form a stratified or terraced pyramid, *fig.* 5*a.*, containing as many cubes of 1 as the double obeliscal area contains squares of 1, or $=\frac{4}{3}n^3-\frac{1}{3}n=\frac{4}{3}\,6^3-\frac{1}{3}6$,

$$=288-2=286.$$

Also content of rectilineal pyramid $=\frac{4}{3}n^3$,

" " stratified pyramid $=\frac{4}{3}n^3-\frac{1}{3}n$.

Double parabolic area $=\frac{4}{3}n^3$,

" obeliscal area $=\frac{4}{3}n^3-\frac{1}{3}n$.

When the squares of 1 are arranged in the order 1, 2, 3, as in *fig.* 7—2, the whole area will $=\frac{1}{2}\overline{n+1}\,.\,n$, which will equal the area of a triangle having its height $=n$, and base $=\overline{n+1}$, or $=\frac{1}{2}n^2+\frac{1}{2}n$.

When the squares of 1, 2, 3 become strata of the depth of 1, and formed into a terraced pyramid, the content of the pyramid will $=\frac{1}{3}\overline{n+1}\,.\,n\,.\,\overline{n+\frac{1}{2}}$, which will = the content of a rectilineal pyramid having the sides of the base $=\overline{n+1}$. by $\overline{n+\frac{1}{2}}$, and height $=n$.

These obeliscal series of solids are expressed in terms of the axes, as sum of $1^3+2^3+3^3=\overline{\text{axis}}^2$.

$$2^3+4^3+6^3=2\,\overline{\text{axis}}^2.$$

$$1^3+3^3+5^3=2\,\overline{\text{axis}}^2-\text{axis}.$$

For when the obeliscal solid of the 1st series $=\overline{\text{axis}}^2$, the content is represented by a stratum, or by an area $=\overline{\text{axis}}^2$, where for each square of 1, a cube of 1 is substituted, so that a stratum having an area $=\overline{\text{axis}}^2$, and thickness that of unity, will form an obeliscal series of cubes having a content $=\overline{\text{axis}}^2$, = the sum of $1^3+2^3+3^3$.

Two strata, the area of each $=\overline{\text{axis}}^2$ of the 2nd series, will form the second series of cubes.

Two strata, the area of each being $=\overline{\text{axis}}^2$ of the 3rd series, less a line of cubes of unity = the axis in length, will form the 3rd series of obeliscal cubes.

The solid obelisk $= \frac{1}{2}\,\overline{\text{axis}}^2 - \frac{1}{6}$ axis, equals a triangular stratum having the height and side of base each = the axis and a depth of 1, less a line of cubes of 1 equal in length $\frac{1}{6}$ axis.

In order to find the sum of the series $1^4+2^4+3^4+4^4+5^4+6^4$, let the squares of 1^2, 4^2, 9^2, 16^2, 25^2, 36^2 be placed in obeliscal order, so that the sides of the squares shall form an axis $= 1+4+9+16+25+36$, *Fig.* 22., which axis will $=\frac{1}{3}\,\overline{n+1}\,.\,n\,.\,\overline{n+\frac{1}{2}}$, and the area of the series of squares will $=\frac{1}{6}(\overline{n+1}\,.\,n)^2\,.\,\overline{n+\frac{1}{2}}$ less $\frac{1}{6}$ of $\frac{1}{3}\,\overline{n+1}\,.\,n\,.\,\overline{n+\frac{1}{2}}$.

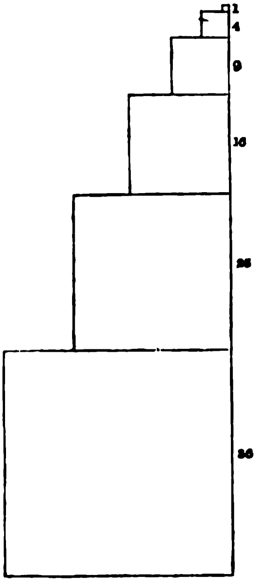


Fig. 22.

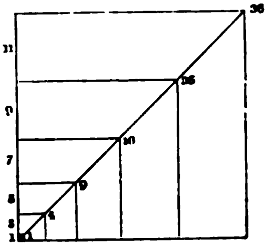


Fig. 23.

For $\frac{1}{6}\,(\overline{n+1}\,.\,n)^2\,.\,\overline{n+\frac{1}{2}}$ equals

$\frac{1}{6}\,\overline{2\times 1}^2 \times 1\frac{1}{2} = \frac{6}{6}$ when $n=1$
$\overline{3\times 2}^2 \times 2\frac{1}{2} = 18$ $=2$
$\overline{4\times 3}^2 \times 3\frac{1}{2} = 100\frac{1}{6}$ $=3$
$\overline{5\times 4}^2 \times 4\frac{1}{2} = 360$ $=4$
$\overline{6\times 5}^2 \times 5\frac{1}{2} = 990$ $=5$
$\overline{7\times 6}^2 \times 6\frac{1}{2} = 2293\frac{1}{6}$ $=6.$

$$\begin{array}{rrcrrr}
1^2= & 1 & \text{and} & 0+ & 1= & 1 \\
4^2= & 16 & ,, & 1+ & 16= & 17 \\
9^2= & 81 & ,, & 17+ & 81= & 98 \\
16^2= & 256 & ,, & 98+ & 256= & 354 \\
25^2= & 625 & ,, & 354+ & 625= & 979 \\
36^2= & 1296 & ,, & 979+ & 1296= & 2275 \\
\cline{2-2}
 & 2275 & & & &
\end{array}$$

Next find a formula for the difference between

$$\tfrac{1}{5}(\overline{n+1}\,.\,n)^2\,.\,\overline{n+\tfrac{1}{2}},$$

and the corresponding series of $1^2+4^2+9^2$, or between $2293\tfrac{1}{5}$ and 2275.

$$\begin{array}{rcrl}
\tfrac{6}{5}- & & 1= & \tfrac{1}{5} \\
18 & - & 17= & 1 \\
100\tfrac{4}{5}- & & 98= & 2\tfrac{4}{5} \\
360 & - & 354= & 6 \\
990 & - & 979= & 11 \\
\multicolumn{4}{c}{2293\tfrac{1}{5}-2275=18\tfrac{1}{5}}
\end{array}$$

$$\begin{array}{rlcr}
\tfrac{1}{5}\times 5= 1 & & & 1-0=1 \\
1\times 5= 5 & & & 5-1=4 \\
2\tfrac{4}{5}\times 5=14 & & & 14-5=9 \\
6\times 5=30 & & & 30-14=16 \\
11\times 5=55 & & & 55-30=25 \\
18\tfrac{1}{5}\times 5=91 & & & 91-55=36 \\
\cline{4-4}
 & & & 91
\end{array}$$

Since $18\tfrac{1}{5}\times 5=91=$ sum of $1+4+9$, &c. $=\tfrac{1}{3}\overline{n+1}\,.\,n\,.\,\overline{n+\tfrac{1}{2}}=91$, when $n=6$, and $\tfrac{1}{5}91=18\tfrac{1}{5}$.

Therefore the sum of the series of squares, or $1^2+4^2+9^2+16^2+25^2+36^2$,

$$\begin{aligned}
&=\tfrac{1}{5}(\overline{n+1}\,.\,n)^2\,.\,\overline{n+\tfrac{1}{2}}\text{ less }\tfrac{1}{5}\text{ of }\tfrac{1}{3}\overline{n+1}\,.\,n\,.\,\overline{n+\tfrac{1}{2}}\\
&=\tfrac{1}{5}\overline{7\times 6}^{\,2}\times 6\tfrac{1}{2}\text{ less }\tfrac{1}{5}\text{ of }\tfrac{1}{3}\,7\times 6\times 6\tfrac{1}{2},\\
&=\quad 2293\tfrac{1}{5}\text{ less }18\tfrac{1}{5}=2275,\text{ when }n=6.
\end{aligned}$$

The sum of the series

$$1^6+2^6+3^6+4^6+5^6+6^6$$

$$\text{or }\ 1^3+4^3+9^3+16^3+25^3+36^3,$$

$$=\tfrac{1}{7}(\overline{n+1\,.\,n}^{\,3}\,.\,\overline{n+\tfrac{1}{2}}-\overline{n+1\,.\,n}^{\,2}\,.\,\overline{n+\tfrac{1}{2}}+\tfrac{1}{3}\overline{n+1}\,.\,n\,.\,\overline{n+\tfrac{1}{2}}).$$

For $\frac{1}{7}\overline{n+1\,.\,n}^{2}\,.\,\overline{n+\frac{1}{2}}$, when $n=1, 2, 3, 4, 5, 6,$

$=\frac{1}{7}\,.\,\overline{2\times1}^{3}\times1\frac{1}{2}=\frac{1}{7}\,.\quad 12=\quad 1\frac{5}{7}.$

$\overline{3\times2}^{3}\times2\frac{1}{2}=\quad 540=\quad 77\frac{1}{7}.$

$\overline{4\times3}^{3}\times3\frac{1}{2}=\quad 6048=\quad 864.$

$\overline{5\times4}^{3}\times4\frac{1}{2}=\quad 36000=\quad 5142\frac{6}{7}.$

$\overline{6\times5}^{3}\times5\frac{1}{2}=\quad 148500=\quad 21214\frac{2}{7}.$

$\overline{7\times6}^{3}\times6\frac{1}{2}=\quad 481572=\quad 68796.$

$1^3=\quad 1 \qquad 0+\quad 1=\quad 1.$

$4^3=\quad 64 \qquad 1+\quad 64=\quad 65.$

$9^3=\quad 729 \qquad 67+\quad 729=\quad 794.$

$16^3=\ 4096 \qquad 794+\ 4096=\ 4890.$

$25^3=15625 \qquad 4890+15625=20515.$

$36^3=\underline{46656} \qquad 20515+46656=67171.$

67171

Find a formula for the difference between $\frac{1}{7}\overline{n+1\,.\,n}^{3}\,.\,\overline{n+\frac{1}{2}}$ and the corresponding series of cubes of 1, 4, 9, &c., or between 68796 and 67171, which = 1625.

$1\frac{5}{7}-\quad 1=\quad \frac{5}{7}.$

$77\frac{1}{7}-\quad 65=\quad 12\frac{1}{7}.$

$864\ -\quad 794=\quad 70.$

$5142\frac{6}{7}-\ 4890=\ 252\frac{6}{7}.$

$21214\frac{2}{7}-20515=\ 699\frac{2}{7}.$

$68796\ -67171=1625.$

$\frac{5}{7}\times\frac{7}{5}=\quad 1=\quad 0+\quad 1=\quad 1$

$12\frac{1}{7}\quad =\quad 17=\quad 1+\quad 16=\quad 17.$

$70\quad =\quad 98=\quad 17+\quad 81=\quad 98.$

$252\frac{6}{7}\quad =\ 354=\quad 98+\ 256=\ 354.$

$699\frac{2}{7}\quad =\ 979=\ 354+\ 625=\ 979.$

$1625\quad =2275=\ 979+\underline{1296}=2275.$

2275

Thus $\frac{5}{7}$ of $(1+16+81+256+625+1296)$,

or $\frac{5}{7}\,.\,(1^2+4^2+9^2+16^2+25^2+36^2)$,

$=\frac{5}{7}\ 2275=1625.$

$=\frac{5}{7}\left(\frac{1}{5}\overline{n+1\,.\,n}^{2}\,.\,\overline{n+\frac{1}{2}}-\frac{1}{5}\text{ of }\frac{1}{3}\overline{n+1}\,.\,n\,.\,\overline{n+\frac{1}{2}}\right).$

$=\frac{1}{7}\left(\overline{n+1\,.\,n}^{2}\,.\,\overline{n+\frac{1}{2}}-\frac{1}{3}\overline{n+1}\,.\,n\,.\,\overline{n+\frac{1}{2}}\right).$

Hence the sum of the series $1^3+4^3+9^3$, &c., or of $1^6+2^6+3^6$, &c., will

$$=\tfrac{1}{7}(\overline{n+1\;.\;n}^{\,3}\;.\;\overline{n+\tfrac{1}{2}}-\overline{n+1\;.\;n}^{\,2}\;.\;\overline{n+\tfrac{1}{2}}+\tfrac{1}{3}\overline{n+1}\;.\;n\;.\;\overline{n+\tfrac{1}{2}})$$
$$=\tfrac{1}{7}(42^3\times6\tfrac{1}{2}-42^2\times6\tfrac{1}{2}+\tfrac{1}{3}\;42\times6\tfrac{1}{2})$$
$$=\tfrac{1}{7}(481572-11466+91)$$
$$=\tfrac{1}{7}470197$$
$$=\;66171 \text{ when } n=6.$$

Thus the sums of the 2nd, 4th, and 6th powers of 1, 2, 3, and 2, 4, 6, will be

$$1^2+2^2+3^2=\text{axis}=\tfrac{1}{3}(\overline{n+1}\;.\;n\;.\;\overline{n+\tfrac{1}{2}})$$
$$1^4+2^4+3^4=\text{area}=\tfrac{1}{5}(\overline{n+1\;.\;n}^{\,2}\;.\;\overline{n+\tfrac{1}{2}}-\tfrac{1}{3}\overline{n+1}\;.\;n\;.\;\overline{n+\tfrac{1}{2}})$$
$$1^6+2^6+3^6=\text{solid}=\tfrac{1}{7}(\overline{n+1\;.\;n}^{\,3}\;.\;\overline{n+\tfrac{1}{2}}-\overline{n+1\;.\;n}^{\,2}\;.\;\overline{n+\tfrac{1}{2}}$$
$$+\tfrac{1}{3}\overline{n+1}\;.\;n\;.\;\overline{n+\tfrac{1}{2}}).$$

$$2^2+4^2+6^2=\text{axis}\;=\tfrac{4}{3}(\overline{n+1}\;.\;n\;.\;\overline{n+\tfrac{1}{2}})$$
$$2^4+4^4+6^4=\text{area}=\frac{4^2}{5}(\overline{n+1\;.\;n}^{\,2}\;.\;\overline{n+\tfrac{1}{2}}-\tfrac{1}{3}\overline{n+1}\;.\;n\;.\;\overline{n+\tfrac{1}{2}})$$
$$2^6+4^6+6^6=\text{solid}=\frac{4^3}{7}(\overline{n+1\;.\;n}^{\,3}\;.\;\overline{n+\tfrac{1}{2}}-\overline{n+1\;.\;n}^{\,2}\;.\;\overline{n+\tfrac{1}{2}}$$
$$+\tfrac{1}{3}\overline{n+1}\;.\;n\;.\;\overline{n+\tfrac{1}{2}}).$$

Having found by trial the sums of these series, let us next arrange them along the axes (*Figs.* 7. and 22.), and find the sums in the terms of the axis and ordinate.

$$1^2+2^2+3^2=\text{axis}=\tfrac{1}{3}\overline{n+1}\;.\;n\;.\;\overline{n+\tfrac{1}{2}}.$$
$$1^4+2^4+3^4=\text{area, here axis}=\tfrac{1}{3}\overline{n+1}\;.\;n\;.\;\overline{n+\tfrac{1}{2}}.$$
$$\text{ordinate}\quad=\overline{n+1}\;.\;n,\text{ or }=n^2+n.$$
$$\text{axis}\times\text{ordinate}=\tfrac{1}{3}\overline{n+1}\;.\;n\;.\;\overline{n+\tfrac{1}{2}}\;.\;\overline{n+1}\;.\;n$$
$$=\tfrac{1}{3}\overline{n+1\;.\;n}^{\,2}\;.\;\overline{n+\tfrac{1}{2}}.$$

$$\text{Sum of series}=\tfrac{3}{5}\text{ axis }\times\text{ ordinate }-\tfrac{1}{5}\text{ axis,}$$
$$=\tfrac{3}{5}\text{ of }\tfrac{1}{3}\overline{n+1\;.\;n}^{\,2}\;.\;\overline{n+\tfrac{1}{2}}-\tfrac{1}{5}\text{ of }\tfrac{1}{3}\overline{n+1}\;.\;n\;.\;\overline{n+\tfrac{1}{2}})$$
$$=\tfrac{1}{5}(\overline{n+1\;.\;n}^{\,2}\;.\;\overline{n+\tfrac{1}{2}}-\tfrac{1}{3}\overline{n+1}\;.\;n\;.\;\overline{n+\tfrac{1}{2}}),$$
$$\text{or}\qquad=\tfrac{1}{5}\overline{n+\tfrac{1}{2}}(\overline{n+1\;.\;n}^{\,2}-\tfrac{1}{3}\overline{n+1}\;.\;n,$$
$$=\tfrac{1}{5}\,6\tfrac{1}{2}(\overline{7\times6}^{\,2}-\tfrac{1}{3}\overline{7\times6})\text{ when }n=6,$$
$$=2275\text{ squares of }1.$$

Next sum $1^6+2^6+3^6=$ solid,

Axis $=\frac{1}{3}\,\overline{n+1}\cdot n\cdot\overline{n+\frac{1}{2}}$ and ordinate$^2=\overline{n+1\cdot n}^2$.

Axis × ordinate$^2=\frac{1}{3}\,\overline{n+1}\cdot n\cdot\overline{n+\frac{1}{2}}\cdot\overline{n+1\cdot n}^2$.

$=\frac{1}{3}\,\overline{n+1\cdot n}^3\cdot\overline{n+\frac{1}{2}}$,

$\frac{3}{7}$ axis × ordinate$^2=\frac{3}{7}$ of $\frac{1}{3}\,\overline{n+1\cdot n}^3\cdot\overline{n+\frac{1}{2}}$.

Sum of series $=\frac{3}{7}$ of $\frac{1}{3}\,\overline{n+1\cdot n}^3\cdot\overline{n+\frac{1}{2}}$,

$-\frac{3}{7}$ of $\frac{1}{3}\,\overline{n+1\cdot n}^2\cdot\overline{n+\frac{1}{2}}+\frac{1}{7}$ axis

$=\frac{3}{7}$ of $\frac{1}{3}\,\overline{n+1\cdot n}^3\cdot\overline{n+\frac{1}{2}}-\frac{3}{7}$ of $\frac{1}{3}\,\overline{n+1\cdot n}^2\cdot\overline{n+\frac{1}{2}}$

$+\frac{1}{7}$ of $\frac{1}{3}\,\overline{n+1}\cdot n\cdot\overline{n+\frac{1}{2}}$,

$=\frac{1}{7}\,\overline{n+\frac{1}{2}}\,(\overline{n+1\cdot n}^3-\overline{n+1\cdot n}^2+\frac{1}{3}\,\overline{n+1}\cdot n)$,

$=\frac{1}{7}\,6\frac{1}{2}\,(\overline{7\times 6}^3-\overline{7\times 6}^2+\frac{1}{3}\,7\times 6)$ when $n=6$,

$=\frac{1}{7}\,6\frac{1}{2}\,(74088-1764+14)=67171$ cubes of 1.

$=\frac{1}{7}\,\overline{n+\frac{1}{2}}$ (ordinate$^3-$ordinate$^2+\frac{1}{3}$ ordinate.)

Sum of 4th series = 4 times sum of 1st series.

„ 5th „ $=4^2$ „ 2nd „

„ 6th „ $=4^3$ „ 3rd „

1st series $=1^2+2^2+3^2=$ axis.

2nd „ $=1^4+2^4+3^4=\frac{3}{5}$ axis × ordinate $-\frac{1}{5}$ axis.

3rd „ $=1^6+2^6+3^6=\frac{3}{7}$ axis × ordinate$^2-\frac{3}{7}$ axis × ordinate $+\frac{1}{7}$ axis;

or 1st $=\frac{1}{3}\,\overline{n+\frac{1}{2}}\cdot\overline{n+1}\cdot n$,

2nd $=\frac{1}{5}\,\overline{n+\frac{1}{2}}\cdot(\overline{n+1\cdot n}^2-\frac{1}{3}\,\overline{n+1})$,

3rd $=\frac{1}{7}\,\overline{n+\frac{1}{2}}\cdot(\overline{n+1\cdot n}^3-\overline{n+1\cdot n}^2+\frac{1}{3}\,\overline{n+1}\cdot n)$,

4th $=\frac{4}{3}\,\overline{n+\frac{1}{2}}\cdot\overline{n+1}\cdot n$,

5th $=\frac{4^2}{5}\,\overline{n+\frac{1}{2}}\,(\overline{n+1\cdot n}^2-\frac{1}{3}\,\overline{n+1}\cdot n)$.

6th $=\frac{4^3}{7}\,\overline{n+\frac{1}{2}}\,(\overline{n+1\cdot n}^3-\overline{n+1\cdot n}^2+\frac{1}{3}\,\overline{n+1}\cdot n)$.

When ordinate ∝ axis$^{\frac{2}{3}}$, area $=\frac{3}{5}$ circumscribing parallelogram, or ordinate^3 ∝ axis^2

Axis^2 ∝ $(\overline{n+1}\cdot n\cdot\overline{n+\frac{1}{2}})^2$.

Ordinate^3 ∝ $(\overline{n+1}\cdot n)^3$.

$(\overline{n+1}\cdot n)^3$ is less than $(\overline{n+1}\cdot n\cdot\overline{n+\frac{1}{2}})^2$,

by $\frac{1}{4}n^4+\frac{1}{2}n^3+\frac{1}{4}n^2$

which $=441$, when $n=6$.

$$(\overline{n+1} \,.\, n)^3 = 42^3 = 74088$$

add 441

$$\overline{74529}$$

$$(\overline{n+1} \,.\, n \,.\, \overline{n+\tfrac{1}{2}})^2 = 273^2 = 74529$$

so that the ordinate^3 should $= (42{\cdot}08\ \&c.)^3$

when $(\overline{n+1} \,.\, n \,.\, \overline{n+\tfrac{1}{2}})^2 = 273^2$.

Then ordinate^3 would $\propto$ axis^2, and the curvilinear area, $\frac{3}{5}$ axis × ordinate, would = sum of the series of squares + $\frac{1}{4}$ axis.

Thus when the ordinate^3 $\propto$ axis^2, the ordinate^3 will $\propto$ $(\overline{n+1} \,.\, n + {\cdot}08333)^3$ when axis^2 $\propto$ $(\overline{n+1} \,.\, n \,.\, \overline{n+\tfrac{1}{2}})^2$.

Or, ordinate^3 will $\propto$ $(\overline{n+1} \,.\, n + \frac{1}{12}\ \text{unity})^3$.

As when $n=3$,

$$\text{axis} = \tfrac{1}{3}\overline{n+1} \,.\, n \,.\, \overline{n+\tfrac{1}{2}} = 14$$

$$\text{ordinate} = \overline{n+1} \,.\, n + \tfrac{1}{12} = 12{\cdot}083333,$$

$$\tfrac{3}{5}\,\text{axis} \times \text{ordinate} = \tfrac{3}{5}\ 14 \times 12\ 083333,$$

or curvilinear area $= 101{\cdot}49999$,

say $= 101{\cdot}5$

subtract $\frac{1}{4}$ axis, $\frac{1}{4}$ 14 $= 3{\cdot}5$

98

Sum of n, or 3 squares,

$$= 1^2 + 4^2 + 9^2 = 1 + 16 + 81 = 98.$$

Again, when $n=6$,

axis $= 91$, ordinate $= 42{\cdot}083333$

$\frac{3}{5}$ axis × ordinate $= \frac{3}{5}\,91 \times 42{\cdot}083333$

or curvilinear area $= 2297{\cdot}749999$

subtract $\frac{1}{4}$ axis, $\frac{1}{4}$ 91 $22{\cdot}75$

2275

Sum of n, or 6 squares,

$$= 1^2 + 4^2 + 9^2 + 16^2 + 25^2 + 36^2 = 2275.$$

Hence the sum of n squares of the series $1^2 + 4^2 + 9^2$ will $= \frac{3}{5}$ axis × ordinate $- \frac{1}{4}$ axis, when ordinate^3 $\propto$ axis^2.

Or the sum of n squares, when the ordinate$^2 = (\overline{n+1} \,.\, n)^2$ will $= \frac{3}{5}$ axis × ordinate $- \frac{1}{5}$ axis.

Straight lines drawn from the extremities of the ordinates, each ordinate being $= \overline{n+1} \,.\, n$, or $n^2 + n$, will form a series of obeliscal sectional areas bounded by straight lines; the triangular part cut off from the top of each square will = the triangular part added at the lower part of the square, so that the obeliscal sectional areas will together = the sum of the series of squares.

But the curvilinear area exceeds the series of squares by $\frac{1}{4}$ axis, and each curvilinear sectional area exceeds the square by $\frac{1}{4}$ the sectional axis, or $\frac{1}{4}$ the side of the square. Therefore each curvilinear area will exceed the corresponding obeliscal sectional area by $\frac{1}{4}$ the sectional axis, since the obeliscal area = the sum of the series of squares, as the axis to the 1st ordinate = 1, so $\frac{1}{4}$ 1 = $\frac{1}{4}$, to the 2nd ordinate = 1 + 4 = 5, so $\frac{1}{4}$5 = 1$\frac{1}{4}$.

Thus $\frac{1}{4}$1 = $\frac{1}{4}$.
5 = 1$\frac{1}{4}$.
14 = 3$\frac{1}{2}$.
30 = 7$\frac{1}{2}$.
55 = 13$\frac{3}{4}$.
91 = 22$\frac{3}{4}$.

From $\frac{1}{4}$, 1$\frac{1}{4}$, 3$\frac{1}{2}$, 7$\frac{1}{2}$, 13$\frac{3}{4}$, 22$\frac{3}{4}$.
take $\frac{1}{4}$, 1$\frac{1}{4}$, 3$\frac{1}{2}$, 7$\frac{1}{2}$, 13$\frac{3}{4}$,

difference $\frac{1}{4}$, 1, 2$\frac{1}{4}$, 4, 6$\frac{1}{4}$, 9.

Thus we have $\frac{1}{4}$ to be added to the 1st *obeliscal* sectional area to make it a curvilinear area, 1 to the 2nd sectional axis, 2$\frac{1}{4}$ to the 3rd, &c. The whole addition to the series will = 36 one-fourth squares of 1, or 9 squares of 1, equal to $\frac{1}{4}$ axis.

$\frac{1}{4}$, 1, 2$\frac{1}{4}$, 4, 6$\frac{1}{4}$, 9,
= $\frac{1}{4}$ of 1, 4, 9, 16, 25, 36,
= $\frac{1}{4}$ of 1^2, 2^2, 3^2, 4^2, 5^2, 6^2.

The sum of $1^2 + 3^2 + 5^2$ has been found.

From 1^4, 2^4, 3^4, 4^4, 5^4, 6^4,
take 2^4, 4^4, 6^4.

Difference 1^4, 3^4, 5^4.

The sums of the 1st and 2nd series are known; therefore the sum of 1^4, 3^4, 5^4, their difference, may be found, as the sum of the series 1^3, 3^3, 5^3 was determined.

$$\begin{array}{ll} \text{Again from} & 1^6,\ 2^6,\ 3^6,\ 4^6, \\ \text{take} & \quad\ 2^6, \quad\ 4^6 \\ \hline \text{Difference} & 1^6 \quad 3^6. \end{array}$$

Since the sums of the 1st and 2nd series are known, the sum of 1^6, 3^6, 5^6 may be found.

Or the sum of 6 terms of the series

$$\begin{array}{ll} & 1^4+2^4+3^4+4^4+5^4+6^4=2275 \\ \text{and} & \quad 2^4 \quad +4^4 \quad +6^4=1568 \\ \hline & 1^4 \quad +3^4 \quad +5^4 \quad = 707. \end{array}$$

The sum of $\frac{1}{2}6$, or 3 terms of 1st series $=1^4+2^4+3^4=98$, and $16 \times 98=1568=$ sum of 3 terms of the 2nd series, which subtracted from the sum of 6 terms of the 1st series $=707=$ the sum of 3 terms of the 3rd series.

Hence the sum of $\frac{1}{2}n$ terms of the 1st series × by 16 = the sum of $\frac{1}{2}n$ terms of the 2nd series, which subtracted from the sum of n terms of the 1st series = the sum of $\frac{1}{2}n$ terms of the 3rd series.

When the series $1+2+3$, &c. is squared, as $1^2+2^2+3^2$, &c., the sum of $\frac{1}{2}n$ terms of this, the 1st series, × by 4, or 2^2, = the sum of $\frac{1}{2}n$ terms of the 2nd series, $2^2+4^2+6^2$.

When cubed, as $1^3+2^3+3^3$, the sum of $\frac{1}{2}n$ terms × by 8, or 2^3 = the sum of $\frac{1}{2}n$ terms of the 2nd series, $2^3+4^3+6^3$.

In the series $1^4+2^4+3^4$, the sum of $\frac{1}{2}n$ terms × by 16, or 2^4 = the sum of $\frac{1}{2}n$ terms of the 2nd series, $2^4+4^4+6^4$.

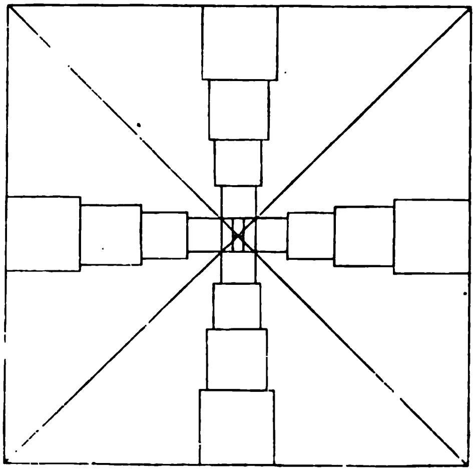


Fig. 25.

Thus from the sum of the series 1+2+3 to the power of 2, 3, or 4, the sum of the series 1+3+5, to the power of 2, 3, or 4 may be found.

Also in the series 1+2+3, &c. $\frac{1}{2}n$ terms of the 1st series × by 2, or 2′, = $\frac{1}{2}n$ terms of the 2nd series, 2+4+6.

Fig. 25. Four series of the cubes of 1, 2, 3, 4, 5 are arranged star-like, radiating from a common centre, their axes being at right angles to each other.

As each series $=\overline{\text{axis}}^2$,
$=\frac{1}{4}$ the circumscribing square,
$=\frac{1}{4}\,\overline{2\,\text{axis}}^2$,
= the circumscribing triangle,

∴ the 4 series of cubes will $=\overline{2\,\text{axis}}^2=$ the circumscribing square stratum of the depth of unity.

Fig. 26. When the axes of two series of cubes of 2, 4, 6, 8,

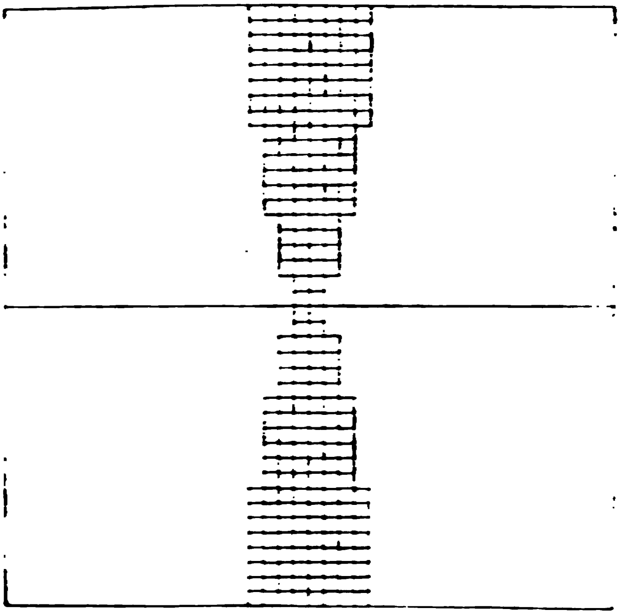


Fig. 26.

are in the same straight line, the sum of each series will $=2\,\overline{\text{axis}}^2$, and the sum of both series $=4\,\overline{\text{axis}}^2=\overline{2\,\text{axis}}^2=$ the circumscribing square stratum having each side = twice the axis.

Fig. 27. Let the two series of cubes of 2, 4, 6, 8, be each divided into 2 equal parts, then they will form 4 solid radiations from a common centre.

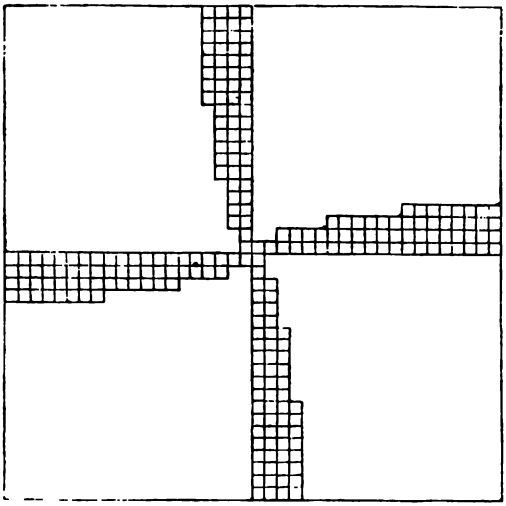


Fig. 27.

The content of the 4 radiations will = the content of two series of cubes of 2, 4, 6, 8 = $\overline{2\ \text{axis}}^2$ = the circumscribing square stratum having a depth of unity; and the side = 2 axis.

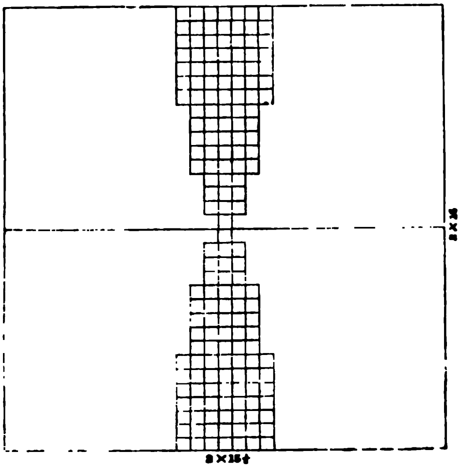


Fig. 28.

Fig. 28. If two series of the cubes of 1, 3, 5, 7, have their axes in the same straight line; then as each series $= 2\overline{\text{axis}}^2 - \text{axis}$, the two series will $= \overline{2\,\text{axis}}^2 - 2$ axis.

Let one side of the circumscribing rectangular stratum = 2 axis, and the other side = 2 axis − 1, then the area of the rectangle will $= \overline{2\,\text{axis}}^2 - 2\,\text{axis}$ = the content of the two series of cubes of 1, 3, 5, 7.

In the *fig.* one side of the rectangle = 2 × 16, and the other = 2 × 15½.

Fig. 29. represents 4 radiations, each formed of two single

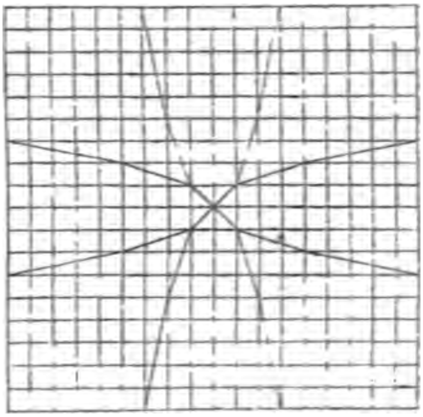


Fig. 29.

obelisks, so that each ray represents 2 obelisks, or each ray represents the breadth of 2 and the depth of 1 obelisk.

Content of a single obelisk $= \frac{1}{2}\overline{\text{axis}}^2 - \frac{1}{6}\,\text{axis}$,

$\therefore$ 8 obelisks $= 4\,\overline{\text{axis}}^2 - \frac{4}{3}\,\text{axis}$,

$= 2\,\overline{\text{axis}}^2 - \frac{4}{3}\,\text{axis}$,

$= \overline{2\,\text{axis}}^2 - \frac{2}{3}2\,\text{axis}$.

The side of the circumscribing square of the 4 radiations = 2 axis. Let this square form a stratum of the depth of unity,

Then $\overline{2\,\text{axis}}^2 - \frac{2}{3}\,2\,\text{axis}$ = square stratum less a line of single cubes of unity extending $\frac{2}{3}$ 2 axis, or $\frac{2}{3}$ side of square: as when axis $= 9$, 2 axis $= 18$, and $18^2 - \frac{2}{3}\,18 = 324 - 12 = 312$ cubes of unity.

When the 4 solid obeliscal series of radiations become 4 solid parabolic series.

Then each parabolic solid will $= \frac{1}{2}\overline{\text{axis}}^2$, and $8 = 4\,\overline{\text{axis}}^2 = \overline{2\,\text{axis}}^2 =$ the circumscribing square stratum having its side $= 2$ axis.

Let $m = 2$ axis, the side of the square stratum circumscribing the series of cubes, obeliscal and parabolic solids.

Then content of 2 series of cubes $= m \times \overline{m-1} = m^2 - m$.

Content of the 4 obeliscal radiations $= m \times \overline{m - \frac{2}{3}} = m^2 - \frac{2}{3}m$.

Content of the 4 parabolic radiations $= m \times m = m^2$.

Fig. 30. In the common multiplication table, called the Pythagorean, the compartments are squares.

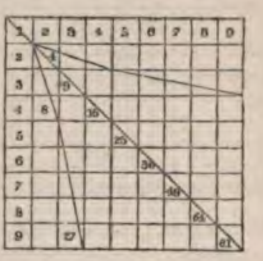


Fig. 30.

The numbers 1, 2, 3, along the top represent the ordinates corresponding to the axes 1, 4, 9 along the side, which $= \overline{\text{ordinate}}^2$.

The numbers 1, 8, 27, at the extremities of the ordinates 1, 2, 3, represent the $\overline{\text{ordinate}}^3$; and 1, 16, 81, along the diagonal, represent the $\overline{\text{ordinate}}^4$.

Sum of $1 + 8 + 16 + 24 = \overline{2n-1}^2 = 7^2 = 49$.

Fig. 31. equals $7^2 = 49$ squares of unity, which square of 7 is composed of the series $7^2 - 5^2$, $5^2 - 3^2$, $3^2 - 1^2$, $1^2 - 0$,
or 24 , 16 , 8 , 1 ,
and sum of $1 + 8 + 16 + 24 = \overline{2n-1}^2 = \overline{8-1}^2 = 7^2 = 49$.

Sum of the series $4+12+20+28=\overline{2n}^2=8^2$.

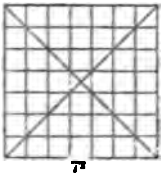


Fig. 31.

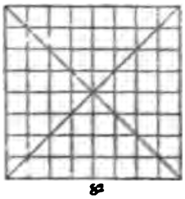


Fig. 32.

Fig. 32. equals $8^2=64$ squares of unity, which square of 8 is composed of the series 8^2-6^2, 6^2-4^2, 4^2-2^2, 2^2-0,

or 28 , 20 , 12 , 4 ,

and sum of $4+12+20+28=\overline{2n}^2=8^2=64$, which also equals the sum of the series $4\ (1+3+5+7)$,

$$=4\times n^2=4\times 4^2=64.$$

Draw the axis and ordinates of *fig.* 7. *a.* like those of *figs.* 1. or 7. Then draw the ordinate at the apex $=6$, the greatest ordinate at the base. By joining this ordinate with the ordinates 1, 2, 3, 4, 5, 6 by lines parallel to the axis, another series of ordinates will be formed, between which will be included the areas

$$1,\ 4,\ 9,\ 16,\ 25,\ 36,$$
$$\text{or } 1^2,\ 2^2,\ 3^2,\ 4^2,\ 5^2,\ 6^2,$$

sum of the series $=\frac{1}{3}\overline{n+1}\,.\,n\,.\,\overline{n+\frac{1}{2}}$.

The series of areas along the axis will equal 0, 3, 10, 21, 36, 55, which are formed by rectangles of the sectional axes and ordinates.

As term $1=\qquad 0,$
$2=\ 3\times1=\ 3,$
$3=\ 5\times2=10,$
$4=\ 7\times3=21,$
$5=\ 9\times4=36,$
$6=11\times5=55.$

The circumscribing rectangled parallelogram including both series will $=$ axis $\times$ ordinate $=\overline{\text{ordinate}}^3=6^3=216$.

The rectangled parallelogram n^3, less the sum of the series $\frac{1}{3}\overline{n+1}\,.\,n\,.\,\overline{n+\frac{1}{2}}$, will $=n$ terms of the series 0, 3, 10.

When $n=6$,
$n^3-\frac{1}{3}\overline{n+1}\,.\,n\,.\,\overline{n+\frac{1}{2}}$ will $=\frac{2}{3}n^3-\frac{1}{2}n^2-\frac{1}{6}n = 216-91 = 125$ $=6$ terms of the series $0+3+10+21+36+55=125$.

Fig. 7. The complementary area of the obeliscal series of squares of 1, 2, 3, 4, 5, 6, 7, 8, formed by rectangles parallel to the axis $=1+3+6+10+15+21+28$, or

$$\begin{array}{r}
1=1\\
1+2=3\\
3+3=6\\
6+4=10\\
10+5=15\\
15+6=21\\
21+7=\underline{28}\\
84=\text{squares of unity.}
\end{array}$$

Here the number of squares $=8$, and complementary rectangles $=7$.

$$\begin{aligned}
\text{The axis} &= \tfrac{1}{2}\overline{n+1}\,.\,n, \text{ here } n=8,\\
&= \tfrac{1}{2}9\times 8=36,\\
\text{and ordinate} &= 8, \text{ the side of 8th square.}
\end{aligned}$$

$\therefore$ the circumscribing rectangled parallelogram

$$\begin{aligned}
&=\tfrac{1}{2}\overline{n+1}\,.\,n\,.\,n,\\
&=36\times 8=288,
\end{aligned}$$

and area of the series of 8 squares

$$=\tfrac{1}{3}\overline{n+1}\,.\,n\,.\,\overline{n+\tfrac{1}{2}}=\tfrac{1}{3}9\times 8\times 8{\cdot}5 = 204,$$

$\therefore$ complementary area

$$\begin{aligned}
&=\tfrac{1}{2}\overline{n+1}\,.\,n\,.\,n-(\tfrac{1}{3}\overline{n+1}\,.\,n\,.\,\overline{n+\tfrac{1}{2}}),\\
&= \quad 288 \quad - \quad 204=84.
\end{aligned}$$

Fig. 8. The complementary area of the obeliscal series of squares of 2, 4, 6, 8, 10, formed by rectangles parallel to the axis $=4+12+24+40$.

$$\begin{array}{lr} \text{As} & 4 \\ & 4+\ 8=12 \\ & 12+12=24 \\ & 24+16=40 \\ \hline & 80. \end{array}$$

Here the number of squares $=5$, and rectangles $=4$.

The axis $=2+4+6+8+10$,
$=\overline{n+1}\,.\,n$, here $n=5$,
$=6\times5 \quad =30$,
and ordinate $=2n=2\times5=10$.

$\therefore$ the circumscribing rectangled parallelogram

$$=\overline{n+1}\,.\,n\,.\,2n,$$
$$=30\times10=300.$$

And area of the series of squares

$$=2^2+4^2+6^2+8^2+10^2,$$
$$=\tfrac{1}{3}\overline{n+1}\,.\,n\,.\,\overline{2n+1}=\tfrac{1}{3}\,6\times5\times11=220.$$

$\therefore$ the complementary area

$$=\overline{n+1}\,.\,n\,.\,2n-(\tfrac{1}{3}\overline{n+1}\,.\,n\,.\,\overline{2n+1}),$$
$$=\quad 300 \quad - \quad 220 \quad = \quad 80.$$

The complementary area $=\ 4+12+24+40$,
$=4\,(1+\ 3+\ 6+10)$.

Fig. 7. The complementary area of the obeliscal series of squares of 1, 2, 3, 4, 5, 6, 7, 8, formed by rectangles parallel to the ordinates equals

$$\begin{array}{r} 1\times7=\ \ 7 \\ 2\times6=12 \\ 3\times5=15 \\ 4\times4=16 \\ 5\times3=15 \\ 6\times2=12 \\ 7\times1=\ \ 7 \\ \hline 84. \end{array}$$

Here the number of squares are 8, and the sides of the 7 rectangles parallel to the axis increase by 1, while the other sides parallel to the ordinates decrease by 1. The differences of the series

7, 12, 15, 16, 15, 12, 7,
are 5, 3, 1, 1, 3, 5.

The complementary area of the obeliscal series of squares of 1, 2, 3, to 12 will be

$$\begin{aligned}
1 \times 11 &= 11 \\
2 \times 10 &= 20 \\
3 \times 9 &= 27 \\
4 \times 8 &= 32 \\
5 \times 7 &= 35 \\
6 \times 6 &= 36 \\
7 \times 5 &= 35 \\
8 \times 4 &= 32 \\
9 \times 3 &= 27 \\
10 \times 2 &= 20 \\
11 \times 1 &= 11 \\
\hline
&\quad 286.
\end{aligned}$$

The differences between the terms of the series

11, 20, 27, 32, 35, 36, 35, 32, 27, 20, 11,
are 9, 7, 5, 3, 1, 1, 3, 5, 7, 9.

Hence, when the first term of the complementary series, which $= n - 1$, is an odd number, the series of differences decreases by the odd numbers from $n - 3$ to unity, and then recommences from unity and increases to $n - 3$.

The area of such a complementary increasing and decreasing series will $= \frac{1}{2}\overline{n+1} \,.\, n \,.\, n \;-\; (\frac{1}{3}\overline{n+1} \,.\, n \,.\, \overline{n+\frac{1}{2}})$,

$$\begin{aligned}
&= \tfrac{1}{2}13 \times 12 \times 12 - \tfrac{1}{3}13 \times 12 \times 12{\cdot}5, \\
&= \quad 936 \quad - \quad 650 \quad = 286, \\
&= \text{axis} \times \text{ordinate} - \text{series of squares.}
\end{aligned}$$

Let n, the number of squares, $= 11$.

Then $n - 1 = 10$, an even number,

and

$$\begin{array}{rcl}
1\times 10 &=& 10\\
2\times 9 &=& 18\\
3\times 8 &=& 24\\
4\times 7 &=& 28\\
5\times 6 &=& 30\\
6\times 5 &=& 30\\
7\times 4 &=& 28\\
8\times 3 &=& 24\\
9\times 2 &=& 18\\
10\times 1 &=& 10\\
\hline
 & & 220
\end{array}$$

The differences between the terms of the series

10, 18, 24, 28, 30, 30, 28, 24, 18, 10

are 8, 6, 4, 2, 0, 2, 4, 6, 8.

Here the complementary series of rectangles $=n-1=10$, an even number, and all the terms are even.

The series of differences begins with $n-3=8$, an even number, and all the terms are even, each in succession decreasing by 2 to 0, and then increasing by 2 to $n-3$, or 8.

The sums of the second series of differences of the odd and even differential numbers are equal;

as 9, 7, 5, 3, 1, 1, 3, 5, 7, 9,

2. difference $=2+2+2+2+0+2+2+2+2=16$,

and 8, 6, 4, 2, 0, 2, 4, 6, 8,

2. difference $=2+2+2+2+2+2+2+2=16$.

When the number of rectangles are odd and $=11$, then $6\times 6=36$ is equidistant from both extremes, being the middle term.

When the number of rectangles are even and $=10$, then $5\times 6=30$ and $6\times 5=30$ are the two nearest the middle, and equidistant, one from one extreme and the other from the other extreme.

Sum of the 11 squares of 1, 2, $3=\frac{1}{3}\overline{n+1}\,.\,n\,.\,\overline{n+\frac{1}{2}}$.

Circumscribing rectangled parallelogram = axis × ordinate $=\frac{1}{2}\overline{n+1}\,.\,n\,.\,n$.

Complementary series of 10 rectangles

$$=\tfrac{1}{2}\overline{n+1}\,.\,n\,.\,n-\tfrac{1}{3}\overline{n+1}\,.\,n\,.\,\overline{n+\tfrac{1}{2}},$$
$$=\tfrac{1}{2}12\times 11^2-\tfrac{1}{3}11\times 11\times 11\cdot5=220.$$

The complementary area of the obeliscal series of squares of 2, 4, 6, 8, 10, formed by the rectangles parallel to the ordinates *fig.* 8. are

$$\begin{array}{r}
2\times 8=16\\
4\times 6=24\\
6\times 4=24\\
8\times 2=16\\
\hline
80
\end{array}$$

Here the number of squares $=5$, and rectangles $=4$.

The complementary area

$$=\overline{n+1}\,.\,n\,.\,2n-(\tfrac{2}{3}\overline{n+1}\,.\,n\,.\,\overline{2n+1})\text{ when }n=5,$$
$$=\quad 300\quad -220=80.$$

When $n=10$, the number of squares, the last term of the series 2, 4, 6 will be 20, and 9 the number of rectangles that form the complementary area, as

$$\begin{array}{r}
2\times 18=\ 36\\
4\times 16=\ 64\\
6\times 14=\ 84\\
8\times 12=\ 96\\
10\times 10=100\\
12\times\ 8=\ 96\\
14\times\ 6=\ 84\\
16\times\ 4=\ 64\\
18\times\ 2=\ 36\\
\hline
660
\end{array}$$

Thus the series of rectangles are formed by each being made equal to the two numbers equally distant from the extremes, or the mean of the series

2, 4, 6, 8, 10, 12, 14, 16, 18.

When n, the number of squares, $=11$, the last term of the series 2, 4, 6, &c. will be 22, and 10 the number of rectangles that form the complementary area, as

$$\begin{array}{r}
2\times 20 = 40\\
4\times 18 = 72\\
6\times 16 = 96\\
8\times 14 = 112\\
10\times 12 = 120\\
12\times 10 = 120\\
14\times 8 = 112\\
16\times 6 = 96\\
18\times 4 = 72\\
20\times 2 = 40\\
\hline
880
\end{array}$$

Sum of 11 squares of 2, 4, $6=\frac{2}{3}\overline{n+1}\,.\,n\,.\,\overline{2n+1}$.

Circumscribing rectangled parallelogram = axis × ordinate $=\overline{n+1}\,.\,n\,.\,2n$.

Complementary series of 10 rectangles

$$\begin{aligned}
&=\overline{n+1}\,.\,n\,.\,2n-\tfrac{2}{3}\overline{n+1}\,.\,n\,.\,\overline{2n+1}\\
&=12\times 11\times 22-\tfrac{2}{3}12\times 11\times 23\\
&=2904-2024=880.
\end{aligned}$$

Or generally the series will be

$$\begin{array}{l}
2\times(2n-2),\\
4\times(2n-4),\\
6\times(2n-6),\\
8\times(2n-8),\ \&c.;
\end{array}$$

and the sum $=\overline{n+1}\,.\,n\,.\,2n-(\frac{2}{3}\overline{n+1}\,.\,n\,.\,\overline{2n+1})$, where n = the number of squares of 2, 4, 6, &c. that form the obeliscal series, and $\overline{n-1}$ the number of rectangles that form the complementary area.

Fig. 22. If the obeliscal series were formed of $1^4, 2^4, 3^4, 4^4, 5^4, 6^4$, the axis would $=1^2+2^2+3^2+4^2+5^2+6^2$, and the area of the series of squares

$$=\tfrac{1}{5}(\overline{n+1\,.\,n}^{\,2}\,.\,\overline{n+\tfrac{1}{2}}-\tfrac{1}{3}\overline{n+1}\,.\,n\,.\,\overline{n+\tfrac{1}{2}}),$$

the circumscribing rectangled parallelogram would = axis × ordinate; here ordinate $=6^2=n^2$.

Therefore the complementary area would be known, which, if formed by a series of 5 rectangles between the ordinates, would be

$$\begin{array}{ll} 1^2\times(6^2-1^2) & \text{or } 1\times 35=\ 35 \\ 2^2\times(6^2-2^2) & \quad 4\times 32=128 \\ 3^2\times(6^2-3^2) & \quad 9\times 27=243 \\ 4^2\times(6^2-4^2) & \quad 16\times 20=320 \\ 5^2\times(6^2-5^2) & \quad 25\times 11=245 \end{array}$$

or generally

$$\begin{array}{l} 1^2\times(n^2-1^2) \\ 2^2\times(n^2-2^2) \\ 3^2\times(n^2-3^2) \\ 4^2\times(n^2-4^2) \\ 5^2\times(n^2-5^2),\ \&c. \end{array}$$

where $n=$ the number of squares that form the obeliscal series, and $n-1$ the number of rectangles that form the complementary obeliscal area.

If the obeliscal series of squares were 2^4, 4^4, 6^4, 8^4, the axis would $=2^2+4^2+6^2+8^2$, the area of the series of squares would

$$=\tfrac{8}{5}(\overline{n+1\,.\,n}^2\,.\,\overline{2n+1}-\tfrac{1}{3}\overline{n+1}\,.\,n\,.\,\overline{2n+1}),$$

and circumscribing rectangled parallelogram = axis × ordinate; consequently the complementary area would be known, which may be formed by a series of rectangles between the ordinates equal to

$$\begin{array}{lll} 2^2\times\overline{8^2-2^2} & \text{or generally} & 2^2\times(2n^2-2^2) \\ 4^2\times\overline{8^2-4^2} & & 4^2\times(2n^2-4^2) \\ 6^2\times\overline{8^2-6^2} & & 6^2\times(2n^2-6^2),\ \&c. \end{array}$$

where $n=$ the number of squares forming the obeliscal series, and $n-1$ the number of rectangles that form the complementary area. The ordinate will $=\overline{2n}^2$.

Fig. 5. The complementary area of the obeliscal series of squares of 1, 3, 5, 7, 9, 11, to 6 terms, formed by 5 rectangles parallel to the axis $=2+8+18+32+50$,

for
$$\begin{aligned} 1\times 2 &= 2=1^2\times 2,\\ 4\times 2 &= 8=2^2\times 2,\\ 9\times 2 &= 18=3^2\times 2,\\ 16\times 2 &= 32=4^2\times 2,\\ 25\times 2 &= 50=5^2\times 2. \end{aligned}$$

The axis $=n^2$, and ordinate $=2n-1$, therefore circumscribing rectangled parallelogram $=n^2 . \overline{2n-1}$, here $n=6$,

$$=36\times 11=396,$$

and area of the series of 6 squares,

or $1^2+3^2+5^2+7^2+9^2+11^2=\frac{4}{3}n^3-\frac{1}{3}n,$

$$=\tfrac{4}{3}6^3-\tfrac{1}{3}6=286.$$

Therefore the complementary area

$$\begin{aligned} &=n^2 . \overline{2n-1}-(\tfrac{4}{3}n^3-\tfrac{1}{3}n),\\ &=\quad 396 \quad -286=110. \end{aligned}$$

Or the area of the series of 6 squares

$$=1^2+3^2+5^2+7^2+9^2+11^2=286.$$

The complementary area $=2(1^2+2^2+3^2+4^2+5^2)=110.$ Therefore the area of the circumscribing rectangled parallelogram $=1^2+3^2+5^2+7^2+9^2+11^2,$

$$+2\ (1^2+2^2+3^2+4^2+5^2)=286+110+396.$$

Fig. 5. The complementary area of the obeliscal series of squares of 1, 3, 5, 7, 9, 11, when formed by a series of rectangles parallel or between the ordinates are

$$\begin{aligned} 1\times 10 &= 10\\ 3\times 8 &= 24\\ 5\times 6 &= 30\\ 7\times 4 &= 28\\ 9\times 2 &= 18\\ & \overline{110} \end{aligned}$$

1, 3, 5, 7, 9, 11, being the sides of the 6 squares parallel to the axis; 2, 4, 6, 8, 10, numbers between them; and 10, 8 6, 4, 2, the sides of the 5 rectangles parallel to the ordinates.

Here each rectangle is formed by an odd and even number; the number of squares $= n$, and number of rectangles $= n - 1$. Sum of the series $= n^2 (2n - 1) - (\frac{4}{3}n^3 - \frac{1}{3}n)$ = rectangled parallelogram less series of squares,

$$= 6^2 \times 11 - (\tfrac{4}{3}6^2 - \tfrac{1}{3}6),$$
$$= 396 - 286 = 110.$$

The duplicate ratio.

1st power	1,	2,	3,	4,	5,	6.
2nd "	1,	4,	9,	16,	25,	36.
3rd "	1,	8,	27,	64,	125,	216,
4th, &c.						

In each of the powers—

The first term : the 4th in the duplicate ratio of the first to the second.

As $1 : 4 :: 1 : 2^2$, also 1st : 2nd :: 2nd : 4th,
$1 : 16 :: 1 : 4^2$, as $1 : 2 :: 2 : 4$,
$1 : 64 :: 1 : 8^2$. $1 : 4 :: 4 : 16$,
$1 : 8 :: 8 : 64$.

The 1st : the 9th in the duplicate ratio of the 1st : 3rd.

The 1st : the 16th in the duplicate ratio of the 1st : 4th.

In the geometrical progression of 1, 2, 4, 8, 16.

1st : last :: first^2 : mean^2.
1st : 2nd :: 2nd : 3rd.
1st : 3rd :: 3rd : 5th.
1st : 4th :: 4th : 8th.

The 1st : 3rd in the duplicate ratio of the 1st : 2nd. The 1st : 5th in the duplicate ratio of the 1st : 3rd. The 1st : 8th in the duplicate ratio of the 1st : 4th.

To construct the Pylonic Curve that shall have its Ordinate varying inversely as $D^{\frac{1}{2}}$, *from the Apex of the Obelisk, and the same Axis common to the Curve and the Obelisk.*

Fig. 34. Let the common axis of the obelisk and the curve = 81; then the last ordinate of the obelisk will = 9.

Make the first ordinate of the curve at the apex of the obelisk = 9, which will represent the mean time in which the first unit, or sectional axis 1, is described in the first second, so the

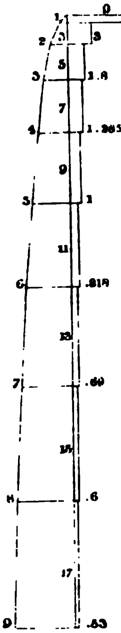


Fig. 34.

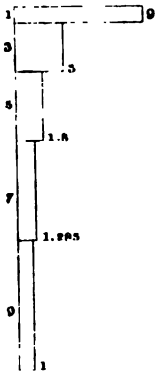


Fig. 34. *a.*

axis 1 will represent the velocity of the first second, then $v \times t = 1 \times 9 = 9 =$ a rectangled parallelogram having an area $= 9$. As the sectional axes of the obelisk are as 1, 3, 5, 7, &c., the distances described in each successive second, those axes will denote the velocities during those seconds, since $v \propto D^{\frac{1}{2}} \propto$ ordinate obelisk, and each of these axes being = the two ordinates by which it is bounded, = twice the mean ordinate of each section, = the mean velocity of each second, or the distance described in each successive second when a body falls freely near the earth's surface.

As $t \propto \frac{1}{v} \propto \frac{1}{D^{\frac{1}{2}}} \propto \frac{1}{\text{ordinate obelisk}} \propto$ ordinate of curve, $v \times t$ will always equal a constant quantity $= 9$, the area of the first rectangled parallelogram. Hence the ordinates of the curve corresponding to the sectional axes 1, 3, 5, 7, 9, &c., will be as 9, $\frac{9}{3}$, $\frac{9}{5}$, $\frac{9}{7}$, $\frac{9}{9}$, &c.

So that these ordinates of the curve will $\propto$ inversely as the sectional axes 1, 3, 5, &c.

During the descent, the velocity with which unity is described along the axis 1, will be to the velocity with which unity is described along the 5th axis $=9$, as $1 : 9$. So that the velocity through axis 9 will be 9 times greater than the velocity through axis 1.

The time t corresponding to these velocities will $\propto$ inversely as the velocities, or as $9 : 1$. So that the time of describing unity along the axis 1 will be 9 times greater than the time of describing unity with the mean velocity of the 5th second along the axis 9.

The central unit of each sectional axis 1, 3, 5, 7, &c. will be described with the mean velocity of the corresponding second, and the time of describing any central unit will be the mean of the times in which the units along that sectional axis are described.

Since time $t \propto \dfrac{1}{\mathrm{V}}$

and T the time of descent $\propto$ V

$$\therefore \mathrm{T} \propto \frac{1}{t}$$

Or t the time of describing unity at any distance $\propto$ inversely as T, the time of descent to that distance.

If an ordinate t, at the 1st axis 1, be made $= 9$ to represent the time t in which unity is described in the 1st section 1, an ordinate $t=1$ will represent the time t of describing one of the nine units in the 5th sectional axis 9 with the mean velocity of that section.

The 1st ordinate $t=9$ and $\mathrm{V}=1$
5th ,, $=1$ and $\mathrm{V}=9$

In the 1st section $t \times \mathrm{V}=9\times 1=9$.
5th ,, $t \times \mathrm{V}=1\times 9=9$.

Or time t of describing unity in the 1st section : time t of describing unity in the 5th section :: 9 : 1; and velocity with which unity is described in the 1st section : velocity with which unity is described in the 5th section :: 1 : 9.

When, as in this *Fig.* 34, the 1st ordinate $t =$ the last ordinate of obelisk $= 9$, the sectional axes 1, 3, 5, 7, &c. will $= 9$ in number, and axis of obelisk $= \overline{\text{ordinate}}^2 = 9^2 = 81$.

The mean time t in describing unity in any sectional axis will $= 9$ divided by that axis.

When the 1st time t ordinate = the n^{th} ordinate of the obelisk $= n$, the time t of describing unity in the 1st sectional axis will be to the time t of describing unity in the last sectional, or n^{th} axis,

as $$\frac{n}{1} : \frac{n}{2n-1},$$

as $$2n-1 : 1.$$

The times t and corresponding velocities will be represented by a series of equal rectangled parallelograms described along the sectional axes, so that each of the sectional axes 1, 3, 5, 7, &c., will represent the velocity, and the corresponding t ordinates the mean time t in which unity is described in a section, and $t \times \text{V}$ will always $= 9$.

In 1st sectional axis $\text{V} = 1$ and ordinate $t = \frac{9}{1} = 9$
2 " $= 3$ " $= \frac{9}{3} = 3$
3 " $= 5$ " $= \frac{9}{5} = 1{\cdot}8$
4 " $= 7$ " $= \frac{9}{7} = 1{\cdot}285$
5 " $= 9$ " $= \frac{9}{9} = 1$
6 " $= 11$ " $= \frac{9}{11} = {\cdot}818$
7 " $= 13$ " $= \frac{9}{13} = {\cdot}69$
8 " $= 15$ " $= \frac{9}{15} = {\cdot}6$
9 " $= 17$ " $= \frac{9}{17} = {\cdot}53$

Since velocity $\propto \text{D}^{\frac{1}{2}}$, the sectional axes 1, 3, 5, &c., are described in equal times; hence the mean ordinate t, which $\propto$ inversely as the sectional axes, will describe equal areas, or equal rectangled parallelograms in equal times.

At the 9th ordinate the series of rectangled parallelograms described will $= 9$, and the area of the whole $= 9 \times 9 = 9^2 = 81$

= the square of the 9th ordinate of obelisk, or 1st ordinate $t = \frac{1}{9}$ the circumscribing rectangled parallelogram which equals axis × ordinate $= 9^2 \times 9 = 9^3$.

The series of rectangled parallelograms, when placed one above another, will form an Egyptian or Cyclopian door, gateway, vaulted roof, or arch, and each rectangled parallelogram will extend beyond the one below by a distance $=2$. By making the first ordinate $t=n$, a variety of such arches may be formed.

Since the t ordinate $\propto$ inversely as the sectional axes 1, 3, 5, &c., and each sectional axis = twice the mean ordinate of the obelisk.

Therefore t ordinate will vary inversely as the mean ordinate, $\overline{\text{axis}}^{\frac{1}{2}}$, or $\text{D}^{\frac{1}{2}}$.

If each rectangled parallelogram along the sectional axes be supposed to be described uniformly, each unit of a sectional axis would be described in equal times, corresponding to the mean t ordinate of the section. But the t ordinate at the beginning of each section, reckoning from the apex of the obelisk, will be greater than the m t ordinate, and at the end of the section the t ordinate will be less than the m t ordinate, since velocity continually increases.

During the descent by the action of gravity, the T ordinate, or ordinate of the obelisk will $\propto \text{D}^{\frac{1}{2}}$ and describe a curvilinear obeliscal or parabolic area. So the t ordinate, which $\propto$ inversely as the ordinate of obelisk, will describe a curvilinear or pylonic area, in which each sectional area will have a greater breadth at the end nearer the apex, and a less breadth at the end further from the apex than the length of the m t ordinate.

The obeliscal sectional areas $\propto 1^2, 3^2, 5^2$, &c. The series of rectangled parallelograms along the pylonic sectional axes are equal.

So the obeliscal series will $\propto$ directly as the squares of the sectional axes. The pylonic series of rectangled parallelograms will vary both directly and inversely as the sectional axes, $\propto 1$.

The obeliscal series $=\frac{2}{3}n^3-\frac{1}{6}n$.

The pylonic series$=n^2=$the square of the 1st t ordinate, or the last ordinate of the obelisk.

The ordinate of obelisk $\propto \overline{\text{axis}}^{\frac{1}{2}}$ will, during the descent, generate a curvilinear obeliscal or parabolic area; while the t ordinate will generate a curvilinear area, similar to the outline, or section of the massive curved cornice projecting from an Egyptian propylon. Hence the curve traced by the t ordinate may be called the pylonic curve.

If the last ordinate of the obelisk $=$ the first ordinate of the pylonic area, the common axis will $= n^2$, and the area of the series of rectangled parallelograms along the sectional axes will $= n^2$.

The circumscribing rectangled parallelogram of the obelisk or pylonic area will $= n^3$.

In *Fig.* 34. the series of rectangled parallelograms have been constructed to the 9th ordinate, the end of the common axis, but they may be continued along the produced axis. Thus the areas of the rectangled parallelograms, how numerous soever they may be, will all be equal.

The pylonic curve will be continually approaching to the axis, and to each other if a similar curve were constructed on the other side of the axis, while the sides of two obeliscal areas will be continually receding from each other and from the axis, but still continually approaching to parallelism with each other, with the axis, and with the pylonic curve.

As the sectional axes 1, 3, 5, 7, &c., are described in equal times, the series of equal rectangled parallelograms or equal areas, along the sectional axes, would be described in equal times by the m t ordinates of the sections. But during the descent the time t ordinate continually varies, so the area described will be curvilinear.

Generally, when the last ordinate n of the obelisk is made the first ordinate t of the pylonic area, each rectangled parallelogram will $= n$ squares of unity, $=$ a line of squares of unity of the length of the ordinate $n=n\times 1=n$.

Sum of the series of rectangled parallelograms $= n^2 = \overline{\text{ordinate}}^2 =$ axis, or a line of square units of the length of the axis.

Circumscribing rectangled parallelogram = axis × ordinate $= n^2 \times n = n^3$.

An area of square units the length of the axis and breadth of the ordinate; or an area of square units $= n$ times the $\overline{\text{ordinate}}^2$.

Circumscribing square = an area of square units n times the circumscribing rectangled parallelogram $= n$ times the axis × ordinate $= n \times n^2 \times n = n^2 \times n^2 = n^4 = \overline{\text{axis}}^2$.

Hence we may say,

area of a rectangled parallelogram $= n = \text{ordinate} = \overline{\text{axis}}^{\frac{1}{2}}$
series of rectangled parallelograms $= n^2 = \overline{\text{ordinate}}^2 = \text{axis}$
circumscribing rectangled parallelogram $= n^3 = \overline{\text{ordinate}}^3 = \overline{\text{axis}}^{\frac{3}{2}}$
circumscribing square $= n^4 = \overline{\text{ordinate}}^4 = \overline{\text{axis}}^2$

It follows that when $\text{V} \propto \frac{1}{t} \propto \text{D}^{\frac{1}{2}}$,

$$\text{T} \propto \frac{\text{D}}{\text{V}} \propto \text{D} \times t$$

$$\text{D} \propto \text{T} \times \text{V} \propto \frac{\text{T}}{t}$$

and $\text{T} \propto \text{V}$.

Having found the sum of the series of squares of 1, 4, 9, 16, 25, 36, or of $1^4, 2^4, 3^4, 4^4, 5^4, 6^4$, when placed along an axis, *fig.* 22.

Let each square in this series be made a square stratum of the depth of unity, and placed in the order 36, 25, 16, 9, 4, 1, such a series of square strata will form a solid like a teocalli, *fig.* 23.; the height, 6, will = the square root of the side of the base or of the lowest terrace, 36, and

content $= \frac{1}{5}(\overline{n+1\,.\,n}^2\,.\,\overline{n+\frac{1}{2}} - \frac{1}{3}\overline{n+1}\,.\,n\,.\,\overline{n+\frac{1}{2}})$ cubes of unity.

The content of a pyramid having its base = the side of a cube, and the height or axis = the length of the side of the cube, will $= \frac{1}{3}$ the content of the cube.

A cube has 6 square sides all equal. Suppose 6 axes to radiate from the centre, and the axes to be rectangular to each other, or perpendicular to the sides of the supposed cube.

Then let 6 pyramids having axes of equal length be generated by square ordinates $\propto \overline{\text{axis}}^2$, or $\overline{\text{distance}}^2$ from that central point or common apex; these 6 pyramids will have equal square bases and equal heights, so they will be equal to each other, and these 6 bases will form the 6 sides of a cube having a content = the content of the 6 pyramids.

Let the cube be divided into 2 equal rectangled parallelopipeds by a plane parallel to one of the sides of the cube; then each rectangled parallelopiped will = the content of 3 pyramids, one of which pyramids will be entire.

As each pyramid $= \frac{1}{6}$ the content of the cube, this pyramid will $= \frac{1}{3}$ the content of the rectangled parallelopiped $= \frac{1}{3}$ area of base of rectangled parallelopiped multiplied by the height.

So the content of pyramid having the same base and height as the rectangled parallelopiped will $= \frac{1}{3}$ the content of the circumscribing rectangled parallelopiped.

Or a pyramid having the same base and twice the height will $= \frac{1}{3}$ the circumscribing cube.

The horn of Jupiter Ammon, like the ammonite, represents the spiral obelisk, and is typical of infinity.

PART II.

HYPERBOLIC SERIES.—SERIES OF $1, \frac{1}{2}, \frac{1}{3}$, &C., $1, \frac{1}{2}, \frac{1}{2^2}, \frac{1}{2^3}$, &C., $1, \frac{1}{3}, \frac{1}{3^2}, \frac{1}{3^3}$, &C.—HYPERBOLIC RECIPROCAL CURVE FROM WHICH IS GENERATED THE PYRAMID AND HYPERBOLIC SOLID, THE ORDINATES OF WHICH VARY INVERSELY AS EACH OTHER, THAT OF THE PYRAMID VARIES AS D^2, THAT OF THE HYPERBOLIC SOLID VARIES AS $\frac{1}{D^2}$.—SERIES $1^2, 2^2, 3^2$, &C., AND $1, \frac{1}{2^2}, \frac{1}{3^2}$, &C.—THE HYPERBOLIC SOLID WILL REPRESENT FORCE OF GRAVITY VARYING AS $\frac{1}{D^2}$ OR VELOCITY VARYING AS $\frac{1}{D^2}$.—TIME t WHICH VARIES AS D^2 WILL BE REPRESENTED BY THE ORDINATE OF PYRAMID, OR BY THE SOLID OBEBISK.—GRAVITY REPRESENTED SYMBOLICALLY IN HIEROGLYPHICS BY THE HYPERBOLIC SOLID.—THE OBELISK REPRESENTS THE PLANETARY DISTANCES, VELOCITIES, PERIODIC TIMES, AREAS DESCRIBED IN EQUAL TIMES, TIMES OF DESCRIBING EQUAL AREAS AND EQUAL DISTANCES IN DIFFERENT ORBITS HAVING THE COMMON CENTRE IN THE APEX OF THE OBELISK.—THE ATTRIBUTES OF OSIRIS SYMBOLISE ETERNITY.

Hyperbolic Areas and Solids.

LET *fig.* 37. be a series of 6 rectangled parallelograms, all of equal areas and rising from the side or base of the 1st, which is a square, and the side of the square to $=6$, then the area will $=36$; the height of the second rectangled parallelogram $=2 \times 6$, and breadth $=\frac{1}{2}6$, then $12 \times 3 = 36$; 3rd rectangled parallelogram $=18 \times 2$; 4th, $=24 \times 1{\cdot}5$; 5th, $=30 \times 1{\cdot}2$; 6th, $=36 \times 1$.

Or 1st, $=6 \times 6$; 2nd, $=2 \times 6 \times \frac{1}{2}6$; 3rd, $=3 \times 6 \times \frac{1}{3}6$; 4th, $=4 \times 6 \times \frac{1}{4}6$; 5th, $=5 \times 6 \times \frac{1}{5}6$; 6th, $=6 \times 6 \times \frac{1}{6}6$.

So that the axis of each rectangled parallelogram $\propto$ D, and ordinate $\propto \frac{1}{D}$.

The area of each rectangled parallelogram = 6.

Hence it follows that the ordinates will be bounded on one side by the asymptote, and on the other by the hyperbolic curve; or the series of rectangled parallelograms will be an hyperbolic series. The 1st ordinate will = 6, the whole axis or asymptote $= 6^2$, and the area of the series of rectangled parallelograms $= 6^2 \times 6 = 6^3$, the circumscribing rectangled parallelogram.

Next take the areas between every two of these ordinates in succession, and let $n = 6$.

These different areas so cut off will form another series of rectangled parallelograms, which will be as 1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$ of n^2. So that if

$n^2 = \ldots\ldots\ 6^2 = \ldots\ldots\ 36$
„ „ $\frac{1}{2}6^2 = \ldots\ldots\ 18$
„ „ $\frac{1}{3}6^2 = \ldots\ldots\ 12$
„ „ $\frac{1}{4}6^2 = \ldots\ldots\ 9$
„ „ $\frac{1}{5}6^2 = \ldots\ldots\ 7{\cdot}2$
„ „ $\frac{1}{6}6^2 = \ldots\ldots\ \underline{6}$
$88{\cdot}2$

Or the area of the series of rectangled parallelograms $= 88{\cdot}2$.

Let the equal sides of such a series of rectangled parallelograms be placed in the same straight line or axis, *fig.* 36. Then

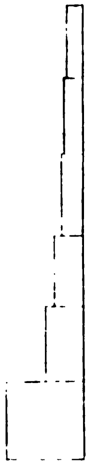


Fig. 36.

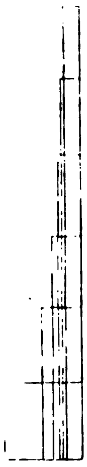


Fig. 37.

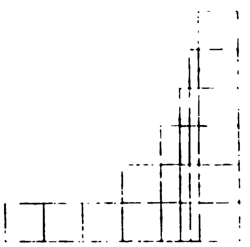


Fig. 38.

as in *fig.* 37. the rectangle contained by each of the ordinates $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}$ of 6, and the corresponding axes 1, 2, 3, 4, 5, 6, will be equal, and the ordinates will $\propto$ inversely as the axes. Hence this series of rectangled parallelograms will $\propto$ as the series of rectangled parallelograms inscribed in an hyperbolic area between the curve and the asymptote, when the asymptotes are rectangular, *fig.* 39.

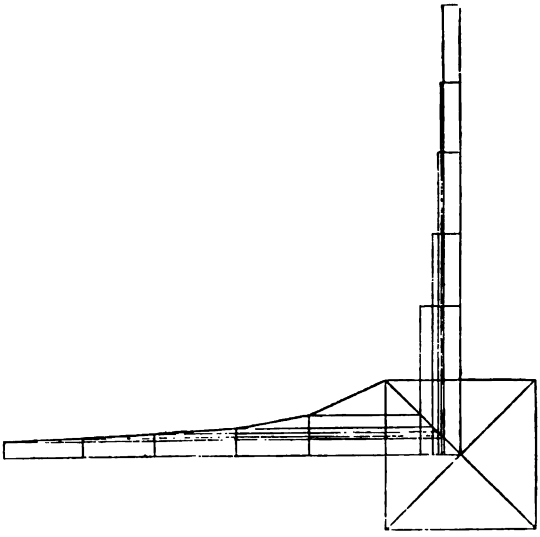


Fig. 39.

In order to approximate the series of rectangled parallelograms nearer to that of the hyperbolic area, it will be necessary to add a series of 5 triangles between the ordinates. Now the sum of the bases of these triangles will $=6-1=5$, and the height of each triangle $=6$.

$\therefore$ area of the triangles will $=\frac{1}{2}5\times 6=15=\frac{1}{2}\overline{n-1}\,.\,n$ generally.

The area of the 6 rectangled parallelograms, or 6 rectangles + the area of 5 triangles will = the rectangular area when the angular recesses are filled up, as *fig.* 39.

Thus the rectilinear area, like the obeliscal area bounded by straight lines, will = the series of 6 rectangled parallelograms and 5 triangles $=n$ rectangled parallelograms $+\overline{n-1}$ triangles less $\frac{1}{2}$ the 1st square, $\frac{1}{2}n^2$, the square is common to both series along the two rectangular asymptotes. But the area of the triangles $= \frac{1}{2}\overline{n-1} \,.\, n = \frac{1}{2}n^2 - \frac{1}{2}n$ can never $= \frac{1}{2}n^2$, $\frac{1}{2}$ the 1st square, though the series of triangles will continually approach to equality with $\frac{1}{2}n^2$ as n increases.

Hence the series of n rectangled parallelograms, which includes the whole square, will be the limit to which the rectilinear area, including $\overline{n-1}$ triangles, $\overline{n-1}$ rectangled parallelograms, and $\frac{1}{2}n^2$, or $\frac{1}{2}$ the 1st square, continually approaches as n, the number of terms of the series $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}$, &c., of n^2, increases.

Thus the rectilinear area, which includes $\overline{n-1}$ triangles, $\overline{n-1}$ rectangled parallelograms, and $\frac{1}{2}$ the square, will continually approach to equality with the series of n rectangled parallelograms, which includes the whole square, since the area of the triangles continually approach to $\frac{1}{2}n^2$, or $\frac{1}{2}$ the 1st or central square, common to both series of rectangled parallelograms along the two rectangular asymptotes.

For when $\overline{n-1}$ triangles are included with $\overline{n-1}$ rectangled parallelograms only $\frac{1}{2}$ the square is included.

But when $\overline{n-1}$ rectangled parallelograms are excluded, the whole square is included with $\overline{n-1}$ rectangled parallelograms.

Fig. 38. If a series of rectangled parallelograms have their ordinates as $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}$, &c. of n, and the breadth of each $= 1$, the first ordinate will $= n$, the last $= \frac{1}{n}n = 1$, and the height or axis will $= n \times 1 = n$. The first in the series will be a rectangled parallelogram $= n \times 1 = n$, and the last will be a square, having the side $= \frac{1}{n} \,.\, n = 1$, and area $= 1^2 = 1$.

Let the 1st ordinate $= 6$, then axis $= 6$,
2nd ,, $\frac{1}{2} = 3$
3rd ,, $\frac{1}{3} = 2$
4th ,, $\frac{1}{4} = 1{\cdot}5$
5th ,, $\frac{1}{5} = 1{\cdot}2$
6th ,, $\frac{1}{6} = 1$
$14{\cdot}7$.

Since each rectangled parallelogram has a breadth of 1, the area of the series will $= 14{\cdot}7$.

When $n = 9$, the series of rectangled parallelograms will $= 25{\cdot}46$.

When $n = 12$, the series of rectangled parallelograms will $= 37{\cdot}273$.

When $n = 18$, the series of rectangled parallelograms will $= 61{\cdot}91$.

When $n = 24$, the series of rectangled parallelograms will $= 89{\cdot}816$ by addition.

Also $2 \times (\tfrac{1}{2}\,\overline{n+1}\,.\,n)^{\frac{2}{3}} = 89{\cdot}6$.

Fig. 36. Next, let each rectangled parallelogram have a breadth of n; then the 1st in the series will be a square $= n^2$, and the last a rectangled parallelogram $= 1 \times n = n$.

When $n = 6$, the sum of the series will $= 6 \times 14{\cdot}7 = 88{\cdot}2$.

When $n = 9$, the sum of the series will $= 9 \times 25{\cdot}46$.

When $n = 12$, the sum of the series will $= 12 \times 37{\cdot}273$.

In these series the 1st ordinate $= n$,

nth „ $= \frac{1}{n} \,.\, n = 1$,

axis $= n \times n = n^2$.

The 1st rectangle or square in the series $= n^2 = \overline{\text{greatest ordinate}}^{\,2} =$ axis.

When $n = 9$, the 1st ordinate $= 9$, and the last ordinate $= 1$; the sum of the series $= 25{\cdot}46$.

1st ordinate		$= 9$	9
2nd	„	$= \frac{1}{2}$	4·5
3rd	„	$= \frac{1}{3}$	3
4th	„	$= \frac{1}{4}$	2·25
5th	„	$= \frac{1}{5}$	1·8
6th	„	$= \frac{1}{6}$	1·5
7th	„	$= \frac{1}{7}$	1·285
8th	„	$= \frac{1}{8}$	1·125
9th	„	$= \frac{1}{9}$	1
			25·46

Then $(\tfrac{1}{2}\overline{n+1}\,.\,n)^{\frac{2}{3}} \times 2$

$= (\tfrac{1}{2}\,10 \times 9)^{\frac{2}{3}} \times 2 = 45^{\frac{2}{3}} \times 2 = 12{\cdot}65 \times 2$

$= 25{\cdot}3$, which is less than

$25{\cdot}46$, the sum by addition.

When $n=12$, the 1st ordinate $=12$, the last ordinate $=1$, and the sum of the series by addition $=37{\cdot}273$,

$$\begin{aligned} &\text{and } (\tfrac{1}{2}\,\overline{n+1}\,.\,n)^{\frac{2}{3}}\times 2 \\ &=(\tfrac{1}{2}\,13\times 12)^{\frac{2}{3}}\times 2=78^{\frac{2}{3}}\times 2 \\ &=18{\cdot}25\times 2=36{\cdot}5, \end{aligned}$$

which is less than $37{\cdot}273$.

When $n=18$, the first ordinate $=18$, the last $=1$, and the sum of the series by addition $=61{\cdot}91$.

$$\begin{aligned} \text{Also}\quad &(\tfrac{1}{2}\,\overline{n+1}\,.\,n)^{\frac{2}{3}}\times 2 \\ &=(\tfrac{1}{2}\,19\times 18)^{\frac{2}{3}}\times 2 \\ &=30{\cdot}8\times 2=61{\cdot}6, \end{aligned}$$

which is less than $61{\cdot}91$.

When $n=24$, the first ordinate $=24$, the last $=1$, and the sum of the series by addition $=89{\cdot}816$.

$$\begin{aligned} \text{Also}\quad &(\tfrac{1}{2}\,\overline{n+1}\,.\,n)^{\frac{2}{3}}\times 2 \\ &=(\tfrac{1}{2}\,25\times 24)^{\frac{2}{3}}\times 2 \\ &=300^{\frac{2}{3}}\times 2 \\ &=44{\cdot}8\times 2=89{\cdot}6, \end{aligned}$$

which is less than $89{\cdot}816$.

But $(\tfrac{1}{2}\overline{n+1}\,.\,n)^{\frac{2}{3}}\times 2$ is only an approximation to the sum of the series $1+\frac{1}{2}+\frac{1}{3}$ &c. of n, when n is a low number; for as n increases, the expression fails in giving proximate results. So in order to sum the series, recourse may be had to other methods. Hence, if the area between the asymptote and curve be found, the area between the two asymptotes, less the area between the asymptote and curve, will = the area of the hyperbola.

The asymptote multiplied by the last ordinate = the 1st square, or rectangled parallelogram. Asymptote : 1st ordinate :: 1st ordinate : last ordinate, or $n^2 : n :: n : 1$.

But (*fig.* 38.) the 1st ordinate and whole axis are equal, and the rectangled parallelograms along the ordinate and axis are also equal, for their breadth = 1.

Then asymptote : 1st ordinate :: breadth of 1st rectangled parallelogram : breadth of rectangled parallelogram along the asymptote :: 1 : 1.

The series $1+\frac{1}{2}+\frac{1}{3}$, &c. can be geometrically represented, but we cannot sum it like the others, and are not prepared to show what other methods of calculation were used by the ancients.—(See *Fluxions.*)

From recent researches, the Indians appear to have been particularly attached to the study of algebra, in which they made great progress. Davis and Delambre think the Hindoo method of calculation essentially different from the Grecian. Jones informs us that it is very improbable the Indians should have borrowed anything from the Greeks, as the pride of the Brahmins leads them to despise foreign nations in general, and the Greeks in particular.

To sum the series,

$$1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\frac{1}{64}$$

$$\text{or } 1+\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\frac{1}{2^4}+\frac{1}{2^5}+\frac{1}{2^6}$$

Here the sum of all the terms after the first term will, though indefinitely continued, never equal the first term 1.

$$\text{Since } \frac{1}{2}+\frac{1}{4}=\frac{3}{4}$$

$$\frac{3}{4}+\frac{1}{8}=\frac{7}{8}$$

$$\frac{7}{8}+\frac{1}{16}=\frac{15}{16}$$

$$\frac{15}{16}+\frac{1}{32}=\frac{31}{32}$$

$$\frac{31}{32}+\frac{1}{64}=\frac{63}{64}$$

Thus the sum of

$$\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\frac{1}{64}\right)=\frac{63}{64}$$

and $\frac{63}{64}$ = the sum of 6 terms

$$=\frac{2^6-1}{2^6}=\frac{2^n-1}{2^n} \text{ generally}$$

$$\therefore S. \text{ of } 1 + \left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\frac{1}{64}\right)=1+\frac{63}{64}$$

$$= 1 + \left(\frac{2^n-1}{2^n}\right)$$

Or the sum of all the denominators in the whole series $\frac{1}{1}+\frac{1}{2}+\frac{1}{4}$, &c., except the last, will form the numerator 63, and the denominator of 63 equals the denominator of the last term $\frac{1}{64}$.

Hence the sum of all the terms in the direct series $1+2+4+8+16+32+64$ will = twice the last term less one,

$= 63 + 64 = 127$,

= numerator + denominator of the sum of the series $= \frac{63}{64}$.

The sum of all the terms after the n^{th} term will never equal the n^{th} term.

The sum of the series

$$1+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\frac{1}{81}+\frac{1}{243}+\frac{1}{729}+\frac{1}{2187}$$

$$\text{or } 1+\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\frac{1}{3^4}+\frac{1}{3^5}+\frac{1}{3^6}+\frac{1}{3^7}$$

will never $= 1\frac{1}{2}$, or the sum of all the terms after the first will never $= \frac{1}{2}$, since

$$\frac{1}{3}+\frac{1}{9}=\frac{4}{9}=\frac{12}{27}$$

$$\frac{12}{27}+\frac{1}{27}=\frac{13}{27}=\frac{39}{81}$$

$$\frac{39}{81}+\frac{1}{81}=\frac{40}{81}=\frac{120}{243}$$

$$\frac{120}{243}+\frac{1}{243}=\frac{121}{243}=\frac{363}{729}$$

$$\frac{363}{729}+\frac{1}{729}=\frac{364}{729}=\frac{1092}{2187}$$

$$\frac{1092}{2187}+\frac{1}{2187}=\frac{1093}{2187}=\frac{3279}{6561}$$

$$\frac{3279}{6561}+\frac{1}{6561}=\frac{3280}{6561},\text{ which is less than } \tfrac{1}{2}.$$

Thus sum of all terms after the n^{th} term will never $= \frac{1}{2}$ the n^{th} term.

Sum of n terms of the series $\frac{1}{3}+\frac{1}{9}$, &c., will $= \dfrac{\frac{1}{2}3^n-\frac{1}{2}}{3^n}$

$$=\tfrac{1}{2}\times\frac{3^n-1}{3^n}$$

The reciprocal curve of contrary flexure is determined by the reciprocals of the sines of the quadrant, and the hyperbolic series of parallelograms is formed by the sines and their reciprocals.

Fig. 40. Draw parallel and equidistant lines. At any radius, 9, describe a quadrant; then, where the arc intersects the 8th line, through that point, A, draw a straight line from the centre C, cutting the 9th line in B. Draw DAE parallel to C 9, then by similar triangles,

$$\text{AE} : \text{AC} :: \text{AD} : \text{AB}$$

$$\text{or } 8 : 9 :: 1 : \text{AB}$$

$$\text{AB} = \frac{9}{8} = \frac{1}{8}\text{ of } 9$$

$$\text{and AE} \times \text{AB} = \text{AC} \times \text{AD}$$

$$\text{or } 8 \times \frac{9}{8} = 9 \times 1 = 9,$$

or sine AE multiplied by its reciprocal AB = 9. Similarly

$$\text{F G} = \frac{9}{7} = \frac{1}{7}\text{ of } 9.$$

$$\text{and FG} \times \text{FH} = \frac{9}{7} \times 7 = 9.$$

So the remaining reciprocals, radiating from the centre C, multiplied by their respective sines 6, 5, 4, &c. will each = 9.

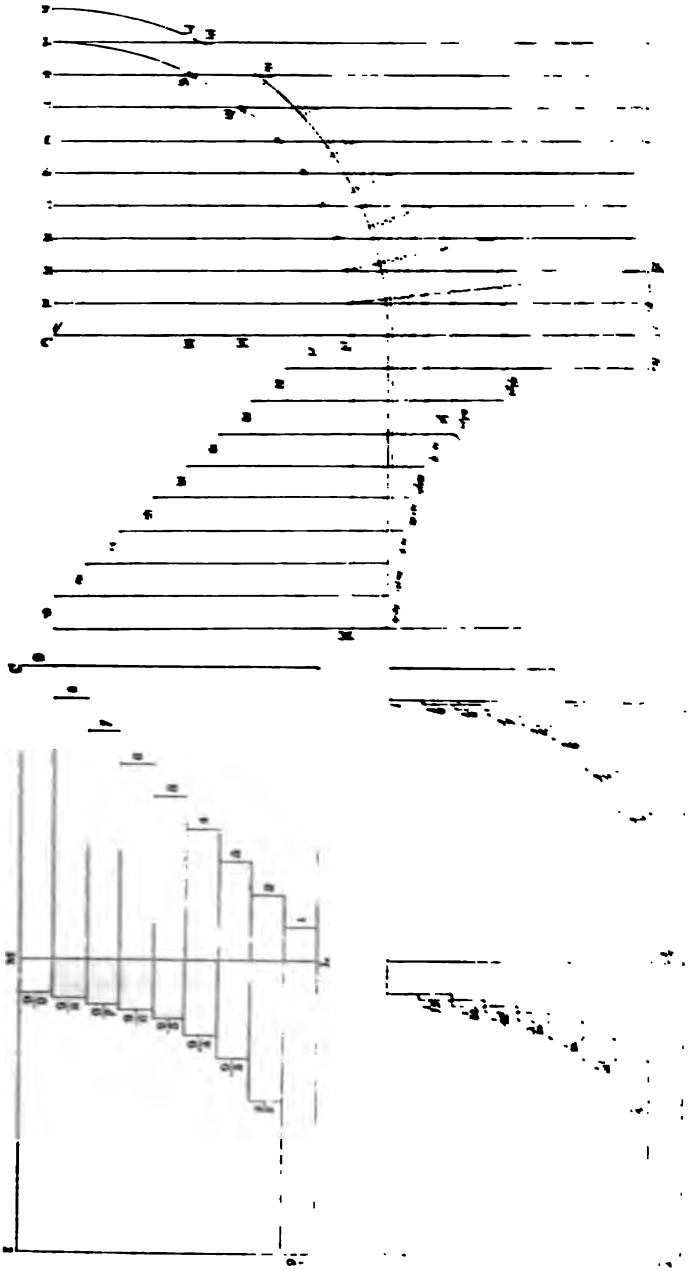


Fig 40.

The extremities of these reciprocal sines will trace a curve of contrary flexure, beginning at $9 + 1$, or 10, and terminating at $\text{CK} = \text{CI} = \text{CL} + \text{LI} = 9 + 9 = 18$, or twice the radius, and K will be in the second line. With radii C 10, CB, CG, &c., describe circular arcs which will cut LI, = 9, at the distances from L of $\frac{1}{9}$, $\frac{1}{8}$, $\frac{1}{7}$, $\frac{1}{6}$, $\frac{1}{5}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$, 1 of 9 or LI.

Let LM be drawn parallel and = C 9, and similarly divided. From the points of division draw lines parallel to LI, which will cut at right angles the straight lines drawn from the points, at $\frac{1}{9}$, $\frac{1}{8}$, $\frac{1}{7}$, &c. of LI, the terminations of the circular arcs; these lines will be respectively as 9, 8, 7, 6, 5, 4, 3, 2, 1, and will form with the lines drawn from LM a series of rectangular parallelograms which will form a hyperbolic area of parallelograms included by the two asymptotes LI, LM, each of which = 9, for the greatest ordinate and greatest axis become asymptotes. The least ordinate at M or I = 1, and the greatest ordinate at N, for this double hyperbolic area, will be 3, the side of the central or angular square; then $1 : 3 :: 3 : 9$,
or least : greatest .: greatest ordinate : asymptote.

The hyperbolic curve will be determined by the series of equal parallelograms inscribed between the curve and the asymptotes. Since the area of each of the 9 parallelograms in the series = 9, their whole area will $= 9 \times 9 = 9^2 =$ the area of the square that circumscribes the series of parallelograms arranged in hyperbolic order. But when so arranged the parallelograms overlap, or partially cover each other, so that the parallelogram along one asymptote, or side of the square, which $= 1 \times 9$, or 9, has only $\frac{1}{9}$ of 9, or 1 square of unity exposed, $\frac{8}{9}$ being concealed below the next parallelogram, and this parallelogram is again partially covered by the next, and so on in succession, the last only being entirely exposed, so that the sum of those exposed, or superficial areas = the area of the hyperbolic series of parallelograms.

Thus a series of parallelograms having each an equal area, and the area of the whole series being equal the square of the asymptote, can be so arranged that the superficial area of the series shall form an hyperbolic area, having the side of the circumscribing square equal the asymptote of the hyperbola. The area of such a series of parallelograms will $= 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}$, &c. of 9. *Fig.* 40.

$$\begin{array}{ccccc} \text{Radius}^2 & - & \text{sine}^2 & - & \text{cosine}^2 \\ 9^2 & - & 8^2 & = & 17 \\ 9^2 & - & 7^2 & = & 32 \\ 9^2 & - & 6^2 & = & 45 \\ 9^2 & - & 5^2 & = & 56 \\ 9^2 & - & 4^2 & = & 65 \\ 9^2 & - & 3^2 & = & 72 \\ 9^2 & - & 2^2 & = & 77 \\ 9^2 & - & 1^2 & = & 80 \\ 9^2 & - & 0^2 & = & 81 \end{array}$$

Cosine² 81, 80, 77, 72, 65, 56, 45, 32, 17

Difference 1, 3, 5, 7, 9, 11, 13, 15

The cosines² decrease from the arc towards the centre, while their differences increase as the odd numbers 1, 3, 5, &c.

If the 9th ordinate of the obelisk represent radius, the remaining 8 ordinates will represent the sines, and the difference between their squares will = 81, 80, 77, &c., = the axes between the ordinates 1, 2, 3, &c. and ordinate 9. Again the difference between the terms of the last series will = the sectional axes 1, 3, 5, &c.

The reciprocal of the sine also $= \left(1 + \frac{\text{cosine}^2}{\text{sine}^2}\right)^{\frac{1}{2}}$

$$\begin{aligned} \text{For radius} &= \text{sine} \times \text{reciprocal}, \\ \text{radius}^2 &= \text{sine}^2 \times \text{reciprocal}^2, \\ \text{and radius}^2 &= \text{sine}^2 + \text{cosine}^2, \\ \therefore \text{reciprocal}^2 &= \frac{\text{sine}^2 + \text{cosine}^2}{\text{sine}^2} \\ &= 1 + \frac{\text{cosine}^2}{\text{sine}^2} \\ \text{reciprocal} &= \left(1 + \frac{\text{cosine}^2}{\text{sine}^2}\right)^{\frac{1}{2}}; \end{aligned}$$

so that when the 9th ordinate of the obelisk is made the radius of the quadrant, the other ordinates, 8, 7, 6, &c., will be as the sines.

Fig. 41. The 9 rectangled parallelograms having their lengths = the cosines, or = the square root of 17, 32, 45, &c., and the breadth of each = unity, will circumscribe the quadrantal arc, and the first 8 of the series of the 9 rectangled parallelograms will be inscribed within the quadrantal arc.

The quadrantal area will = the sum of the series of such inscribed rectangled parallelograms $+\frac{1}{n}$ radius^2. For the difference between the eight inscribed parallelograms and the nine parallelograms that circumscribe the quadrant = the nine parallelograms along the arc = the last parallelogram c 1 $= 9 \times 1 = 9 = \frac{1}{9}\,\overline{\text{radius}}^2$

Let the radius be divided into ninety equal parts, then the difference will $= \frac{1}{90}\,\overline{\text{radius}}^2$; when the radius is divided into 900 equal parts, the difference of the two series of rectangled parallelograms will $= \frac{1}{900}\,\overline{\text{radius}}^2$. Generally, the difference will $= \frac{1}{n}\,\overline{\text{radius}}^2$. For parallelogram c 1 will = radius $\times \frac{1}{n}$ radius $= \frac{1}{9}$ radius^2. Hence, as n increases, the differential series of rectangled parallelograms will become evanescent, and the series of inscribed rectangled parallelograms will approach nearer and nearer to equality with the quadrantal area. Since the quadrantal area = the series of inscribed parallelograms + only half the evanescent series of parallelograms. For the diagonals of the differential parallelograms may ultimately be regarded as portions of the quadrantal arc.

Thus, a Cyclopian arch may be constructed so that the semicircle shall touch the angular projections of the arch. (*Fig.* 41.)

By varying the value of *n* in the hyperbolic series of rectangled parallelograms, different Egyptian or Cyclopian hyperbolic arches may be constructed.

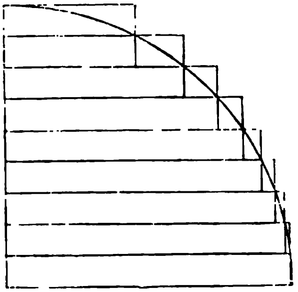


Fig. 41.

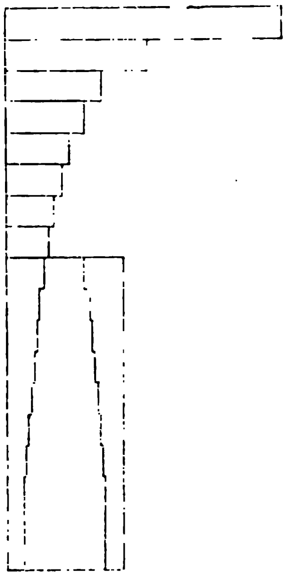


Fig. 42.

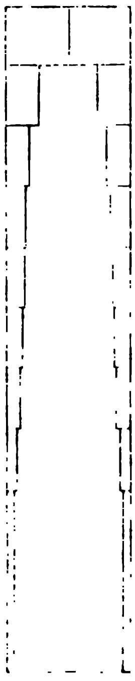


Fig. 43.

Fig. 42. is formed from the lower section of an hyperbolic series of rectangled parallelograms.

Fig. 43. is formed by the hyperbolic series of rectangled parallelograms; the first in the series is a square.

Figs. 42. and 43. form hyperbolic galleries. That of 42. corresponds with the view of a gallery in the interior of the Pyramid of Cheops given by the French writers.

The sides of the hyperbolic series of rectangled parallelograms formed within the square IM (*fig.* 40.) have their sides 1, 2, 3, &c., along the axis LM parallel and equal to the sines; and the other sides of the rectangled parallelograms, which are at right angles to the sines, are equal to the reciprocals of the sines, and form the reciprocals of the sides 1, 2, 3, &c., of the rectangled parallelograms.

Hence, as the reciprocals of the sines, which determine the curve 10 BGK, form the reciprocals of the hyperbolic series of rectangled parallelograms, we may call this curve the hyperbolic reciprocal curve of contrary flexure.

Having shown that the obelisk represents the laws of motion when a body falls near the earth's surface, or when a planet revolves in its orbit, we shall next attempt, by means of the pyramidal and hyperbolic temples, to interpret the ancient theory of the laws of gravitation when a body is supposed to fall from a planetary distance to a centre of force.

With this view the velocity will first be supposed to $\propto \frac{1}{D^2}$, but afterwards in a greater inverse ratio.

The pyramidal may not accord with the Newtonian theory of gravitation. We may not have interpreted the pyramid correctly; but now we are unable to revise what has been done.

The pyramid, like the obelisk, still points to the heavens as an enduring record of the laws of gravitation, though it has ceased to be intelligible for countless ages.

If velocity $\propto \frac{1}{D^2}$, the square of the reciprocal ordinates within the square IM will represent the variation of the velocity; and the square of the sines or ordinates within the square CM will represent the variation of the time t, which $\propto \frac{1}{V} \propto D^2$.

When each of the sines or sides 1, 2, 3, &c. of the rectangled parallelograms and their reciprocals 1, $\frac{1}{2}, \frac{1}{3}$ of 9, or $\frac{9}{1}, \frac{9}{2}, \frac{9}{3}$, &c., are in the same straight line; but divided by the axis LM, common to both series, the line of sines 1, 2, 3, &c., will trace a triangular area = $\frac{1}{2}$ the square CM or IM; and the line of reciprocals the hyperbolic area within the square IM.

The sine on one side of the axis multiplied by its reciprocal on the other side, or the distance from L multiplied by its reciprocal ordinate, will always = a constant quantity 9 = the area of an inscribed hyperbolic rectangled parallelogram.

Hence the axis or distance = the sine, or corresponding ordinate of the triangle, and varies inversely as the ordinate of the hyperbolic area, or the reciprocal of the sine or axis: or ordinate of hyperbolic area $\propto$ inversely as the distance from L.

Suppose the hyperbolic ordinate to be made = the square of the linear ordinate: such a square ordinate will vary inversely as $\overline{\text{ordinate}}^2$ of triangle or inversely as $\overline{\text{distance}}^2$; and $D^2 \times$ hyperbolic $\overline{\text{ordinate}}^2$ will always equal a constant quantity $= 9^2 = \overline{\text{axis}}^2 =$ the circumscribing square CM, or the square IM that contains the hyperbolic series of rectangled parallelograms.

The axis being divided in 9 = parts, let the sphere of attraction have the centre of force in L, and the semi-diameter = 1, one of the 9 equal parts of the axis.

Then if a body descending to the centre of force L, with a velocity $\propto \frac{1}{D^2}$ from L, should, at the distance of 9, or the 9th ordinate from the centre, have a velocity represented as 1^2, and that velocity should be continued uniformly through a semi-diameter = 1, along the axis from the 9th to the 8th ordinate, the solid thus generated by the velocity ordinate $= 1^2$ would be represented by $1^2 \times 1$, 1^3, or a cube of unity.

Since velocity $\propto \frac{1}{\text{D}^2}$, the corresponding t ordinate, the reciprocal of the velocity ordinate will $\propto \text{D}^2$. Hence the square stratum generated by the corresponding t ordinate $\propto \text{D}^2$ on the other side of the axis, will $= 9^2 \times 1 =$ a stratum having an area $= 9^2$ and a depth of 1, $= 81$ cubes of unity. In the descent through each successive semi-diameter, or 1, the rectangle by the velocity ordinate and the t ordinate will $= \overline{\text{axis}}^2 = 9^2 = 81$, and $81 \times 1 = 81$ cubes of 1.

At the distance of 1 from the centre of force the velocity ordinate will be represented by 9^2, and the corresponding time t ordinate by 1^2. If these two ordinates descended to the centre of force with the acquired velocity, continued uniform, then the respective strata so generated would be 81 and 1 cube of unity; but the body cannot descend beyond the surface, or circumference of the spheres, at the distance of 1 from the centre.

The area of the series of rectangular parallelograms, 1, 2, 3, &c., $= \frac{1}{2}\overline{n+1} \,.\, n = \frac{1}{2}\overline{n^2 + \frac{1}{2}n}$, as n increases by subdivision of the same axis or radius, the series will approach to $\frac{1}{2} n^2$, the area of the triangle.

Or the value of n varies inversely as the number of parts into which the same axis or radius is divided; but $\frac{1}{2}n^2$ still $= \frac{1}{2}\overline{\text{axis}}^2 = \frac{1}{2}\overline{\text{radius}}^2$, and $\frac{1}{2}n = \frac{1}{n}$ area of $\frac{1}{2}\ \overline{\text{axis}}^2$; which becomes evanescent as n increases numerically, and vanishes when ordinate of triangle continually $\propto$ axis or distance from the apex. Or $\frac{1}{2}\, n = \frac{1}{2}\, n$ squares of unity (*Fig.* 7-2.).

Hence as the series of rectangular parallelograms approaches to a triangular area, so will the hyperbolic series of parallelograms approach to an hyperbolic area.

In the same manner the series of strata generated by the $\overline{\text{ordinates}}^2$ of pyramid and hyperbolic solid will approach to a rectilinear pyramid and curvilinear hyperbolic solid.

When the ordinate of triangle $\propto$ D, and ordinate of hyperbola $\propto \frac{1}{\text{D}}$, each of their rectangles, or ordinate of triangle × by ordinate of hyperbola = area of the corresponding parallelogram inscribed along the axis = 9. The sum of the areas of the series of parallelograms $= 9 \times \; = 9^2 = \overline{\text{axis}}^2$; and triangle generated by ordinate $\propto \text{D} = \frac{1}{2} \overline{\text{axis}}^2$.

When the ordinate of pyramid $\propto \text{D}^2$, and ordinate of hyperbolic solid $\propto \frac{1}{\text{D}^2}$, their product = the circumscribing square $= 9^2 = \overline{\text{axis}}^2$, and as each of the 9 square strata has a depth of unity, the sum of the series of square strata will $= 9^2 \times 9 = 9^3 = \overline{\text{axis}^3}$; and $\frac{1}{3} \overline{\text{axis}}^3$ = pyramid generated by *t* ordinate $\propto \text{D}^2$ from the apex.

The content of the stratified pyramid $= 1^2 + 2^2 + 3^2 + 4^2$, &c.

$$= \tfrac{1}{3} \overline{n+1}\,.\; n.\; \overline{n+\tfrac{1}{2}} = \tfrac{1}{3} n^3 + \tfrac{1}{2} n^2 + \tfrac{1}{6} n$$

$$= \text{pyramid} + \text{triangle} + \tfrac{1}{6}\,\text{axis}.$$

For $\frac{1}{3} n^3$ = content of the rectilinear pyramid.

$\frac{1}{2} n^2$ = content of the triangular stratum of the depth of 1.

$\frac{1}{6} n$ = a line or column of cubes of $1 = \frac{1}{6}$ axis in length.

If the same axis be continually divided, or n continually increased, the triangular stratum will become thinner, and so will the line of cubes $= \frac{1}{6}$ axis. Thus they will ultimately become evanescent as the content of the stratified pyramid approaches to equality with the rectilinear pyramid, $\frac{1}{3} n^3$, $\overline{\text{or } \tfrac{1}{3} \text{axis}}^3$, and vanish when the ordinate continually $\propto$ as $\overline{\text{axis}}^2$.

The solid = the $\overline{\text{axis}}^3 = n^3$ will always remain the same how much soever the axis be subdivided.

A pyramid having the sides of the rectangular base as $\overline{n+1}$ by $\overline{n+\frac{1}{2}}$, and axis $= n$, will = the stratified pyramid $= \frac{1}{3} \overline{n+1}\,.\, n\,.\, \overline{n+\frac{1}{2}}$, $= 1^2 + 2^2 + 3^2 + 4^2$, &c., each stratum having the depth of 1.

(*Fig.* 43. *a.*) The two triangles are similar, equal and invariable, each having the axis divided into 9 = parts; the distance between the apices = 1. The circumscribing triangle includes 8 parallelograms ; the sum of which

$= \frac{1}{2}\overline{n+1} \,.\, n = \frac{1}{2}9 \times 8 = 36$

area triangle $=$ „ $= \frac{1}{2}9^2 = 40{\cdot}5$

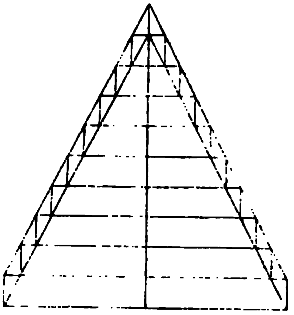


Fig. 43. a.

therefore $40{\cdot}5 - 36 = 4{\cdot}5$, the area of the 2×8 triangles cut off from the series of 8 parallelograms by the lower triangle. Thus the triangular area exceeds the series of 8 parallelograms by 4·5.

Or triangular area : difference of areas :: $\frac{1}{2}9^2 : 4{\cdot}5 :: 40{\cdot}5 : 4{\cdot}5 :: 9 : 1$.

When the axis of each triangle is divided into 81 equal parts, the distance between the apices $= \frac{1}{81}$ axis.

The series of 80 parallelograms $= \frac{1}{2}\overline{n+1} \,.\, n = \frac{1}{2}80 \times 81 = 3240$.

Area triangle $= \frac{1}{2}\text{axis}^2 = \frac{1}{2}81^2 = 3280{\cdot}5$, therefore $3280{\cdot}5 - 3240 = 40{\cdot}5$.

Or triangular area : difference of areas :: $\frac{1}{2}81^2 : 40{\cdot}5 :: 3280{\cdot}5 : 40{\cdot}5 :: 81 : 1$.

The two triangles being always invariable and each $= \frac{1}{2}$ axis^2.

When axis $= 9$, difference of areas $= \frac{1}{9}$ triangle

„ $= 9^2 = 81$ „ $= \frac{1}{9^2}$

„ $= 9^3 = 729$ „ $= \frac{1}{9^3}$

„ $= 9^4 = 6561$ „ $= \frac{1}{9^4}$

„ $= 9^n$ „ $= \frac{1}{9^n}$

When axis $= 9$, distance between apices $= \frac{1}{9}$ axis

„ $= 9^2$ „ $= \frac{1}{9}$

„ $= 9$ „ $= \frac{1}{9^n}$

The distance between the bases of the 2 triangles = the distance between their apices.

Next let a series of 9 instead of 8 parallelograms be described, then the area of the series will exceed that of the triangle.

For area of 9 parallelograms $=\frac{1}{2}\overline{n+1}\,.\,n=\frac{1}{2}10\times 9=45$

Area of triangle „ $=\frac{1}{2}9^2 \quad =40{\cdot}5$

therefore $45-40{\cdot}5=4{\cdot}5$, the area of the 2×9 triangles, the excess of the 9 parallelograms above the triangle $=\frac{1}{2}\text{axis}^2$. So the excess of the parallelograms over the invariable triangle will be $\frac{1}{9}$ triangle when axis = 9

$\frac{1}{9^2}$ „ $=9^2$

$\frac{1}{9^n}$ „ $=9^n$

Hence the more the axis is subdivided the less will be the difference between the parallelograms and triangle, and the apices of the two triangles will approach each other, as will their bases, so that their coincidence will be the limiting ratio of the two series of parallelograms to equality with the invariable triangle $=\frac{1}{2}\text{axis}^2$, or to the triangle generated by ordinate $\propto$ axis and area $=\frac{1}{2}\text{axis}^2$.

Also the triangle circumscribing the 8 parallelograms will ultimately coincide with the triangle described within the 9 parallelograms.

If instead of areas, solids be represented, then we shall have a pyramidal series of strata continually approaching to the content of rectilinear pyramid as their limiting ratio; or to the pyramid generated by the ordinate $\propto t \propto \text{D}^2$.

If the ordinate of obelisk be made the axis, the corresponding square ordinate will vary as D^2, and a pyramid will be generated.

The $\overline{\text{ordinate}}^{\,2}$ of obelisk will represent the ordinate t, which $\propto \text{D}^2$, corresponding to the hyperbolic $\overline{\text{ordinate}}^{\,2}$, which represents the velocity ordinate that $\propto \frac{1}{\text{D}^2}$ (*fig.* 44.)

When the ordinates of obelisk 1, 2, 3, &c. are made the common axis of the pyramid and hyperbolic solid; and the ordinates 1, 2, 3, at right angles to the axis are also made = the distances 1, 2, 3, then the axes 1, 2, 3, to 9 will represent the distances, and 1^2, 2^2, 3^2 will represent the square ordinates corresponding to these distances. Therefore these ordinates will $\propto$ as the $\overline{\text{distance}}^2$.

So that if a body fall along the axis with a velocity $\propto \frac{1}{D^2}$, these ordinates 1^2, 2^2, 3^2, &c., will represent the variation of the time t corresponding to the velocity, which square ordinate t will, during the descent, generate a pyramid; while the corresponding square ordinates of the hyperbola which $\propto \frac{1}{D^2}$ will generate a hyperbolic solid on the opposite side of the common axis.

Here the ordinate t, or ordinate of pyramid, is represented by a square $= \overline{\text{ordinate}}^2$ of obelisk which = axis of obelisk.

Ordinate t may also be represented by a linear ordinate = axis of obelisk, and velocity ordinate, the reciprocal of the ordinate t, will then be represented by a linear ordinate.

The same results may be obtained by representing the square ordinates in lines; then the rectangle of the two lines will $= \overline{\text{axis}}^2$, which multiplied by 1 will form a stratum of the depth of 1 = the sum of the corresponding hyperbolic and pyramidal strata.

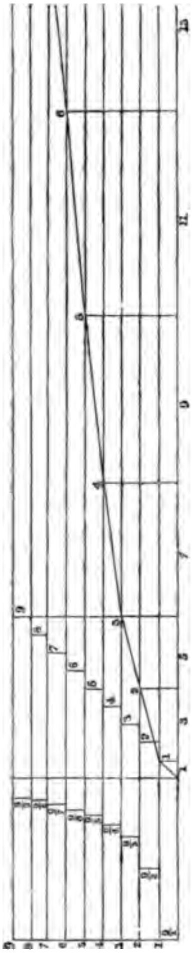


Fig. 44.

Or the lines corresponding to the square ordinates of time and velocity will, in their descent along the axis, generate as many squares of unity as the square ordinates in their descent will generate cubes of unity.

When velocity $\propto \frac{1}{D^2}$, the corresponding *t* ordinate $\propto D^2$. *Fig.* 45.

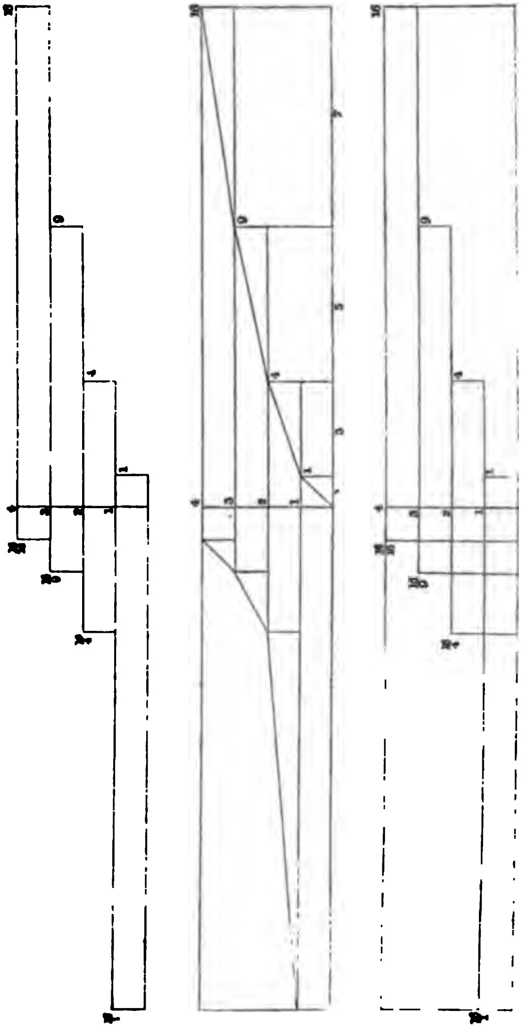


Fig. 45.

Let 1, 2, 3, 4, the distances along the axis common to the velocity and t ordinates, be equal the ordinates 1, 2, 3, 4, of the obelisk, and let 1, 4, 9, 16, the corresponding axes of the obelisk, be the t ordinates of this common axis. So that the ordinates of the obelisk will represent the axes or distances, and the axes of the obelisk will represent the t ordinates which will $\propto$ as the $\overline{\text{distance}}^2$.

The ordinate 16 will represent the t ordinate corresponding to the distance 4; 9 the t ordinate corresponding to the distance 3, and 4, 1, to the distances 2, 1.

Supposing the velocity acquired at the beginning of each distance were continued uniform through the distance of unity, then the corresponding t ordinate will describe the series of rectangled parallelograms 16, 9, 4, 1, and the whole area described will equal $1^2+2^2+3^2+4^2$, or $= \frac{1}{3}\overline{n+1}\,.\,n\,.\,\overline{n+\frac{1}{2}}$ generally.

But as the time t, which $\infty \frac{1}{V}$, is continually varying during the descent, the area described by the t ordinate will be less than the sum of the rectangled parallelograms $1^2+2^2+3^2+4^2$, by the 4 triangles, or by half the sum of $1+3+5+7$, or half the axis × 1, which $=\frac{1}{2}n^2$. To reduce the area to the complementary obeliscal area, a further reduction of $\frac{1}{6}n$ must be made to form the complementary parabolic area, which will be described when the velocity continually $\propto \frac{1}{D^2}$.

As series of rectangled parallelograms $1^2+2^2+3^2+4^2$
$= \frac{1}{3}\overline{n+1}\,.\,n\,.\,\overline{n+\frac{1}{2}} = \frac{1}{3}n^3+\frac{1}{2}n^2+\frac{1}{6}n$
from which take $\frac{1}{2}n^2+\frac{1}{6}n$
then the complementary parabolic area will $=\frac{1}{3}n^3$
or $\frac{1}{3}$ the circumscribing parallelogram.

Hence the whole area described by the t ordinate when it continually varies will = the complementary parabolic area.

The whole time T of descent will $\propto$ the whole area described $\propto$ ultimately as the complementary parabolic area $\propto$ $\frac{1}{3}$ the circumscribing rectangled parallelogram

$\propto$ the whole rectangled parallelogram $\propto$ axis $\times$ ordinate
$\propto n \times n^2 \propto n^3 \propto D^3$
or $\propto D \times t \propto D \times D^2 \propto D^3$.

Or whole time T of descent $\propto$ the cube of the distance described.

On the opposite side of the common axis the velocity ordinate will describe, like the t ordinate, during the descent the series of rectangled parallelograms

$$\tfrac{1}{16}, \tfrac{1}{9}, \tfrac{1}{4}, 1 \text{ of } 16.$$

The whole series thus described by the velocity ordinate will $=1+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}$, or $1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}$of 16, or 4^2, or of n^2 generally.

To make the velocity rectangled parallelograms like the complementary obeliscal area of t, the area of 3 triangles will be required to be added to the velocity rectangled parallelograms at the angles of the series. These 3 triangles will together $=\frac{1}{2}(16-1)=\frac{1}{2}(n^2-1)$.

A further correction will be required to make the velocity area curvilinear, so as to correspond with the complementary parabolic area of t.

The inscribed velocity rectangled parallelograms having their sides each equal unity along the axis, will $=1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}$ of 16.

The inscribed velocity rectangled parallelograms having their sides along the axis $=1, 2, 3, 4$, will equal 1×16, $2 \times \frac{1}{4}$ of 16, $3 \times \frac{1}{9}$ of 16, $4 \times \frac{1}{16}$ of 16, or equal $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ of 16.

The last series of rectangled parallelograms $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ of 16, when they partially cover each other, form the series

$$1, \frac{1}{2^2}, \frac{1}{3^2}, \frac{1}{4^2}, \text{ of } 16,$$

as the series of equal rectangled parallelograms, when they partially cover each other, form the hyperbolic series of rectangled parallelograms $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}$, The greatest time ordinate $=16$, and the least $=1$, at the distance of 1 from the apex or centre of force.

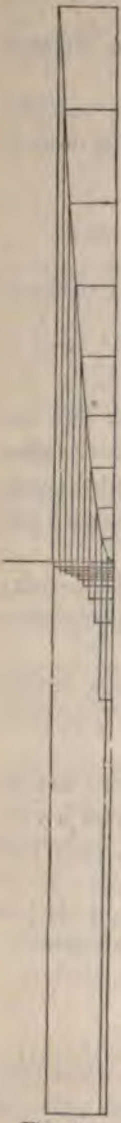


Fig. 46.

The least velocity ordinate $=1$, and the greatest, at the distance of 1 from the apex $=16$.

Fig. 46. The time ordinates form the obelisk like half a canoe, perhaps the sacred boat.

The velocity ordinates form an outline like the section of an architrave and column.

The greatest rectangle, or rectangled parallelogram of one series = the greatest rectangle, or rectangled parallelogram of the other series $=n^2$.

The least rectangle, or square of unity, in one series = the least rectangle, or square of unity in the other series.

The first and greatest rectangled parallelogram in the t series becomes the last in the velocity series. The last and least rectangle, the square of unity, in the t series becomes the first in the velocity series.

The circumscribing rectangled parallelograms of both series are equal.

If instead of the lineal t ordinate, which $\propto D^2$, the t ordinate were a rectangle having the length $\propto D^2$, and the breadth $=$ unity; such an ordinate during the descent would generate a series of rectangled parallelopipedons, the length of each $\propto D^2$ and the breadth and depth of each = unity. These series of rectangled parallelopipedons would each = a square stratum $= \overline{\text{ordinate}}^2$ of obelisk and the depth of unity, and together the series would form a stratified pyramidal solid at the common axis, *fig.* 44., with degrees. The content of the series will $= \frac{1}{3}\overline{n+1} \,.\, n \,.\, \overline{n+\frac{1}{2}}$.

But as the t ordinate will continually vary during the descent, the obeliscal solid will become a rectilinear pyramid, having the height and side of base = the greatest ordinate obelisk and content $= \frac{1}{3}\overline{\text{ordinate}}^3$; which will also = the content of the complementary parabolic stratum when *t* ordinate continually $\propto$ D^2.

Again, if the pyramid were cut by a plane vertical to the base, and made to pass through the apex, the section will represent a triangle, having the height = side of base. A stratum = this triangular area = $\frac{1}{2}\,\overline{\text{axis}}^2$ and having the depth of unity, will = the content of the obelisk having the same height or axis; since content obeliscal parabola = $\frac{1}{2}\overline{\text{axis}}^2$.

Since the obeliscal parabolic area = $\frac{2}{3}\,\overline{\text{axis}}^{\frac{3}{2}}$, let this area become a stratum of the depth of unity. Then such a stratum will = $\frac{2}{3}\,\overline{\text{axis}}^{\frac{3}{2}}$, and the pyramid having the same axis, and side of base = axis, will = $\frac{1}{3}\,\overline{\text{axis}}^3$.

Hence the parabolic stratum will $\propto$ the square root of the content of the pyramid.

The circumscribing parallelogram of the obelisk = axis × ordinate, a rectangled parallelogram, or = $\overline{\text{ordinate}}^3$, a cube.

The stratified rectangled parallelogram of the depth of unity would equal as many cubes of unity as would be contained in the $\overline{\text{ordinate}}^3$.

Thus the circumscribing stratified rectangled parallelogram or $\overline{\text{ordinate}}^3$ would = the square root of the $\overline{\text{axis}}^3$ or of the cube inclosing the pyramid.

Fig. 49. As the obelisk or parabolic solid $\propto \overline{\text{axis}}^2 \propto$ D^2. If the obelisk were placed on its apex like the pyramid, then as the content of obelisk $\propto \overline{\text{axis}}^2 \propto$ D^2 from the apex, wherever the *t* ordinate cuts the obelisk the content of the obelisk intercepted by the *t* ordinate and the apex would $\propto$ D^2 $\propto$ *t* ordinate, or ordinate of pyramid.

The apex of the pyramid or obelisk is placed in the centre of the planet or sun towards which the body falls.

Fig. 47. Let 10 be the side of the central or angular square of a rectangular hyperbolic area. Then axis will = 10^2 = 100, and ordinate of extreme axis, or asymptote, will = unity. The rectangle of asymptote and its ordinate =

100 × 1 = 100, and so will the rectangle of each axis, or distance and its ordinate = 100 = asymptote = 10^2 = central square.

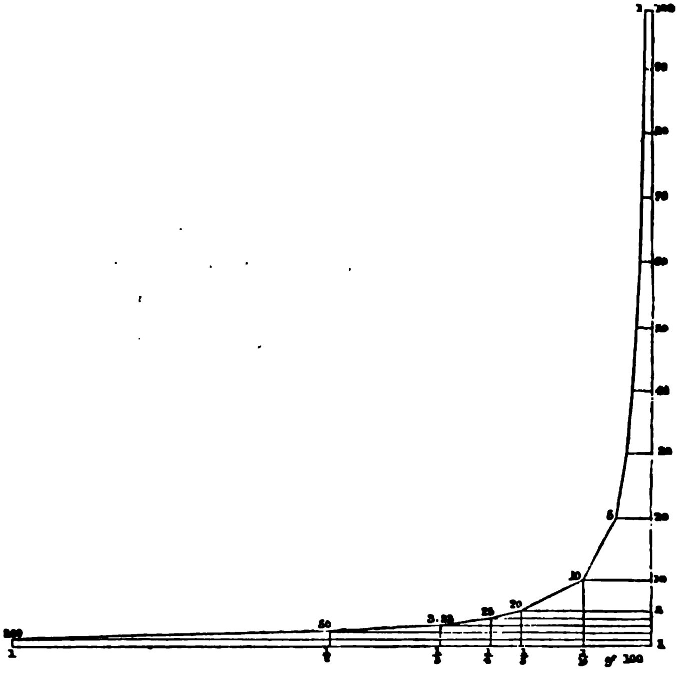


Fig. 47.

This hyperbolic area has 2 asymptotes, one of which is divided into 1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{10}$ of 100; the other is divided into 1, 2, 3, 4, 5, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100 units. The ordinates corresponding to these distances from the centre will be $\frac{100}{1}$, $\frac{100}{2}$, $\frac{100}{3}$, &c. to $\frac{100}{100}$ or 1.

Thus the rectangle of any distance and its ordinate from the centre will = 100.

Also the rectangle of any $\overline{\text{distance}}^2$ and its $\overline{\text{ordinate}}^2$ will = the square of the asymptote = $\overline{100}^2$.

Hence if a body fall from the distance of 100 from the centre, or right angle formed by the asymptotes, with a velocity $\propto \frac{1}{D^2} \propto \overline{\text{ordinate}}^2$, this $\overline{\text{ordinate}}^2$ of the hyperbolic area would during the descent generate a square hyperbolic solid. The *fig.* 47. only represents the outline, or rectilinear hyperbolic area.

The body is supposed to have a velocity of 1 at the distance of 100 from the centre, or angle.

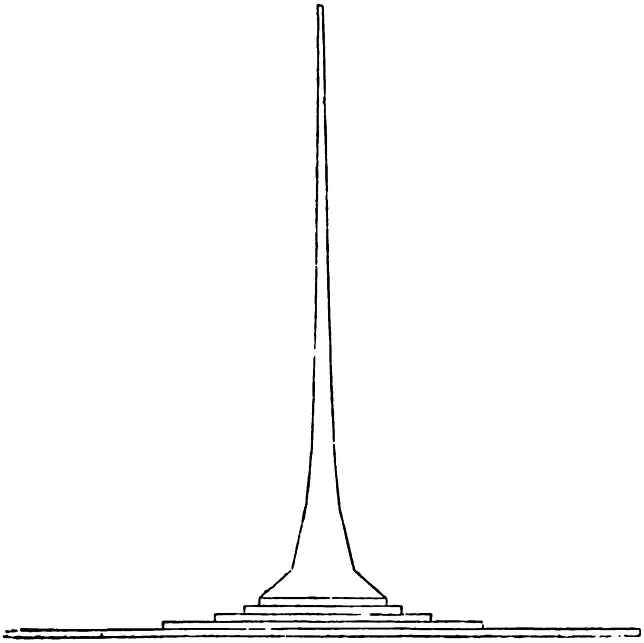


Fig. 48.

If n semi-diameters of the sun were the distance of the earth from the sun, and the velocity. at the earth's orbit equalled 1, the velocity at the surface of the sun would $= n^2$ when velocity $\propto \frac{1}{D^2}$.

For velocity at earth : velocity at sun

$$:: \frac{1}{n^2} : \frac{1}{1^2} :: : 1 : n^2$$

The velocity beyond the earth's orbit will be as $\frac{1}{(n+1)^2}$, $\frac{1}{(n+2)^2}$, &c., or velocity at distance $\overline{n+1}$: velocity at 1, the sun :: $\frac{1}{(n+1)^2} : \frac{1}{1^2} :: 1^2 : \overline{n+1}^2$

The velocity within the earth's orbit will be as $\frac{1}{(n-1)^2}$, $\frac{1}{(n-2)^2}$, &c.

Let one asymptote divide the other asymptote at right angles into 2 equal parts. (*Fig.* 48.) So the ordinate in the descent will also be divided equally by the axis or asymptote; then the solid generated will resemble the outline of a Burmese pagoda with its square terraced base, the sides of the terraces being as 1, $\frac{1}{2}$, $\frac{1}{3}$, of the side of the lowest terrace. The curve begins at the 3rd or 4th terrace, and is continued to the summit of the spire or tee.

These pagodas are solid structures like the pyramids. So that when the velocity $\propto \frac{1}{\text{D}^2}$ the pyramid represents the variation of the time, and the pagoda the variation of the velocity.

Hence both the pyramidal and hyperbolic solid temples have originally been constructed as symbolical of the laws of gravitation.

About one thousand five hundred and ninetieth part of the pyramid of Cheops is occupied by chambers and passages, while all the rest is solid masonry.

Fig. 49. illustrates the velocity $\propto \frac{1}{\text{D}^2}$ in the descent of a body to the centre of force.

The apices of the pyramid and obelisk are both in the centre of force. The ordinate of pyramid, and the solid obelisk itself, both of which vary as D^2 from the centre of force, will both $\propto$ time t; so that the horizontal section or ordinate of pyramid at any point of descent, and the corresponding section of the obelisk intercepted between that point and the apex will both $\propto \text{D}^2 \propto$ time t. The corresponding ordinate of the hyperbolic solid will $\propto \frac{1}{\text{D}^2} \infty$ V corresponding to the time t at the given point.

The hyperbolic solid, a horizontal section of which shows the variation of the velocity, has its base = the base of the pyramid = $\overline{100}^2$, passing through the centre, and its leas ordinate = the square of unity, is in a line with the bases of the pyramid and obelisk. The horizontal section of the pyramid at the orb's surface also equals the square of unity. The axis, common to the obelisk, pyramid, and hyperbolic solid = 100; the side of the base of the pyramid and hyperbolic solid = the common axis = the side of the circumscribing square = 100. The rectangle of the t ordinate and velocity ordinate at any distance = $\overline{100}^2$. At the distance 10 from the centre of force the t and velocity ordinates are equal, each = $\overline{10}^2$, and their rectangle = $\overline{100}^2$ = the area of the circumscribing square. At the beginning of the descent the velocity ordinate × t ordinate = $1^2 \times \overline{100}^2 = \overline{100}^2$. At the surface of the orb, the end of the descent, velocity ordinate × t ordinate = $\overline{100}^2 \times 1^2 = \overline{100}^2$. At 50 from the centre, or half the descent, velocity ordinate × t ordinate = $2^2 \times 50^2 = \overline{100}^2$.

The side of the base, or greatest ordinate of obelisk, = $\overline{\text{axis}}^{\frac{1}{2}}$ = 10 = side of the central or angular square of the hyperbolic area = $\overline{\text{asymptote}}^{\frac{1}{2}} = \overline{\text{axis}}^{\frac{1}{2}} = \overline{\text{side}}^{\frac{1}{2}}$ of the circumscribing square = 10.

The axis × ordinate obelisk = $\overline{\text{ordinate}}^3 = 10^3$ = the circumscribing rectangled parallelogram of the obelisk = $\frac{1}{10}\,\overline{\text{axis}}^2$, or $\frac{1}{10}$ the circumscribing square.

Compare the area of the sections in *fig.* 49. made by a plane, which being at right angles to the sides of the base of the pyramid and obelisk, divides each into two equal parts by passing through their apices.

$$\begin{aligned} \text{Area of obeliscal parabola : area triangle} &:: \tfrac{2}{3}\,\overline{\text{axis}}^{\frac{3}{2}} : \tfrac{1}{2}\,\overline{\text{axis}}^2 \\ &:: \tfrac{2}{3} : \tfrac{1}{2}\,\overline{\text{axis}}^{\frac{1}{2}} \\ &:: 4 : 3\,\overline{\text{axis}}^{\frac{1}{2}} \end{aligned}$$

Fig. 47. The ordinate of the hyperbolic area at the distance of 20 from the apex of the obelisk or centre of force = 5.

At the distance of 25 from the apex the ordinate of the obelisk = 5. Hence the hyperbolic and obeliscal ordinates will become equal between the distances 20 and 25, where the hyperbolic curve will cut the obeliscal or parabolic curve.

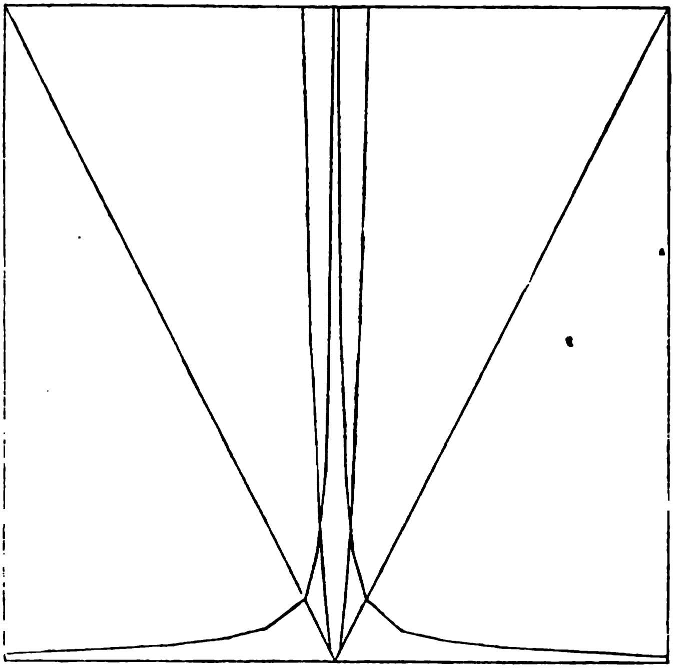


Fig. 49.

Fig. 49. The two hyperbolic curves are continually approaching each other and the common axis; but as the last ordinate of the hyperbolic area $= \frac{1}{n}$ of n; therefore, how great soever the axis n may be supposed, still the ordinate $\frac{1}{n}$ of n will be a definite quantity, and although the curves are continually approaching the axis, and to parallelism with each other, yet they can never meet, nor become parallel.

On the contrary, the sides of the obelisk are continually diverging from each other and the common axis; yet they are continually approaching to parallelism with each other and the axis n; but they never can become parallel, because how far soever they may be extended, still the ordinate $n^{\frac{1}{2}}$ will exceed the ordinate $\overline{n-1}^{\frac{1}{2}}$ of the obelisk.

It follows that although the hyperbolic curves are continually approaching each other, and the sides of the obelisk continually diverging from each other, still the curve and side of the obelisk are continually approaching to parallelism with each other, although they are continually diverging from each other.

The series, if continued beyond n, will become $\frac{1}{n+1}$, $\frac{1}{n+2}$ to $\frac{1}{n+n}$ or $\frac{1}{2n}$, which may again be continued to $\frac{1}{3n}$, and so on to $\frac{1}{n \times n}$ or $\frac{1}{n^2}$, and then again, $\frac{1}{n^3}$, $\frac{1}{n^4}$, &c. Still $\frac{1}{n^4}$ will be greater than 0.

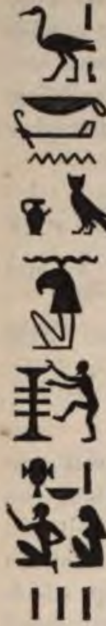
This figure unveils three great enigmas; the obelisk, the pyramid, and hyperbolic solid; temples around which the race who erected them, before history commenced, knelt and looked through Nature up to Nature's God. The Sabæans worshipped these symbols of the laws of gravitation which govern the glorious orb of day, the planetary and astral systems — the grandest and most sublime of the visible works of the Creator. The knowledge of these laws, and of the magnitude, distance, and motion of the heavenly bodies, inspired man with the most exalted feelings of reverence towards the Great First Cause.

The sacred Tau is again represented in *fig.* 49. by the obelisk and hyperbolic solid, as the generators of time, velocity, and distance.

Typhon, the son of Juno, conceived by her without a father, was of a magnitude so vast that he touched the East with one hand and the West with the other, and the heavens with the crown of his head.

If a body be supposed to fall from the earth to the sun, the apex of the obelisk or pyramid would be in the centre of the sun, and the base of the hyperbolic solid, like two arms, would extend from east to west.

The following hieroglyphics, with the translation, is given by Gliddon in his " Ancient Egypt."



May

thy soul

attain (come)

to

KHNUM, (one of the forms of AMON, the creator)

the creator (the idea denoted by a man building the walls of a city)

of all

Mankind, (literally *men* and *women.*)

" May thy soul attain to Khnum, the Creator of all mankind."

Here we find the Creator represented as forming the laws of gravitation, and appears to be in the act of completing a counteracting force, similar, equal, and opposite to the one already made, so that where the central line bisects the distance between the two equal and opposite forces a body would gravitate to neither.

If velocity $\propto \frac{1}{D^2}$, then velocity will $\propto$ ordinate of the hyperbolic column.

If force of gravity $\propto \frac{1}{D^2}$, then force will $\propto$ ordinate of the hyperbolic column.

The effect produced by the action of gravity on a body that begins to fall freely at a distance near the earth's surface is that equal increments of velocity are generated in equal times.

But the effect produced by gravity when a heavy body is freely acted upon by the earth at the distance of the moon from the earth will be different, as unequal increments of velocity will be generated in equal times.

According to Newton the force of gravity varies inversely as the distance squared generally.

Having deduced the properties of the obelisk from the effects produced by gravity acting on a body during its fall near the surface of the earth, let us now endeavour to illustrate the effect produced by gravity generally.

When the body falls from the apex of the obelisk, the distance described is reckoned from the apex, and the velocity acquired, as well as the time, T, elapsed, both vary as $\mathrm{D}^{\frac{1}{2}}$ from the apex.

But the time, t, in describing a small definite distance at any point in the descent $\propto$ inversely as the velocity at that point, or $t \propto \frac{1}{\mathrm{V}}$.

Or t ordinate of the curve $\propto$ inversely as the ordinate of the obelisk.

$$\text{Also } \mathrm{V} \propto \mathrm{D}^{\frac{1}{2}},$$

$$\text{and } t \propto \frac{1}{\mathrm{D}^{\frac{1}{2}}}$$

Newton found that the versed sine of the arc described by the moon in one minute was equal to the distance through which a heavy body at the earth's surface would fall in one second. Therefore the distance through which the latter would fall in one minute would be 3600 times greater than that through which the moon would fall in the same time.

Or, according to Newton, the accelerating force of gravity $\propto \frac{1}{\mathrm{D}^2}$; that is, if the circular motion of the moon were destroyed and the moon descended as a heavy body towards the earth, it would in 1 second describe ·00443 of a foot; a heavy body falling from a state of rest near the earth's surface will describe 16·14 feet in a second.

Now ·00443 × 3600 = 15,948 feet, so that the force of gravity would, at the distance of 60 semi-diameters of the earth from its centre, cause a body to move from a state of rest and describe ·00443 of a foot in one second; while in the same time a body would descend from a state of rest and describe 16·12 feet by the force of gravity at the earth's surface.

Thus gravity is an accelerating force, and is 3600 times greater at the earth's surface than at the distance of the moon. So that if the $\frac{1}{2}$ diameter of the earth be made = unity, this accelerating force will $\propto \frac{1}{\mathrm{D}^2}$.

Hence the figure that represents the velocity at different distances, from the centre of force to the moon's orbit, will also correspond to the force of gravity at the same distances.

According to Newton, the times wherein any bodies would fall to the centre from different distances are between themselves in the sesquialteral proportion of their distances directly. Or time to centre $\propto \mathrm{D}^{\frac{3}{2}}$.

But if instead of the accelerating force of gravity varying $\frac{1}{\mathrm{D}^2}$, the velocity be supposed to $\propto \frac{1}{\mathrm{D}^2}$, then the time to centre will $\propto \mathrm{D}^3$.

Since the force of gravity at the moon : the force of gravity at the surface of the earth :: 1 : 3600, if a body be supposed to fall from a state of rest at the moon and at the earth's surface; the distance (unity) described in 1 second at the moon by the force of gravity : the distance described in 1 second at the surface of the earth by the force of gravity :: 1 : 3600 :: velocity produced by the force of gravity at the distance of the moon : the velocity produced by the force of gravity at the earth's surface.

The time t of describing unity at the distance of the moon : the time t of describing unity at the earth's surface :: 3600 : 1, for $t \times \mathrm{V} = 3600 \propto 1$.

Hence if, at any point of the descent, sections of the hyperbolic solid and pyramid be made perpendicular to the axes, the area of the section of the hyperbolic solid will be proportional to the force of gravity at that point, and to the distance the force at that point would cause the body to fall from a state of rest in 1 second, which will be proportional to the velocity produced from rest, or to the distance described in 1 second by the force of gravity at that point.

The section of the pyramid will be proportional to the time t of describing unity at that point.

$t \times \mathrm{V}$ will $= 3600$, and $\propto 1$.

But supposing the force of gravity be such as to produce a velocity $\propto \frac{1}{\mathrm{D}^2}$, time t will $\propto \mathrm{D}^2$, then we shall be enabled to illustrate these variations by the hyperbolic and pyramidal temples of the ancients.

So calling the distance of the moon from the earth $= 60$ semi-diameters of the earth, we shall have velocity at moon : velocity at the earth's surface $:: \frac{1}{60^2} : \frac{1}{1^2} :: 1^2 : 60^2$

$$:: 1 : 3600,$$

or velocity acquired at the end of the descent will be 3600 times greater than the velocity at the beginning.

In making some experiments we found that we could, without contact or external agency, attract and repel various substances with a velocity that evidently varied in some inverse ratio of the distance; and, as far as the eye could judge, the velocity seemed to vary inversely as the distance squared. The effects were produced by the finger touching the water on which the substances floated.

This caused us to reflect on the laws of gravitation. So the experiments were abandoned, and our attention directed to other subjects mentioned in this work.

Having shown by the obelisk that the time t in describing unity $\propto$ inversely as the velocity at that point, or that $t \times \mathrm{V}$ $=$ a constant quantity,

This relation of t to V will be the same whatever the law of velocity may be, or $t \times \mathrm{V}$ will always equal a constant quantity.

Since the velocity at the earth is 3600 times greater than the velocity at the moon, it follows, that the time t in describing a small definite distance at the moon will be 3600 times greater than the time t in describing the same distance at the earth,

or $t \times V$ at the moon

$$=3600 \times 1=3600,$$

and $t \times V$ at the earth

$$=1 \times 3600=3600.$$

Similarly $t \times V$ at the intermediate distances will $= 3600$.

$$\text{Since } t \propto \frac{1}{V}$$

$$\text{and } \quad V \propto \frac{1}{D^2}$$

$$t \propto D^2.$$

Fig. 49. When the obelisk is placed along with the pyramid, the bases of both being at the moon and their apices at the centre of the earth; then as the ordinate of the pyramid descended as the time T elapsed from the beginning of the descent, the ordinate of the obelisk will correspond with the ordinate of the pyramid. The frustum of the pyramid above the ordinate will denote the time T elapsed during part of descent, and the remaining or lower part of the obelisk included between the descending ordinate and apex will $\propto D^2 \propto$ time t.

At the end of the descent the whole time T elapsed will be represented by the whole pyramid, and time t will vanish with the obelisk.

Or the time T elapsed will increase as the frustum of the pyramid increases, while the time t will decrease as the obelisk decreases, so that at the end of the descent the pyramid will be completed and the obelisk will have vanished, excepting the small portions of the pyramid and obelisk each having an axis$=1$, since the descent of the body would cease at the earth's surface.

Thus great T may be said to have consumed little t, or

Kronos to have devoured his offspring. But supposing the body to be repelled from the centre or apex, then during the ascent the obelisk, which was consumed at the end of the descent, will increase from the apex, so that at the end of the ascent the obelisk will be completed, or the offspring may be said to have attained the heavens.

Again, the time of descent from the beginning to any point of the axis $\propto D^3 - d^3$, D being the whole axis described in the time T, and d the distance remaining to be described from the point in the axis to the apex of the pyramid. At the end of the descent the whole time T will $\propto D^3$, for d^3 will have vanished.

If a body be repelled from the apex, time will $\propto d^3$; at the end of the ascent the whole time T will $\propto \text{axis}^3 \propto d^3 \propto D^3$.

Here during the descent little d is consumed by great D, or Saturn devours his children. But during the ascent little d replaces great D, or Jupiter deposes his father Saturn, or Typhon destroys his brother Osiris. The Titans were brothers of Saturn, one of whom was Typhæus or Typhon. They strove to depose Jupiter from the possession of heaven, but they were beaten and cast down into hell.

$\frac{\text{Kronos}}{\text{Jupiter}} \propto \frac{T}{t} \propto \frac{D^3}{D^2} \propto D \propto \text{axis} \propto$ pyramid deprived of its generating ordinate. Thus Kronos, when divided by his son Jupiter, may be said to be emasculated, as Cælum was by Saturn, and as Osiris by Typhon.

Jupiter Ammon is represented with the horns of a ram.

The ram's horn is symbolical of the spiral obelisk. The content of the obelisk $\propto D^2$.

Kronos and Jupiter may be said to be divided against each other, when Jupiter wars against his father.

Jupiter castrated Saturn or Kronos, as Saturn had castrated his father Cælum before with a sickle.

The sickle may be symbolical of the curved obelisk.

Saturn, like Time, has his scythe.

Should the scythe represent the area of the obelisk, then the scythe of Saturn would be typical of the periodic time of the revolution of planets round the Sun.

Saturn holds in his hand a serpent with the tail in its mouth, forming a circle.

The circular serpent is symbolical of the circular obelisk. The obelisk is typical of infinity or eternity, and the circle the orbit of a planet. So the circular serpent denotes that planets revolve in circular orbits, having their $\text{P.T} \propto$ area obelisk and velocity $\propto \frac{1}{\text{ordinate}}$, and that they will revolve in their orbits to eternity.

The proud Neith says—"I am all that has been—all that shall be—and none among mortals has raised my veil." Neith is gravitation, by which the planets are preserved in their orbits, and supposed to continue their revolutions round the sun to all eternity.

But what is gravitation, that causes planets to revolve in orbits having their $\text{P.T} \propto D^{\frac{3}{2}}$, and to be continually urged with a velocity $\propto \frac{1}{D^{\frac{1}{2}}}$?

To show the variation of the P.T and velocity in terms of the obelisk and circle or orbit,

$$\text{P.T} \propto D^{\frac{3}{2}} \propto \text{area obelisk},$$

$$\text{velocity} \propto \frac{\text{orbit}}{\text{P.T}} \propto \frac{D}{D^{\frac{3}{2}}} \propto \frac{D^{\frac{3}{2}}}{D^{2}} \propto \frac{\text{area obelisk}}{\text{area orbit}};$$

or velocity $\propto$ directly as area obelisk, and inversely as area orbit.

The serpent when coiled, like the ram's horn of Jupiter Ammon, resembles the ammonite, and both are symbolical of the circular obelisk.

Hence when $V \propto D^{\frac{1}{2}}$,

$$\propto \frac{1}{D^{\frac{1}{2}}},$$

$$t\ \text{ordinate} \times \text{axis} \propto \frac{1}{D^{\frac{1}{2}}} \times D \propto D^{\frac{1}{2}},$$

$$\propto T\ \text{ordinate},$$

$$\propto \text{whole time } T.$$

When $V \propto \frac{1}{D^2}$,

t ordinate $\propto D^2$,

t ordinate × axis $\propto D^2 \times D \propto D^3$,

and whole time $T \infty D^3$.

Thus in both instances

t ordinate × axis,

or t ordinate × D or $t \times D$

$\propto$ as whole time T of descent.

The time t of describing a small distance at any point of the descent $\propto D^2 \propto$ axis^2 $\propto$ square ordinate that generates the pyramid.

The whole time T of descent, or $T\,C$ (time to centre) from different distances to the earth $\propto D^3 \propto$ axis^3 $\propto$ content of pyramid.

Thus, by deducing the variation of time and distance described from the effects or velocities produced by the influence of the earth, we have, when the body falls from the moon to the earth, the velocity represented by a square ordinate, which $\propto \frac{1}{D^2}$, and generates the hyperbolic solid, while the t ordinate which $\propto D^2$ generates the pyramidal solid.

As the velocity of planets round the sun vary inversely as the square root of their distance from the sun, the periodic time of a planet's revolution will $\propto$ directly as the orbit described, and inversely as the velocity, when $D =$ the mean distance, and the orbit is supposed to be circular.

For $$P\,T \propto \frac{\text{orbit}}{V} \propto \frac{\text{rad}}{V} \propto \frac{D}{V}$$

$$\propto D \times D^{\frac{1}{2}} \propto D^{\frac{3}{2}} \propto \text{area obelisk.}$$

or $$P\,T^2 \propto D^3.$$

Again, since $D^3 \propto \overline{P\,T}^2$, (Kepler,)

$$D^2 \propto P\,T \propto \frac{\text{orbit}}{V} \propto \frac{D}{V},$$

or $$V \propto \frac{D}{D^{\frac{3}{2}}} \propto \frac{1}{D^{\frac{1}{2}}}.$$

The axis of the obelisk represents D,

area of obelisk „ P T, and velocity $\propto$ inversely as the ordinate obelisk, or directly as the ordinate of the pylonic curve, which $\propto$ inversely as $D^{\frac{1}{2}}$ from the apex of obelisk.

Thus the distances, velocities, and P times of planets are represented by the obelisk.

The area of the obelisk is here supposed to be a curvilinear or parabolic area.

When $V \propto \frac{1}{D^2}$, a horizontal section of the hyperbolic solid, made at any point,. or distance, in the descent, will represent the velocity at that point, and the time *t* corresponding to this velocity will be represented by a horizontal section of the pyramid made at an equal distance, the pyramid having its apex in the centre to which the body falls.

Since velocity at any point will $\propto \frac{1}{t}$, *t* will $\propto \frac{1}{V}$, or $\propto D^2$. Thus during the descent the velocity plane will generate an hyperbolic solid, while the corresponding *t* plane will generate an inverted pyramid.

The time T elapsed at any point in the descent will be represented by the frustum of a pyramid. The whole time T of descent will be represented by a pyramid.

The whole time T of descent from different planetary orbits to the centre or sum will $\propto D^3$. If the whole time T of descent from any orbit to the centre be called T C, or time to centre, then $TC \propto D^3 \times \overline{PT}^2$.

Or times of descent from the planetary orbits to the centre vary as the square of the periodic times of the revolution of planets round that centre, or sun. The time *t* corresponding to the velocity at any point may be represented by that part of the obelisk intercepted between that point and the apex of the obelisk; since the solid obelisk $\propto D^2$ from the apex. But the pyramid will represent the variations of both T and *t*, since the whole time T of descent can be represented by the pyramid, and as the horizontal square section of that pyramid can represent the time t corresponding to the velocity at any point in the descent, for such a section of the pyramid will $\propto$ D^2 from the centre.

Take 60 semi-diameters of the earth to equal the distance of the moon from the earth, and dividing the pyramid, having the side of base = height = 60, into 60 horizontal square sections, each having the depth of 1. Then supposing a body to fall from the orbit of the moon to the earth, and the mean velocity of each of these sections to be continued uniformly through that section, the time consumed in describing each of these semi-diameters will be represented by a square stratum. Thus the pyramid will have 60 steps, and 60 square strata, and each stratum will represent the time consumed while the body descends through a corresponding semi-diameter of the earth. The section of the pyramid next the moon will = a stratum having a surface $=60^2=3600$ and the depth of unity; so this section will contain 3600 cubes of unity, while the section at the earth's surface will be represented by one cube. The solid generated by the velocity plane will represent one cube for the section next the moon, and 3600 cubes for the section at the earth's surface.

Thus 3600 cubes would represent the time consumed during the descent through the first semi-diameter, or that next the moon, and one cube would represent the time remaining to be consumed at the last semi-diameter, that of the earth itself, if the time at the surface of the earth were continued uniform to the centre, but the body cannot descend beyond the surface.

So at the surface of the earth the side of one cube would represent the time, and the surface of a stratum of 3600 cubes the velocity. Thus $t \times$ V at the earth's surface will $= 1 \times 3600 = 3600$, and at the moon $t \times$ V will $= 3600 \times 1 = 3600$.

But through the first semi-diameter from the moon $t \times$ V$=$ $3600 \times 1 = 3600$ cubes of unity; and at all the intermediate semi-diameters to the surface of the earth $t \times$ V will = 3600 cubes. .

Suppose a lamp, like the inverted pyramid having plain sides, were filled with oil, and lighted at the beginning of the descent from the moon, and that equal quantities of oil were consumed in equal times, so that when the body had reached the earth's surface the quantity remaining should be on a level with the square of unity next the apex, or at the distance of unity from the apex.

Thus the quantity of oil consumed during the whole descent would equal the content of the pyramid or $\frac{1}{3}\,\overline{\text{axis}}^{3}$.

The area of the surface during the descent would $\propto$ ordinate of the pyramid $\propto \mathrm{D}^2$ from the apex, or earth's centre, $\propto t \propto$ inversely as velocity or the horizontal section of the hyperbolic solid.

According to different writers there seems to have been a tradition that the pyramid represented a flame.

Since the $\mathrm{PT} \propto \mathrm{D}^{\frac{3}{2}} \propto$ area obelisk, if a stratum of oil similar and equal to the area of the obelisk, and having a depth = unity, were supposed to represent by its axis the distance from the Sun to Uranus, such a stratum would represent the PT of Uranus. Then if the stratum were supposed to stand on its base or greatest ordinate, and a light to be applied to the apex, when if equal quantities were consumed in equal times, then as the flame descended along the axis it would arrive at the several proportional distances of the intermediate planetary orbits; and the oil consumed through each of these planetary distances would be proportional to the PT of each of these planets' revolution round the Sun.

In *fig.* 52. the ordinates of the pylonic curve, having its axis corresponding to that of the obelisk, would, at these several distances, represent the proportional planetary velocities which $\propto \frac{1}{\mathrm{D}^{\frac{1}{2}}}$; and the corresponding ordinates of the obelisk will represent the times t corresponding to these velocities, since $t \propto \frac{1}{\mathrm{V}} \propto \mathrm{D}^{\frac{1}{2}}$.

Should a body fall from the orbit of the moon to the earth, the apex of the pyramid generated by the t ordinate would be in the centre of the earth.

But should a body fall from the orbit of the earth to the sun, the base of the pyramid would touch the earth's orbit, and its apex would be in the centre of the sun.

Near the Ajunta Pass, where the road from Central Hindostan ascends the mural heights supporting the table-land of the Dekhin, is a series of temples excavated out of the solid rock, having the walls and roofs embellished with paintings, among which is seen a much defaced head of Siva with a rich crown, ornamented, among other things, with crosses.

The crux ansata is found in the sculptures of Khorsabad, on ivories, and on cylinders. At Kouyunjik, Layard found the lotus introduced as an architectural ornament upon pavement slabs.

In the latest palace at Nimroud were the crouching sphinxes with beardless human head, supposed to be that of a female. Scarabæi are not unfrequently found in Assyrian ruins.

The crux ansata, or sacred tau, is the symbol of divinity of Osiris; ⊥ is symbolical of time, velocity, and distance, when a body descends near the earth's surface.

⊥ is the symbol of velocity and distance, and ⊤ of time and distance when a body descends from the moon to the earth.

$\underset{\circ}{\perp}$ The ringed tau denotes that the body cannot descend beyond the circumference of the attracting orb or sphere.

Bruce remarks that it is not the extreme height of the mountains in Abyssinia that occasions surprise, but the number of them, and the extraordinary forms they present to the eye. Some of them are flat, thin, and square, in shape of a hearth-stone, or slab, that scarce would seem to have base sufficient to resist the action of the winds. Some are like pyramids, others like obelisks or prisms, and some, the most extraordinary of all the rest, pyramids pitched upon their points, with their base uppermost, which, if it were possible, as it is not, they could have been so formed in the beginning, would be strong objections to our received ideas of gravity.

If these pyramids could not have been so formed originally, have they, like other pyramids, been formed by the ancients to represent the law of the time of a body falling from the moon to the earth?

Some of the great American teocallis would appear to have been natural hills shaped by the hands of man into terraced pyramids.

In these three laws of motion the times T, $P\ T$, $T\ C$, vary directly as the distance and inversely as the velocity.

1. In the descent near the earth's surface.
2. In the revolutions of planets round the sun.
3. In the descent from the planetary orbits to the centre.

1. $T \propto D \times \frac{1}{V} \propto \frac{D}{D^{\frac{1}{2}}} \propto D^{\frac{1}{2}}$

2. $P\ T \propto D \times \frac{1}{V} \propto D \times D^{\frac{1}{2}} \propto D^{\frac{3}{2}}$

3. $T\ C \propto D \times \frac{1}{V} \propto D \times D^{2} \propto D^{3}$.

Since $t \propto \frac{1}{V}$, these times will also vary directly as $D \times t$.

These times, T, $P\ T$, and $T\ C$, as well as their corresponding times t and velocities, can all be geometrically represented.

1. $T \propto D^{\frac{1}{2}} \propto$ ordinate obelisk $\propto n \propto \overline{\text{axis}}^{\frac{1}{2}}$ obelisk.
2. $P\ T \propto D^{\frac{3}{2}} \propto \overline{\text{ordinate}}^{3} \propto n^{3} \propto$ area obelisk.
3. $T\ C \propto D^{3} \propto \overline{\text{axis}}^{3} \propto n^{3} \propto$ content pyramid.
1. $t \propto \frac{1}{D^{\frac{1}{2}}} \propto \frac{1}{\text{ordinate obelisk}} \propto$ ordinate of pylonic curve.
2. $t \propto D^{\frac{1}{2}} \propto$ ordinate obelisk.
3. $t \propto D^{2} \propto$ ordinate pyramid $\propto$ content obelisk.
1. $V \propto D^{\frac{1}{2}} \propto$ ordinate obelisk.
2. $V \propto \frac{1}{D^{\frac{1}{2}}} \propto \frac{1}{\text{ordinate obelisk}} \propto$ ordinate of pylonic curve.
3. $V \propto \frac{1}{D^{2}} \propto \frac{1}{\text{ordinate pyramid}} \propto$ ordinate hyperbolic solid.

The distance D being represented by the axis of the obelisk, pyramid, pylonic curve, or hyperbolic solid.

In the descent from the apex of the obelisk to different distances,

$$\mathrm{T} \propto \mathrm{D}^{\frac{1}{2}}, \text{ and } t \propto \frac{1}{\mathrm{V}} \propto \frac{1}{\mathrm{D}^{\frac{1}{2}}}$$

but

$$t \times \mathrm{D} \propto \frac{1}{\mathrm{D}^{\frac{1}{2}}} \times \mathrm{D} \propto \mathrm{D}^{\frac{1}{2}} \propto \mathrm{T}.$$

In the revolutions of different planets round the same centre,

$$\mathrm{P\,T} \propto \mathrm{D}^{\frac{3}{2}}, \text{ and } t \propto \frac{1}{\mathrm{V}} \propto \mathrm{D}^{\frac{1}{2}},$$

but

$$t \times \mathrm{D} \propto \mathrm{D}^{\frac{1}{2}} \times \mathrm{D} \propto \mathrm{D}^{\frac{3}{2}} \propto \mathrm{P\,T}.$$

In the descents from different orbits to the same centre

$$\mathrm{T\,C} \propto \mathrm{D}^{3}, \text{ and } t \propto \frac{1}{\mathrm{V}} \propto \mathrm{D}^{2}$$

but

$$t \times \mathrm{D} \propto \mathrm{D}^{2} \times \mathrm{D} \propto \mathrm{D}^{3} \propto \mathrm{T\,C}.$$

Thus in the three laws of motion $t \times \mathrm{D}$ will vary as T, P T, and T C.

Hence $\mathrm{D} \propto \frac{\mathrm{T}}{t}$

$$\mathrm{D} \propto \frac{\mathrm{P\,T}}{t}$$

$$\mathrm{D} \propto \frac{\mathrm{T\,C}}{t}$$

In any of the three laws of motion, if the variation of V, T, or t be given, the other variations may be determined.

$$\text{Generally, } \mathrm{T} \times \mathrm{V} \propto \mathrm{D}, \text{ and } t \propto \frac{1}{\mathrm{V}}$$

$$\text{When } \mathrm{V} \propto \mathrm{D}^{\frac{1}{2}},\ \mathrm{T} \propto \frac{\mathrm{D}}{\mathrm{V}} \propto \frac{\mathrm{D}}{\mathrm{D}^{\frac{1}{2}}} \propto \mathrm{D}^{\frac{1}{2}}$$

$$V \propto \frac{D}{T} \propto \frac{D}{D^{\frac{1}{2}}} \propto D^{\frac{1}{2}}$$

$$t \propto \frac{T}{D} \propto \frac{D^{\frac{1}{2}}}{D} \propto \frac{1}{D^{\frac{1}{2}}} \propto \frac{1}{V}$$

$$V \propto \frac{1}{D^{\frac{1}{2}}}, \text{ P T, or } T \propto \frac{D}{V} \propto D \times D^{\frac{1}{2}} \propto D^{\frac{3}{2}}$$

$$V \propto \frac{D}{T} \propto \frac{D}{D^{\frac{3}{2}}} \propto \frac{1}{D^{\frac{1}{2}}}$$

$$t \propto \frac{T}{D} \infty \frac{D^{\frac{3}{2}}}{D} \propto D^{\frac{1}{2}} \propto \frac{1}{V}$$

$$V \propto \frac{1}{D^2}, \text{ T C, or } T \propto \frac{D}{V} \propto D \times D^2 \propto D^3$$

$$V \propto \frac{D}{T} \propto \frac{D}{D^3} \propto \frac{1}{D^2}$$

$$t \propto \frac{T}{D} \propto \frac{D^3}{D} \propto D^2 \propto \frac{1}{V}$$

Generally $T \propto D \times t$

$$t \propto \frac{T}{D}$$

$$D \propto \frac{T}{t}$$

When $V \propto \frac{1}{D^{\frac{1}{2}}}$, P $T^{\frac{2}{3}}$ $\propto D \propto$ axis obelisk

P $T^{\frac{3}{3}}$ $\infty D^{\frac{3}{2}} \propto$ area obelisk

P $T^{\frac{4}{3}}$ $\propto D^2 \propto$ content obelisk

$\propto$ orbicular area

P T^2 $\propto D^3 \propto$ content pyramid.

In the orbicular velocities t, the time of describing unity, $\propto \frac{1}{V} \propto$ ordinate obelisk

or t $\propto$ ordinate obelisk

$t^2 \propto$ axis

$t^3 \propto \overline{\text{axis}}^{\frac{3}{2}} \propto$ area obelisk $\propto$ P T.

$\text{PT}^2 \propto \overline{\text{axis}}^3 \propto$ pyramid or $\frac{1}{3}\,\overline{\text{axis}}^3$

$t^3 \propto \overline{\text{ordinate}}^3 \propto$ pyramid or $\frac{1}{3}\,\overline{\text{ordinate}}^3$

$\text{PT} \propto \overline{\text{ordinate}}^3 \propto$ pyramid or $\frac{1}{3}\,\overline{\text{ordinate}}^3$

The *fig.* 50. represents the pylonic area composed of a series of 6 equal parallelograms along the sectional axes 1, 3, 5, 7, 9, 11, so that each sectional axis multiplied by its mean

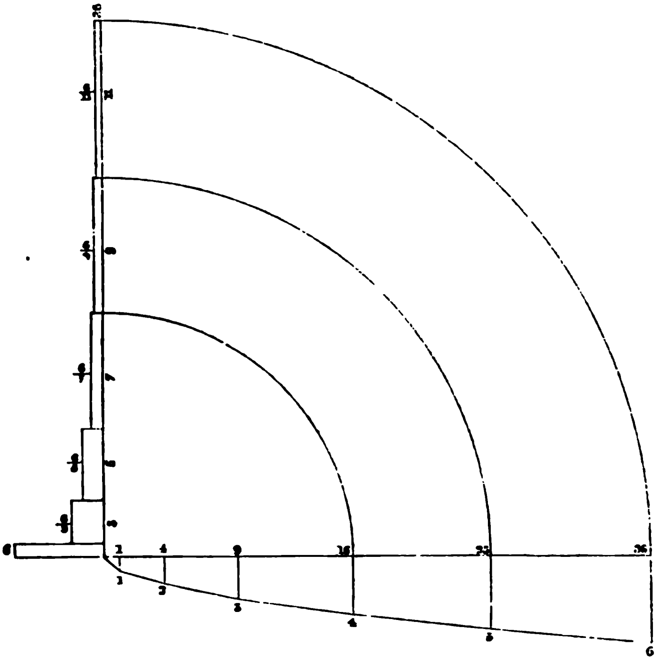


Fig. 50.

ordinate will = 6, which equals the area of the first parallelogram or 6 × 1, 6 being the last ordinate of the obelisk corresponding to its axis 36, and the first ordinate of the pylonic area.

The mean ordinates of 1, 3, 5, 7, 9, 11, the sectional axes, will correspond to the mean ordinates of the obelisk, which will lie between the ordinates 1, 2, 3, 4, 5, 6.

Hence the mean ordinate of obelisk multiplied by the mean ordinate of the pylonic area will = 3, the half of 6, the area of each parallelogram when one side = a sectional axis, or two ordinates of obelisk, and the other side = mean pylonic ordinate.

$$\begin{array}{lll} \text{as} & \frac{1}{2} \times \frac{6}{1} = 3 \text{ or } & \frac{1}{2} \times \frac{6}{1} = 3 \\ & 1\frac{1}{2} \times \frac{6}{3} = 3 & \frac{3}{2} \times \frac{6}{3} = 3 \\ & 2\frac{1}{2} \times \frac{6}{5} = 3 & \frac{5}{2} \times \frac{6}{5} = 3 \\ & 3\frac{1}{2} \times \frac{6}{7} = 3 & \frac{7}{2} \times \frac{6}{7} = 3 \\ & 4\frac{1}{2} \times \frac{6}{9} = 3 & \frac{9}{2} \times \frac{6}{9} = 3 \\ & 5\frac{1}{2} \times \frac{6}{11} = 3 & \frac{11}{2} \times \frac{6}{11} = 3 \end{array}$$

These mean ordinates, $\frac{6}{1}$, $\frac{6}{3}$, &c. of the pylonic area will ∝ inversely as the mean ordinates $\frac{1}{2}$, $1\frac{1}{2}$, $2\frac{1}{2}$, &c. of the obelisk. So that if the orbits passed these two series of ordinates, the rectangle of each two corresponding ordinates would = 3.

Fig. 51. Another series of parallelograms may be inscribed

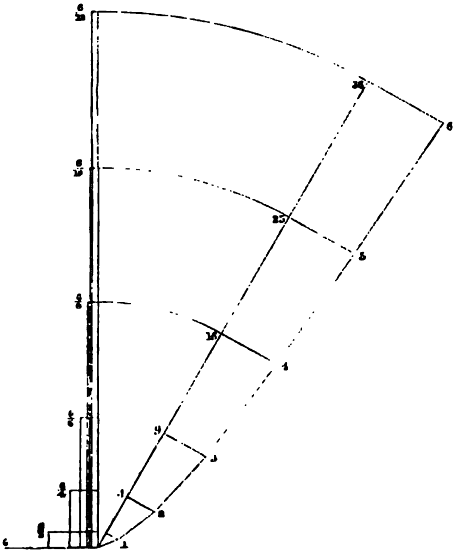


Fig. 51.

between the axis and the curve by making the pylonic ordinates = $\frac{6}{12}$, $\frac{6}{10}$, $\frac{6}{8}$, &c. for one side of each parallelogram, and the corresponding axis 36, 25, 16, &c. for the other sides. The areas of these series of parallelograms along the axis of the curve will be as 18, 15, 12, 9, 6, 3, for the axis multiplied by its corresponding ordinate will be as $36 \times \frac{6}{12} = 18$, $25 \times \frac{6}{10} = 15$, $16 \times \frac{6}{8} = 12$, and the areas 18, 15, 12, &c., will $\propto$ as the corresponding ordinates of obelisk 6, 5, 4, &c., which, it will be seen, is the ratio of the areas described in equal times in different orbits, having their axes or distances as 36, 25, 16, &c., and corresponding velocities as $\frac{6}{12}$, $\frac{6}{10}$, $\frac{6}{8}$, &c. These pylonic ordinates at the distances 36, 25, 16, &c. will $\propto$ inversely as the ordinates of the obelisk 6, 5, 4, &c., or inversely as the square root of the distances, and directly as the velocities.

These series of inscribed parallelograms will be as 3, 6, 9, 12, 15, 18

$$\text{sum will } = 3 \times (\overline{\tfrac{1}{2}n + 1} \, . \, n) = 3 \times \frac{7 \times 6}{2} = 63.$$

by having two series of ordinates for the pylonic area, one the mean ordinates of the sectional axes $\frac{6}{11}$, $\frac{6}{9}$, &c., and the other series $\frac{6}{12}$, $\frac{6}{10}$, &c.; so that the series of lines drawn from the ordinate of one series to the ordinate of the other series will form a succession of straight lines approaching to the pylonic curve.

Fig. 51. The series of inscribed parallelograms along the 6 different axes $1^2, 2^2, 3^2$, &c. $= 3 \times (\overline{\tfrac{1}{2}n + 1} \, . \, n) = 63$.

But the series of parallelograms between the sectional axes 1, 3, 5, 7, 9, 11 will equal

$$\begin{aligned}
1 \times \tfrac{6}{2} &= 6 \times \tfrac{1}{2} = 3 \\
3 \times \tfrac{6}{4} &= 6 \times \tfrac{3}{3} = 4{\cdot}5 \\
5 \times \tfrac{6}{6} &= 6 \times \tfrac{5}{6} = 5 \\
7 \times \tfrac{6}{8} &= 6 \times \tfrac{7}{8} = 5{\cdot}25 \\
9 \times \tfrac{6}{10} &= 6 \times \tfrac{9}{10} = 5{\cdot}4 \\
11 \times \tfrac{6}{12} &= 6 \times \tfrac{11}{12} = \underline{5{\cdot}5} \\
& \qquad\qquad\quad \underline{28{\cdot}65}
\end{aligned}$$

$$\begin{aligned}
&\text{or sum of } \tfrac{1}{2} + \tfrac{3}{4} + \tfrac{5}{6} + \tfrac{7}{8} + \tfrac{9}{10} + \tfrac{11}{12} \text{ of } 6 = 28{\cdot}65 \\
&\text{add sum of } \underline{\tfrac{1}{2} + \tfrac{1}{4} + \tfrac{1}{6} + \tfrac{1}{8} + \tfrac{1}{10} + \tfrac{1}{12} \text{ of } 6 = \ \ 7{\cdot}35} \\
&\text{and sum of } 1 + 1 + 1 + 1 + 1 + 1 \text{ of } 6 = 36
\end{aligned}$$

= the sum of the series of 6 parallelograms each = 6, inscribed between the sectional axes 1, 3, 5, &c.

Sum of the series $\frac{1}{2}+\frac{1}{4}+\frac{1}{6}$, &c. of 6 equals

$$\begin{aligned} \tfrac{1}{2} \text{ of } 6 &= 3 \\ \tfrac{1}{4} \quad &= 1{\cdot}5 \\ \tfrac{1}{6} \quad &= 1 \\ \tfrac{1}{8} \quad &= {\cdot}75 \\ \tfrac{1}{10} \quad &= {\cdot}6 \\ \tfrac{1}{12} \quad &= \underline{{\cdot}5} \\ & 7{\cdot}35 \end{aligned}$$

Fig. 52. By making the least ordinate = 1, then the series of six inscribed parallelograms having

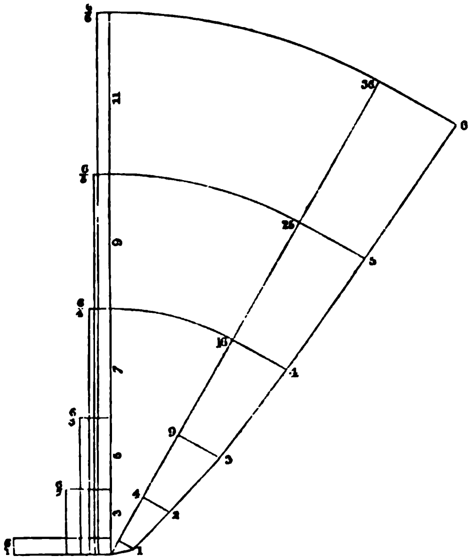


Fig. 52.

the axes	1,	4,	9,	16,	25,	36 for sides
and ordinates	6,	$\frac{6}{2}$,	$\frac{6}{3}$,	$\frac{6}{4}$,	$\frac{6}{5}$,	$\frac{6}{6}$
or	1,	$\frac{1}{2}$,	$\frac{1}{3}$,	$\frac{1}{4}$,	$\frac{1}{5}$,	$\frac{1}{6}$ of 6

for the other sides; thus the series will = 6, 12, 18, 24, 30, 36, or sum $= 6(\overline{\tfrac{1}{2}n+1}\,.\,n) = 6 \times \frac{7 \times 6}{2} = 126$.

Here the ordinates of the series will ∝ inversely as the ordinates of the obelisk, and the areas of the inscribed parallelograms 6, 12, 18, &c. will $\propto$ as the areas described in equal times in different orbits round the same centre.

Hence, if the distances of planets were as the squares of 1, 2, 3, the areas described in equal times would be in arithmetical progression.

$$\begin{array}{ll} \text{Distances} & \propto 1^2,\ 2^2,\ 3^2, \\ \text{velocities} & \propto 1,\ \tfrac{1}{2},\ \tfrac{1}{3} \\ \text{areas} & \propto 1,\ 2,\ 3 \end{array}$$

$$\text{or areas in equal times} \propto \text{ordinate} \propto 1,\ 2,\ 3, \propto D^{\frac{1}{2}} \propto \frac{1}{V}$$

$$\text{distances} \quad \propto \text{ordinate}^2 \propto 1^2,\ 2^2,\ 3^2, \propto D \propto \frac{1}{V^2}$$

$$PT \quad \propto \text{ordinate}^3 \propto 1^3,\ 2^3,\ 3^3, \propto D^{\frac{3}{2}} \propto \frac{1}{V^3}$$

$$\text{orbicular areas} \quad \propto \text{ordinate}^4 \propto 1^4,\ 2^4,\ 3^4, \propto D^2 \propto \frac{1}{V^4}$$

The ordinates of the pylonic area, which $\propto$ inversely as axis$^{\frac{1}{2}}$, will represent the velocities in orbits described between the pylonic area and obelisk. The obelisk will represent the distances and variations of t, PT, areas described in equal times, times of describing equal areas and equal distances in different orbits having the common centre at the apex of the obelisk. Lastly, the solid obelisk will represent the orbicular area.

$$PT \propto D^{\frac{3}{2}} \propto \text{area obelisk}$$

$$\text{velocity} \propto \frac{\text{orbit}}{PT} \propto \frac{D}{D^{\frac{3}{2}}} \propto \frac{D^{\frac{3}{2}}}{D^2} \propto \frac{\text{area obelisk}}{\text{solid obelisk}}.$$

As the orbicular areas described by different planets in equal times $\propto$ radius × velocity $\propto D \times \frac{1}{D^{\frac{1}{2}}} \propto D^{\frac{1}{2}} \propto$ ordinate, the series of parallelograms described along the axis, *fig.* 52., will $\propto D^{\frac{1}{2}} \propto$ ordinate.

Axes are $1^2,\ 2^2,\ 3^2,\ 4^2,\ 5^2,\ 6^2$.

velocity ordinates $\frac{6}{1}$, $\frac{6}{2}$, $\frac{6}{3}$, $\frac{6}{4}$, $\frac{6}{5}$, $\frac{6}{6}$,
areas 6, 12, 18, 24, 30, 36,
which $\propto$ 1, 2, 3, 4, 5, 6 $\propto$ ordinate obelisk.

Ordinate 6 being $36^{\frac{1}{2}}$, let $36 =$ the distance of Mercury from the Sun, and the series be continued to 30 terms; then will $30 = 900^{\frac{1}{2}}$, and $900 =$ the distance of Saturn from the Sun, and the corresponding parallelogram along the axis will $= 900 \times \frac{6}{30} = 180 =$ the area described by Saturn in the same time that $36 \times \frac{6}{6}$, or the area 36, was described by Mercury; these areas are as 180 : 36, or 30 : 6, or as the ordinates 30 and 6. Here the ordinates which represent the velocities of Mercury and Saturn are as $\frac{6}{6} : \frac{6}{30} :: 1 : \frac{1}{5} :: 5 : 1$.

These ordinates 1 and $\frac{1}{5}$ being tangents to the circles and perpendicular to the radii, or distances, represent the velocities corresponding to the distances; being reduced to minute tangents to the circles, they may ultimately be supposed to coincide with their corresponding circular arcs. Since by continually diminishing the velocity ordinates, still their ratio will remain as $1 : \frac{1}{5}$, and so will the ratio of the parallelograms along the axes 1^2, 2^2, 3^2, corresponding to the areas described in equal times in different orbits, continue to $\propto D^{\frac{1}{2}}$, or ordinates as the small tangents and arcs continually approach to coincidence, when ultimately the arc will represent the velocity which $\propto \frac{1}{D^{\frac{1}{2}}}$. So the area described will vary as axis × arc $\propto$ axis × ordinate $\propto$ radius × velocity $\propto D \times \frac{1}{D^{\frac{1}{2}}} \propto D^{\frac{1}{2}} \propto$ ordinate.

Sum of the series of parallelograms

$$6 + 12 + 18 + 24 + 30 + 36 + 126.$$

$$\begin{aligned} \text{Sum} &= 6 \times \tfrac{1}{2}\overline{n + 1}.\, n \\ &= 6 \times \tfrac{1}{2}n^2 + \tfrac{1}{2}n \\ &= 3n^2 + 3n. \end{aligned}$$

When velocity $\propto D^{\frac{1}{2}}$,

$$T \propto D \times t \propto D \times \frac{1}{D^{\frac{1}{2}}} \propto D^{\frac{1}{2}} \propto \text{ordinate obelisk.}$$

$$\propto \text{axis} \times \text{pylonic ordinate.}$$

When velocity $\propto \frac{1}{D^{\frac{1}{2}}}$

$$\text{P T} \propto D \times t \propto D \times D^{\frac{1}{2}} \propto D^{\frac{3}{2}}$$

$$\propto \text{axis} \times \text{ordinate} \propto \text{obeliscal area.}$$

When velocity $\propto \frac{1}{D^2}$

$$\text{T C} \propto D \times t \propto D \times D^2 \propto D^3$$

$$\propto \text{axis} \times \text{ordinate} \propto \text{content pyramid.}$$

$$\therefore \text{T C} \propto \text{P T}^2.$$

Here 1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, &c., of 6, the pylonic or velocity ordinates, are the reciprocals of the ordinates of the obelisk.

The series of parallelograms between the ordinates will be

$$1 \times \tfrac{6}{1},\ 3 \times \tfrac{6}{2},\ 5 \times \tfrac{6}{3},\ 7 \times \tfrac{6}{4},\ 9 \times \tfrac{6}{5},\ 11 \times \tfrac{6}{6},$$

$$\text{or}\quad 6,\quad 9,\quad 10,\quad 10{\cdot}5,\quad 10{\cdot}8,\quad 11.$$

Sum $= 57{\cdot}2$.

Let n be the number of terms, and 1st ordinate $= 6$; then the last sectional axis will $= 2n - 1$, and the last parallelogram of the series will $= 2n - 1 \times \frac{6}{n} = \frac{2n-1}{n} \times 6$ $= \left(2 - \frac{1}{n}\right) \times 6$; but $\frac{2n-1}{n}$ can never equal 2. So the last parallelogram of the series can never equal 12.

Hence this area by reciprocal ordinates will continually approach to the pylonic area, where the parallelograms inscribed along the sectional axes, 1, 3, 5, 7, &c., are all equal. When the ordinate continually $\propto \frac{1}{\text{axis}^{\frac{1}{2}}} \propto \frac{1}{D^{\frac{1}{2}}}$ the curvilinear area described may be called the pylonic area.

The pylonic area is described by ordinates which are the reciprocals of the ordinates of the obelisk.

The hyperbolic area, or the area between the asymptote and the curve, is described by ordinates which are the reciprocals of the ordinates of the triangle.

By making the ordinate $\propto \frac{1}{D^{\frac{1}{3}}}$, a series of rectangled parallelograms may be described between the curve and the axis, and the sum of the series will $= n\ (\frac{1}{3}\ \overline{n+1}\ .\ n\ .\ \overline{n+\frac{1}{2}})$ when n is the first ordinate of the series of parallelograms.

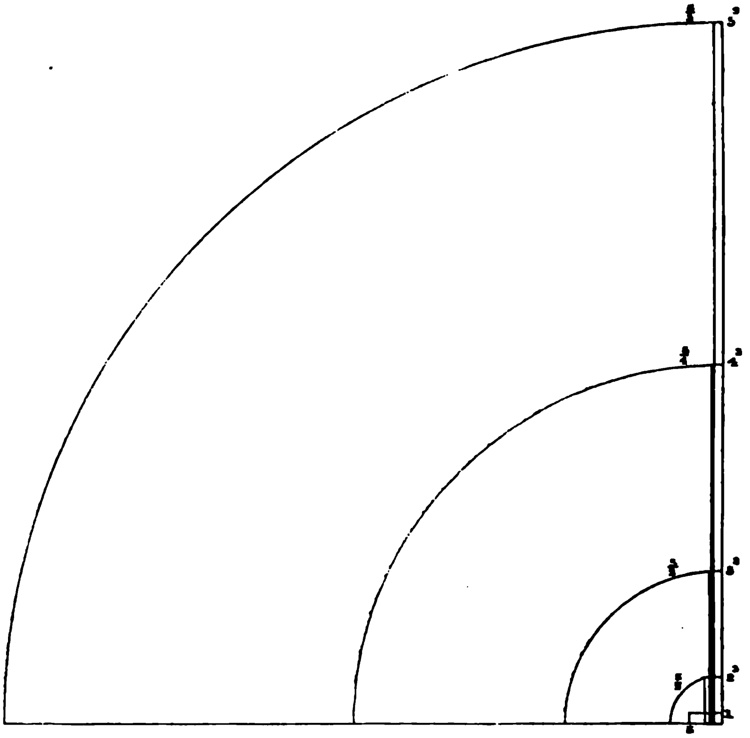


Fig. 53.

Fig. 53. is a series of five inscribed parallelograms having their ordinates inversely as the cube root of their axes. The series of axes being 1^3, 2^3, 3^3, 4^3, 5^3,

their ordinates will be $\frac{5}{1}$, $\frac{5}{2}$, $\frac{5}{3}$, $\frac{5}{4}$, $\frac{5}{5}$,

sum of the series $=5\ (1^2+2^2+3^2+4^2+5^2)$,

generally sum $=n\ (\frac{1}{3}\overline{n+1}\ .\ n\ .\ \overline{n+\frac{1}{2}})$,

when $n=5$. Sum $=5\times 55=275$,
whole axis $=n^3=5^3=125$.

Next, the series of 5 rectangled parallelograms along the sectional axes will be

$$5,\ 17{\cdot}5,\ 31{\cdot}66,\ 46{\cdot}25,\ \ 61$$

$$S=5\ (1+3{\cdot}5+6{\cdot}33+9{\cdot}25+12{\cdot}2).$$

If the series be continued to n terms, and the 1st ordinate remain $=5$, the sum will

$$=5\ \left(1+3{\cdot}5+6{\cdot}33+9{\cdot}25\ldots\ldots+\overline{3n-3+\frac{1}{n},}\right)$$

for the nth parallelogram will $=$ ordinate $\times$ axis

$$=\frac{5}{n}\times(n^3-\overline{n-1}^3)=5\left(3n-3+\frac{1}{n}\right).$$

So as n increases, the last parallelogram will continually approach to

$$5\,(3n-3)=15\times\overline{n-1}.$$

When $n=1,\quad 3n-3=0,$
and first rectangled parallelogram

$$=5\left(3n-3+\frac{1}{n}\right)=5\times\frac{1}{n}=5\times\frac{1}{5}=1.$$

So $\frac{1}{n}$ is incomparably greater than $3n-3$, being as $1:0$; but as n continually increases, $3n-3$ becomes vastly great compared with $\frac{1}{n}$. Since $\frac{1}{n}$ varies inversely as n, or as n increases $\frac{1}{n}$ decreases, much more will $3n$ increase while $\frac{1}{n}$ diminishes.

If the ordinate $\propto\frac{1}{D^{\frac{1}{4}}}$, and $n=5=$ the 1st ordinate of the series of 5 parallelograms, the several axes will be

$$1^4,\ 2^4,\ 3^4,\ 4^4,\ 5^4,$$

and ordinates $\quad\frac{5}{1},\ \frac{5}{2},\ \frac{5}{3},\ \frac{5}{4},\ \frac{5}{5}.$

Sum of the series of parallelograms will be

$$5\,(1^3+2^3+3^3+4^3+5^3)$$
$$=5\,(\tfrac{1}{2}\,\overline{n+1}\,.\,n)^2,$$
$$=5\times 15^2=1125,$$

axis of the series $=5^4=625$.

The series of 5 parallelograms along the sectional axes will be

5, 37·5, 108·33, 218·75, 369,

or 5 (1, 7·5, 21·66, 43·75, 73·8).

Let the series be continued to n terms while the 1st ordinate remains $=5$. The series will be

$$5\,(1,\ 7{\cdot}5,\ 21{\cdot}66 \ldots\ldots\ldots 4\,n^2-6\,n+4-\frac{1}{n},$$

since the nth parallelogram will $=$ ordinate $\times$ axis

$$=\frac{5}{n}\,(n^4-\overline{n-1}^4)=5\left(4n^2-6\,n+4-\frac{1}{n}\right).$$

When velocity $\propto \frac{1}{D^{\frac{1}{2}}}$, $\overline{P\,T}^2$ will $\propto D^3$.

Since the orbits of planets are supposed to be circular, and the velocity in each orbit uniform, the distance described will $\propto$ as the time, and the whole time T, or periodic time of a revolution, will $\propto$ directly as the orbit, or whole distance described, and inversely as the velocity,

$$\text{or } P\,T \propto \frac{\text{orbit}}{\text{velocity}} \propto \text{radius} \times D^{\frac{1}{2}} \propto D \times D^{\frac{1}{2}} \propto D^{\frac{3}{2}}$$

but area obelisk $\propto \overline{\text{axis}}^{\frac{3}{4}} \propto D^{\frac{3}{2}}$,

consequently $P\,T$ will $\propto$ area obelisk $\propto D^{\frac{3}{2}}$

or $\overline{P\,T}^2 \propto D^3$.

Hence, knowing the variation of the P T in different orbits round the same centre, the areas described in equal times by the radius vector in different orbits may be found.

Areas described in equal times by the radius vector in different orbits will $\propto$ directly as the orbicular area, and inversely as the P T $\propto \frac{\overline{\text{radius}}^{2}}{\text{P T}} \propto \frac{D^{2}}{D^{\frac{3}{2}}} \propto D^{\frac{1}{2}}$.

Or area described $\propto \frac{\overline{\text{radius}}^{2}}{\text{P T}} \propto \frac{\overline{\text{axis}}^{2}}{\overline{\text{axis}}^{\frac{3}{2}}} \propto \overline{\text{axis}}^{\frac{1}{2}} \propto$ ordinate of obelisk.

Hence the area of obelisk from the apex to the ordinate, corresponding to any axis, radius, or distance will represent the P T of a body revolving in the orbit of that radius: and the ordinates themselves, corresponding to the different distances or axes, will represent the variation of the areas described in equal times in different orbits. *Figs.* 50, 51.

By the tables the distance of Mercury from the Sun = 36,841,468 miles: that of Saturn = 907,956,130.

The periodic time of Mercury = 88 days nearly.

The periodic time of Saturn = 10,759 days.

Taking 36 and 900, in round numbers, as the distances of Mercury and Saturn from the Sun, the corresponding ordinates will be $36^{\frac{1}{2}}$ and $\overline{900}^{\frac{1}{2}}$ or 6 and 30.

So the area of Mercury's orbit will be to the area of Saturn's orbit :: $36^{2} : \overline{900}^{2}$ :: 1296 : 810,000. Then areas described in equal times by Mercury and Saturn will be as $\frac{\text{orbicular area}}{\text{P T}}$ as $\frac{D^{2}}{\text{ordinate}^{3}}$ as $\frac{36^{2}}{6^{3}} : \frac{\overline{900}^{2}}{30^{3}}$ :: 6 : 30, which is the ratio of their ordinates and the inverse ratio of their velocities.

The times of describing equal areas in different orbits $\propto$

$$\frac{\text{P T}}{\text{orbicular area}} \propto \frac{D^{\frac{3}{2}}}{D^{2}} \propto \frac{1}{D^{\frac{1}{2}}} \propto \frac{1}{\text{ordinate}}$$

So times of describing equal areas in the orbits of Mercury and Saturn will be as $\frac{\text{ordinate}^{3}}{D^{2}}$ as $\frac{6^{3}}{36^{2}} : \frac{30^{3}}{\overline{900}^{2}}$ :: 30 : 6 :: 5 : 1, which is inversely as their ordinates and directly as their velocities.

Thus in equal times the area described by Saturn with a velocity 1 will be to the area described by Mercury with velocity 5 :: 5 : 1.

So that Mercury may describe an area equal to what Saturn describes in a given time as 1 second; the time required by Mercury will be 5 times greater than the time required by Saturn; though Mercury moves with a velocity 5 times greater than that of Saturn.

Or area described by Saturn in 1 second $= \frac{1}{2}$ D × velocity $= \frac{1}{2}\, 900 \times 1 = \frac{1}{2}\, 900$; area described by Mercury in 1 second $= \frac{1}{2}$ D × velocity $= \frac{1}{2}\, 36 \times 5 = \frac{1}{2}\, 180$; in 5 seconds $= \frac{1}{2}\, 180 \times 5 = \frac{1}{2}\, 900 =$ area described by Saturn in 1 second.

As the areas described in circular orbits in a small portion of time, 1 second, $\propto$ radius × velocity $\propto$ D × velocity; the areas described in a greater portion of time will $\propto$ D × V; for the latter areas will be equal multiples of the small areas.

Or, as velocity is the distance described in a given time, it may be represented by a straight line, or the arc of a circle.

For the area of circle $= \frac{1}{2}$ the rectangle of the radius × circumference.

According to Archimedes a circle is equal to a right-angled triangle having one of the sides equal to the radius, and the other equal to the circumference of the circle.

So the area described in a circular orbit can be represented by a rectangle $\frac{1}{2}$ D × velocity.

Otherwise, calling the distance of Mercury and Saturn 36 and 900, since P T $\propto$ ordinate^3, P T of Mercury : P T of Saturn :: $6^3 : 30^3$. The orbicular area of Mercury : orbicular area of Saturn :: $36^2 : 900^2$.

Therefore orbicular area of Mercury $= \frac{36^2}{900^2} = \frac{1^2}{25^2} = \frac{1}{625}$, the orbicular area of Saturn.

So the time of describing an area in Saturn's orbit = the area of Mercury's orbit will be as $\frac{\text{P T}}{625} = \frac{30^3}{625} = 43{\cdot}2$.

Hence the times of describing equal areas in the orbits of Mercury and Saturn will be as

$$\begin{aligned} 6^3 : 43\ 2 &:: 216 : 43{\cdot}2 \\ &:: \ \ 5 : 1 \\ &:: 30 : 6, \end{aligned}$$

which are inversely as their ordinates, or directly as their velocities.

Since the velocity in each orbit is uniform, the distances described in equal times in different orbits will $\propto$ velocities $\propto \dfrac{1}{\text{ordinate}}$.

Also as time t of describing unity,

$$\propto \frac{1}{v} \propto \text{ordinate},$$

.·. the times of describing any number of units, or equal distances, in different orbits will $\propto$ ordinates.

Mercury describes a million of miles in its orbit in 57 hours.

Saturn describes a million in 285 hours.

And $57 : 285 :: 1 : 5 :: 6 : 30$; or the times of describing equal distances in the orbits of Mercury and Saturn are as $6 : 30$, which is the ratio of their ordinates.

Hence the distances described in equal times will be in the inverse ratio of the times of describing equal distances.

The times $t \propto$ ordinate, or are as $1 : 5$

The orbits $\propto \overline{\text{ordinate}}^2$, „ $1 : 5^2$

The P times $\propto \overline{\text{ordinate}}^3$, „ $1 : 5^3$

The orbicular areas $\propto \overline{\text{ordinate}}^4$, $1 : 5^4$.

Thus P T $\propto t^3$.

Distances described in equal times in different orbits $\propto \dfrac{1}{\text{ordinate}}$.

P T $\propto$ ordinate^3.

.·. P T $\propto$ inversely as the cube of the distances described in equal times in different orbits.

The mean distance of Jupiter is somewhat more than a fourth of the distance of Uranus from the Sun. Suppose the distances to be 4 and 16.

Then ordinates $= \sqrt{4}$ and $\sqrt{16}$, or 2 and 4.

And velocity of Jupiter will be to the velocity of Uranus $:: \frac{1}{2} : \frac{1}{4} :: 2 : 1$.

Their P T will be $:: 2^3 : 4^3$
$:: 8 : 64 :: 1 : 8$.

Areas described in equal times will be as $2 : 4 :: 1 : 2$.

Times of describing equal areas will be as $\frac{1}{2} : \frac{1}{4} :: 2 : 1$.

Times t of describing a unit, or equal distances, will be as $2 : 4$, or $1 : 2$.

Hence the times of describing equal areas will $\propto$ directly as the velocities or distances described in equal times, or inversely as the areas described in equal times, or inversely as $\text{P T}^{\frac{1}{3}}$.

The areas described in equal times will $\propto$ directly as $\text{P T}^{\frac{1}{3}}$; or directly as the times of describing equal distances, or inversely as the times of describing equal areas.

Or, when the times are equal, the areas described will be as $1 : 2$.

When the areas are equal, the times of describing them will be as $2 : 1$.

Or areas described in equal times $\propto$ ordinate $\infty \frac{1}{\text{V}}$.

Times of describing equal areas $\propto \frac{1}{\text{ordinate}} \propto \text{V}$.

Times t of describing equal distances, a unit, $\infty \frac{1}{\text{V}} \propto$ ordinate.

$\therefore$ Time t of describing equal distances, a unit, $\propto$ inversely as the times of describing equal areas.

So the times of describing equal areas $\propto \frac{1}{\text{ordinate}} \propto \text{V}$ $\propto \frac{1}{t} \propto \frac{1}{\text{PT}^{\frac{1}{3}}}$.

Or the times of describing equal areas $\propto$ inversely as the times t of describing equal distances, a unit, $\propto$ inversely as the cube root of the P T.

Fig. 54. When the axis bisects the obeliscal area, and another straight line drawn from the apex represents the

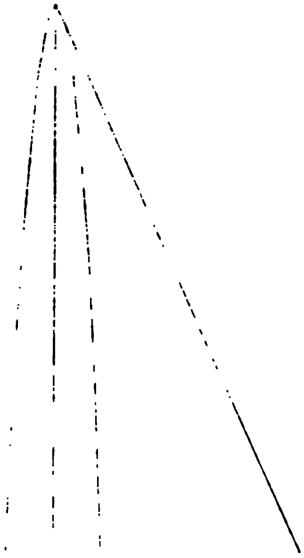


Fig. 54.

axis of the pylonic area, we have what is commonly called the flail or whip of Osiris, an emblem of divinity, which he often holds in one hand, while in the other hand, crossed, he holds the crosier or curve of Osiris; sometimes the crux ansata, or sacred tau. So that this geometrical obeliscal representation of the laws of gravity becomes, in place of the whip, one of the most exalted emblems that the genius of man can assign to a divinity.

The obelisk was called "the finger of God." It now appears that the obelisk indicates the laws by which the universe is governed, and the granitic durability of this monolith is typical of the eternity of these laws and the monolith of unity. As such a symbol it was held in the greatest veneration, and placed within and at the entrance of the temples.

Nebuchadnezzar, who invaded and ravaged Egypt, erected in the plain of Dura a golden image, which he commanded the people to worship. From its dimensions, height 60 cubits, and breadth 6, it might have been an obelisk covered with gilded plates of metal.

In the Hippodrome at Constantinople there is a structure, or kind of obelisk, built with pieces of stone, said to be 94 feet high, "which was formerly covered with plates of copper, as we learn from the Greek inscription on its base." The pieces of copper were fastened together by iron pins, which were secured by lead; the holes in the stone are still visible. This obelisk, according to Bellonius, had the copper plates gilded so as to appear of gold.

Herodotus informs us that Pheron, after recovering his sight, erected, as an offering in the temple of the Sun, two obelisks, the height of each monolith being 100 cubits and breadth 8.

The golden thigh of Pythagoras was probably a small circular obelisk, by means of which he acquired a knowledge of the true solar system of the ancients; but Europe was not sufficiently enlightened in the age of Pythagoras to admit its truth, which he revealed only to a few of his select disciples.

The Chinese pagoda and Mahomedan minaret are varied, but false, forms of the obelisk, being devoid of the true principle of construction. Both these imitations of the obelisk continue to be dedicated to religion in the East. Probably some of the most ancient Chinese pagodas may be found to be true obelisks.

This sacred type of the eternal laws appears to have become more and more obscure as the days of science declined, till ultimately it ceased to be intelligible; when, instead of this spiritual symbol, a physical one, palpable to the senses and adapted to the capacity of the unlearned, was substituted, and so the Phallic worship became embodied and revered in the religious rites of Egypt, India, Greece, and Rome.

Squire concludes, from the American monuments, that this form of worship extended over that vast territory.

When the sacred tau, the symbolical generator of time, velocity, and distance, ceased to be understood as a spiritual type, it was also adopted as a physical emblem.

It would seem that these types were properly understood, and most probably first associated with religion, by the Sabæans.

In the latest Assyrian palaces are frequent representations of the fire-altar in bas-reliefs and on cylinders, so that Layard thinks there is reason to believe that a fire-worship had succeeded the purer forms of Sabæanism.

The worship of planets formed a remarkable feature in the early religion of Egypt, but in process of time it fell into desuetude.—(*Jablonski.*)

To form the series of hyperbolic parallelograms $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, ...1 of 9.

(*Fig.* 40.) Series of inscribed parallelograms is

$$1, \tfrac{1}{2}, \tfrac{1}{3}, \tfrac{1}{4}, \tfrac{1}{5}, \tfrac{1}{6}, \tfrac{1}{7}, \tfrac{1}{8}, \tfrac{1}{9} \quad \text{of } 9.$$

Difference $\frac{1}{2}$, $\frac{1}{6}$, $\frac{1}{12}$, $\frac{1}{20}$, $\frac{1}{30}$, $\frac{1}{42}$, $\frac{1}{56}$, $\frac{1}{72}$ of 9.

Hence the series of inscribed rectangled parallelograms at right angles to 1, $\frac{1}{2}$, $\frac{1}{3}$, &c., will be $\frac{1}{2}$, twice $\frac{1}{6}$, three times $\frac{1}{12}$, &c. For 1st superficial rectangled parallelogram $= \frac{1}{2}$ of 9

2nd „ $= 2 \times \frac{1}{6}$ „ $= \frac{1}{3}$
3rd „ $= 3 \times \frac{1}{12}$ „ $= \frac{1}{4}$
4th „ $= 4 \times \frac{1}{20}$ „ $= \frac{1}{5}$
5th „ $= 5 \times \frac{1}{30}$ „ $= \frac{1}{6}$
6th „ $= 6 \times \frac{1}{42}$ „ $= \frac{1}{7}$
7th „ $= 7 \times \frac{1}{56}$ „ $= \frac{1}{8}$
8th „ $= 8 \times \frac{1}{72}$ „ $= \frac{1}{9}$
9th „ $= 9 \times 1$ „ $= 1.$

In this hyperbolic series, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$.................. 1 of 9, the greatest parallelogram $= 9$ is placed the last.

But in the series 1, $\frac{1}{2}$, $\frac{1}{3}$, &c. of 9, the greatest parallelogram is placed the first.

This last series of parallelograms overlap each other from M to I L.

The series $\frac{1}{2}$, $\frac{1}{3}$ &c., overlap one another from I to L M.

Also, as in *fig.* 37., when the first of the series is a square, the last will be a rectangled parallelogram.

But, as in *fig.* 38., when the first is a rectangled parallelogram, the last of the series will be a square.

By taking the difference of the series of rectangled parallelograms, 1, $\frac{1}{2}$, $\frac{1}{3}$, &c. in one square, we have the series of rectangled parallelograms, $\frac{1}{2}$, $\frac{1}{6}$, $\frac{1}{12}$, &c. formed in the other square.

The sum of the series $\frac{1}{2}+\frac{1}{6}+\frac{1}{12} \dots + \frac{1}{72}$ of 9 to 8 terms will by construction $= \overline{9-1} \times 1 = 8$.

So when 1, $\frac{1}{2}$, $\frac{1}{3}$, &c. of n is continued to n terms, the sum of the differential series $\frac{1}{2}+\frac{1}{6}+\frac{1}{12}$, &c. of n to $\overline{n-1}$ terms will $= \overline{n-1}$.

The series $\frac{1}{2}$, $\frac{1}{6}$, $\frac{1}{12}$, &c. to $\frac{1}{\overline{n-1} \cdot n}$ of n may also be formed from the series 1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\dots \frac{1}{n}$ of n, by multiplying the 1st term by the 2nd, the 2nd by the 3rd, the 3rd by the 4th, and the $\overline{n-1}$ by n, as

$$\begin{aligned} 1 \times \tfrac{1}{2} &= \tfrac{1}{2} \\ \tfrac{1}{2} \times \tfrac{1}{3} &= \tfrac{1}{6} \\ \tfrac{1}{3} \times \tfrac{1}{4} &= \tfrac{1}{12} \\ \frac{1}{n-1} \times \frac{1}{n} &= \frac{1}{\overline{n-1} \,.\, n} \end{aligned}$$

The sum of this series to $\overline{n-1}$ terms will $= \overline{n-1}$.

By construction, it will be seen that the differential series

$$\begin{array}{rl} & \tfrac{1}{2}, \tfrac{1}{6}, \tfrac{1}{12}, \tfrac{1}{20}, \tfrac{1}{30}, \tfrac{1}{42}, \tfrac{1}{56}, \tfrac{1}{72} \text{ of } 9 \\ \text{to 8 terms} \times \text{by} & 1, 2, 3, 4, 5, 6, 7, 8 \\ \hline = & \tfrac{1}{2}, \tfrac{1}{3}, \tfrac{1}{4}, \tfrac{1}{5}, \tfrac{1}{6}, \tfrac{1}{7}, \tfrac{1}{8}, \tfrac{1}{9} \text{ of } 9. \end{array}$$

Thus the sum of this series to 8 terms + 9 for the 9th term = the hyperbolic series of rectangled parallelograms.

The sum of the direct series

$$0+2+6+12+20, \text{\&c.},$$

which is formed from n^2-n, will be seen to $= \frac{1}{3} n^3 - \frac{1}{3} n$

$\therefore 0 + \frac{1}{2} + \frac{1}{6} + \frac{1}{12} \ldots + \frac{1}{\overline{n-1}\,.\,n}$ of n to n terms will $= n-1$ $= \frac{1}{n}$ of $\overline{n^2-n}$, from which the direct series 0, 2, 6, 12, 20, &c. is formed. The last term of the series $= \frac{1}{\overline{n-1}\,.\,n'}$ hence the sum of the series, $\overline{n-1}$, will $= \frac{1}{n}$ of the denominator of the last term.

The more the radius of the quadrant is subdivided the nearer will the hyperbolic reciprocal curve approach its axis and the quadrantal arc, but still the axis of the curve will = twice radius = twice the axis of the hyperbolic series of rectangled parallelograms within the square.

The hyperbolic area will also continually diminish as the area of the curve approaches to the area of the quadrant. For suppose the radius of the quadrant to be divided into 900 instead of 9 equal parts, then the axis of the hyperbola will = 900, and the area of the central or angular square L N = 900 = 30^2.

So the side of the central square will be to the axis of the hyperbola or radius of the quadrant, as 30 : 900 :: 1 : 30.

But when the axis of the hyperbola = 9 = radius, the side of the central square, 3 : axis of the hyperbola :: 3 : 9 :: 1 : 3.

When 6 hyperbolic parallelograms are inscribed in the square = $\overline{\text{axis}}^2$ of curve = 6^2 = 36, the area of the series = 14·7. When 36 parallelograms are inscribed in the same $\overline{\text{axis}}^2$, now = 36^2, the area of the series = 150·3.

$\therefore$ area of 6 parallelograms : $\overline{\text{axis}}^2$ :: 14·7 : 36

area of 36 parallelograms : $\overline{\text{axis}}^2$ :: 150·3 : 36^2 :: 4·17 : 36

Thus 6 inscribed parallelograms will = 14·7 of 6^2, or $\overline{\text{axis}}^2$.

And 36 inscribed parallelograms will only = 4·17 of the same square. First parallelogram in the series 36 will = $\frac{1}{6}$ of the first parallelogram in the series 6. And first 6 parallelograms in series 36 will $= \frac{1}{6}$ of the first 6 parallelograms in series 6, $= \frac{1}{6}\, 14{\cdot}7 = 2{\cdot}45$, but whole series of 36 parallelograms $= 4{\cdot}17$ of $\overline{\text{axis}}^2$, or of 6^2.

$\therefore\ 4{\cdot}17 - 2{\cdot}45 = 1{\cdot}72$ for the area of the remainder of the 36 parallelograms.

Hence when the radius of the quadrant is divided into 6 equal parts, the area of the 6 hyperbolic parallelograms described in the square $= \overline{\text{axis}}^2$, will $= 14{\cdot}7$.

When the same radius is divided into 36 equal parts, the area of the 36 hyperbolic parallelograms described in the same square will $= 4{\cdot}17$ of the $\overline{\text{axis}}^2$, or of 6^2.

As n increases the more the radius is subdivided, the more will the angle B C 9 of the first or primitive triangle decrease, and the sine of the triangle will approach to equality with the hypothenuse, or radius, and the curvilinear area to that of the quadrant. The difference between the hypothenuse or radius C B and the sine that subtends the angle at C of the primitive triangle will always equal unity in the series 1, 2, 3, to n terms. Hence the radius will be to this sine as $n : n - 1$; also twice the hypothenuse of the triangle = diameter of the circle = the axis of the reciprocal curve = the two asymptotes of the hyperbola.

Thus the hypothenuse of the primitive triangle determines the radius of the quadrant: the angle at C determines unity in the radius. These also determine the reciprocal curve, and the series of hyperbolic parallelograms as well as the series of parallelograms which form the triangular area.

The outline of a dome is formed by the hyperbolic reciprocal curve, or the dome itself is formed by the revolution of the curve on its axis.

Hodges thus describes his visit to the mosque of Mounheyr, twenty miles distant from Patna, the capital of the province of Bahar. This edifice is not large, but very beautiful. A majestic dome rises in the centre, the line of whose curve is not broken, but is continued by a reverse curve till it terminates in a crescent. This appears to our author infinitely more beautiful than the European system of crowning the dome by some object making an angle with it.

Area of quadrant $= 9^2 \times \cdot7854$
„ $= 63\cdot6174$
which × by 2 $= 127\cdot2348$.

Hence, the radius continuing the same, as n increases, the curvilinear reciprocal area will continually approach to equality with that of the quadrant.

The hyperbolic area, as n increases, will also continually decrease, when the same quadrant has its radius continually subdivided into equal parts for determining the reciprocals of the sines, which determine the hyperbolic area.

The high cap having the hyperbolic reciprocal curve for the outline is one of the insignia of divinity or royalty (for kings shared the attributes of gods). Such a cap is sometimes seen on the head of Osiris, and on the colossal statues at the entrance of the Luxor.

Sometimes the top of the cap or helmet, like the hyperbolic area, terminates in a point; such are found in Egypt, at Nimroud, and at Babylon. Also, in the Nimroud sculptures two archers have caps or helmets truncated at the top, like that in the constructed curvilinear area.

The more truncated the top, the less will the radius be divided. The more pointed the top, the more will the same radius be subdivided. The two arches that have the truncated-like caps have both curled beards of the obeliscal form, like the Egyptians.

The sphent may represent the hyperbolic area. The beards, or their casings, as seen in the Egyptian statues, are of the obeliscal form, typical of infinity. Similar beards are seen in the Assyrian sculptures.

The hair of the head is frequently arranged in parabolic curved lines; the focus being placed lower down than the crown of the head, over that part called by phrenologists the love of offspring.

This parabolic arrangement of the hair is also symbolical of infinity. The focus may be supposed to be the sun, and the parabolic curves the paths of the comets. Or they may together be supposed to represent a comet itself, or Stella crinita.

The impression of Buddha's foot is like this parabolic or cometary system; but with the addition of circular orbs placed round the focus, or sun, indicative of the planetary orbits.

So that the foot-mark of Buddha represents both the cometary and planetary systems: the sun being placed in the centre of the heel, having concentric planetary circles; the cometary parabolic paths extend to the toes, having the sun in the focus.

The lower part of one form of Egyptian cap, as it rises from the head, is sometimes curved outwards, probably intended to denote the hyperbolic curve; from this lower part rises the crown, of an egg-like shape. Such a combination is on the head of a colossal statue of polished red granite in the British Museum. The whole height of the statue is supposed to have been about 26 feet English, which would equal 37 Babylonian cubits.

The egg-shaped part of the cap may represent the parabolic or hyperbolic conoid,—both being typical of eternity.

Or, if an ellipse revolve on its less axis, an oblate spheroid will be generated, like the figure of the earth.

If the same ellipse revolve on its greater axis, an oblong spheroid will be generated, like the mundane egg.

But the oblate spheroid, being the greater, would contain the oblong spheroid.

So the world might be said to contain the mundane egg.

We have since met with the reciprocal hyperbolic cap on a figure, supposed that of a priest, sculptured on stone, which Rich found at Hillah. He also informs us, "that among the gardens a few hundred yards to the west of the Husseinia gate, is the Mesjid-ess-hems, a mosque built on the spot where popular tradition says a miracle similar to that of the prophet Joshua was wrought in favour of Ali; and from this the mosque derives its appellation. It is a small building, having instead of a minaret an obelisk, or rather hollow cone, fretted on the outside like a pine-apple, placed on an octagonal base. This form, which is a very curious one, I have observed in several very old structures; particularly the tomb of Zobeide, the wife of Haroun-al-Raschid, at Bagdad; and I am informed it cannot now be imitated. On the top of the cone is a mud cap, elevated on a pole, resembling the cap of liberty. This, they say, revolves with the sun; a miracle I had not the curiosity to verify."

The exaltation of the horn, an expression so frequent in scripture, is explained by the practice still existing in the East, of employing the horn in the head-dress.

This is particularly the case among the Druses of Lebanon, where the horn is a tin or silver conical tube, about twelve inches long, and the size of a common post horn. The wife of an emir is distinguished by a gold horn enriched with precious stones. This ornament of female attire is worn on the head in various positions, distinguishing their several conditions. A married woman has it affixed to the right side of the head, a widow to the left, and a virgin is pointed out by its being placed on the very crown: over this silver projection the long veil is thrown, with which they so completely conceal their faces as rarely to have more than one eye visible. A similar horn is in use among the Christian women at Tyre; and ornaments of this kind are worn in some parts of the Russian territories. In Abyssinia Bruce found these horns worn by men: they attracted his particular attention in a cavalcade, when he observed that the governors of provinces were distinguished by this head-dress. It consists of a broad fillet tied behind, from the centre of which projects a horn or conical piece of silver-gilt, about four inches long, and very much in its general appearance resembling a candle-extinguisher. It is called *kirn* (as in Hebrew), and is worn after a victory or on great public occasions.

The hyperbolic reciprocal curve formed by the 4 quadrants will resemble a winged circle, which may be the origin of the winged globe or planet urged forward in its orbit by its reciprocal wings—typical of positive and negative electricity.

The semicircle and reciprocal wings may represent the outline of Mercury's cap, which is hemispherical with wings attached to the sides. To his ankles the winged sandals, or talaria, are attached. The winged caduceus that he holds in his hand is entwined by two serpents in opposite directions, which may also denote positive and negative electricity.

The Egyptians painted his face partly black and dark, and partly clear and bright, because he is supposed to converse sometimes with the celestial, and sometimes with the infernal gods. Or he may be regarded as flying by the aid of electrical wings, and so like an electrical telegraph communicating with heaven and earth. The positive and negative electric powers may have been indicated by his face being partly dark and partly bright.

Nared, the son of Brahma, was, like Hermes or Mercury, a messenger of the gods.

The wings of Mercury being hyperbolic and electrical, they denote that planetary distances would be traversed with the speed of electricity.

The velocity of Mercury, which is nearest the sun, is greater than that of any other planet.

But we suppose the wings of the globe to be symbolical of the obelisk, the exponent of the laws that urge a planet onwards with a velocity $\propto \frac{1}{\text{ordinate}}$, and $\text{P T} \propto$ area obelisk.

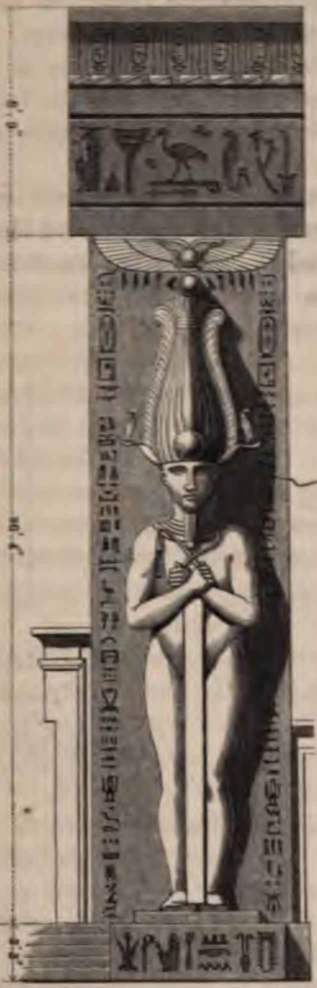
The motive power of the two wings by which the planet is propelled forward and preserved in its orbit may be positive and negative magnetism, galvanism, or electricity; all of which have recently been discovered to be modifications of the same law of nature.

By this agency the planet, like a bird, is supposed to fly with two electrical wings, which urge it forward and prevent its falling to the earth.

Two serpents belong to the winged globe. The serpent is typical of the circular obelisk, or infinity.

But the large expanded wings of the globe resemble the outline of an obeliscal or parabolic area, which denotes the periodic time of a planet.

The serpent when formed into a circle with the tail in its mouth, denotes the orbit in which the planet will revolve to eternity. Or if the serpent be supposed to eat its tail, the orbit will diminish so that the planet would ultimately fall to the centre of force, — the sun.



A caryatid pilaster, at Medinet-abou, 24 feet high, including the high cap, has the hands at the lower part of the chest resting upon a support rising from between the feet, and presenting in front a parallelogram having a breadth about $\frac{1}{18}$ length.

If the parallelogram were divided into $\frac{1}{2}$, $\frac{1}{2^2}$, $\frac{1}{2^3}$, &c., the sum of all the parts, how far soever continued, would never equal the parallelogram itself; so the parallelogram would be symbolical of infinity or eternity.

One hand holds the whip, or outline of the obelisk, the other the crosier or curve of Osiris. The beard is obeliscal.

Above the cap is the globe and serpents with obeliscal or parabolic wings.

The high cap itself is bounded on each side at the lower part by two serpents, and with feathered-like appendages of uniform breadth and of contrary flexure, extending the whole length of the sides.

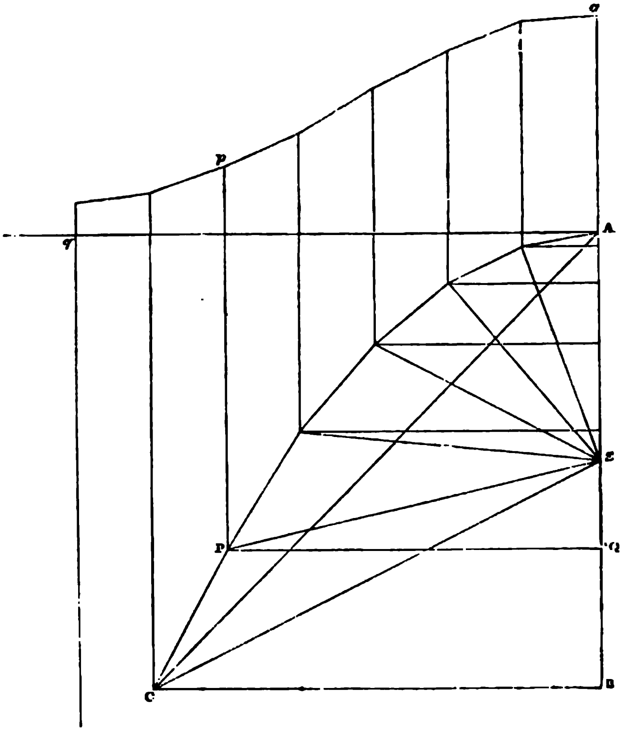


Fig. 55.

Fig. 55. If the focal distance AS of the parabola $= Aa = SB = \frac{1}{2}\,BC = \frac{1}{2}$ latus rectum $= \frac{1}{2}\,36$.

So $AB = BC =$ latus rectum $= 6 \times 6 = 36$. Parabolic area $= 6$ times area of obelisk.

The ordinates $Pp = SP$ at the different sections, ap will be a curve of contrary flexure traced by p.

$$\begin{aligned} SP^2 &= SQ^2 + PQ^2 \\ &= (AQ - AS)^2 + PQ^2 \\ &= (\text{axis} - \tfrac{1}{2}\,L)^2 + \text{ordinate}^2 \\ &= \text{axis}^2 - L \times \text{axis} + \tfrac{1}{2}\,L^2 + \text{ordinate}^2 \\ &= \text{axis}^2 + \tfrac{1}{4}\,L^2 \end{aligned}$$

$\therefore$ SP and Pp will always be greater than the axis, and the curve of contrary flexure ap will continually approach to, but can never touch the axis Aq.

Hence the curve ap will be infinite, and the high cap of Osiris will be symbolical of eternity.

The two feathered-like appendages along the curved side of the cap denote that the breadth of the cap will increase as the focal distance AS increases.

If the focal distance were increased, the feathered-like appendages would become more like the curve which Osiris holds in his left hand. Thus the curve of Osiris will be typical of the parabolic curve of contrary flexure, or of infinity.

When SP is above S,

$$SP^2 = \text{ordinate}^2 + (\tfrac{1}{2}\,L - \text{axis})^2$$

When SP is below S,

$$SP^2 = \text{ordinate}^2 + (\text{axis} - \tfrac{1}{2}\,L)^2.$$

The top of the cap and feathers being rounded off may denote their infinite extension.

The serpents on the sides of the cap are typical of the obelisk or of infinity.

The serpent here represented is perhaps the most common of all the Egyptian hieroglyphics. It is known by its erect position, swollen neck, and the entwining folds of the lower part of the body. Denon has given a sketch of this serpent in the same attitude as we see it on the sculptured stone. It is the Naia Haje, a most venomous snake, which the ancient Egyptians assumed as the emblem of Cneph or the Good Deity. It is also a mark of regal dignity, and is seen on the fore part of the tiara of almost all Egyptian statues of deities and kings.

This serpent in the erect position with its swollen neck resembles the parabolic curve of contrary flexure, the same as that of the cap, and the curve in one hand of Osiris.

The Ibis, like the Naia Haje, may have been held sacred from its head and long beak having a resemblance to the parabolic curve of contrary flexure.

In the other hand Osiris holds the obeliscal whip, by means of which he urges the heavenly bodies onwards in their orbits. Hence the myth of Phaeton driving the chariot of his father Sol. The Sun was worshipped by the Egyptians under the name of Osiris.

The sun is the centre of force round which the planets revolve with velocity $\propto \frac{1}{\text{ordinate}}$, and $\text{PT} \propto$ area obelisk, that is, the planets are urged onwards in their orbits by laws indicated by the obelisk; or, metaphorically, they are driven by Sol or Osiris with the obeliscal whip.

As the focal distance increases, the parabola increases, which is denoted by the feathered-like side of the cap; for the short lines made by a series of increasing parabolas will be more inclined as they recede from the axis of the parabola, and thus give the outside of curve of contrary flexure a feathered appearance. The axis of the curve $\propto$ ordinate of parabola, and ordinate of curve $\propto \text{SP} -$ axis of parabola. The revolution of the curve on its axis would generate a solid like the cap.

The obeliscal beard typifies eternity.

If equal parabolas, having their axes in the same straight line and their apices coinciding in A, but on opposite sides of Ap, then the parabolas described on one side of Ap will feather the curve generated by the parabolas on the opposite side of Ap.

Again, if the apex of each parabola passed through the focus of the other, the sun would be in the axis of the curve, like the globe over the forehead of the figure; then the two parabolas would represent the paths of two comets describing parabolas or ellipses round the sun as the common focus. The other globe on the top of the cap might denote a fixed star, or another sun placed beyond any definite distance from the sun.

The Egyptian deities, when in a state of repose, are seated on hyperbolic steps, which decrease as 1, $\frac{1}{2}$, $\frac{1}{3}$, &c. So that the legs and thighs form a right angle, like the side and top of the seat; the thighs and trunk form another right angle, like the top and back of the seat; the arms also form a right angle, like the back and top of the seat. This hyperbolic attitude, which is typical of infinity, gives them a constrained appearance.

Buddha, in the attitude of sitting cross-legged, assumes the form of the hyperbolic solid; the Virginian Okee also assumes the same form; so that by their constrained positions they may be said to represent infinity or eternity.

Wilkinson remarks that the same veneration for ancient usage, and the stern regulations of the priesthood, which forbade any alteration in the form of the human figure, particularly in subjects connected with religion, fettered the genius of the Egyptian artists, and prevented its development. The same formal outline, the attitudes and postures of the body, the same conventional mode of representing the different parts, were adhered to, at the latest as at the earliest periods: no improvements resulting from experience and observation were admitted in the mode of drawing the figure; no attempt was made to copy nature, or to give proper action to the limbs. Certain rules, certain models, had been established by law, and the faulty conceptions of early times were copied and perpetuated by every successive artist. For, as Plato and Synesius inform us, sculptors were not suffered to attempt anything contrary to the regulations laid down regarding the figures of the gods; they were forbidden to introduce any change, or to invent new subjects and habits; and thus the art, and the rules which bound it, always remained the same.

Some of the drawings of the Irish round towers represent them expanding towards the base, like a section of the hyperbolic solid.

PART III.

TOWER OF BELUS. — DESCRIPTION BY HERODOTUS. — CONTENT $\frac{1}{24}$ CIRCUMFERENCE OF THE EARTH. — CUBE OF SIDE OF ENCLOSURE EQUAL TO THE CIRCUMFERENCE OF THE EARTH. — THE EQUIVALENT OF THE STADE, ORGYE, CUBIT, FOOT, AND PALM OF HERODOTUS IN TERMS OF THE EARTH'S CIRCUMFERENCE AND THE STATURE OF MAN. — THE FRENCH MEASUREMENT OF THE EARTH'S CIRCUMFERENCE. — THE CIRCUIT OF LAKE MŒRIS, SIXTY SCHÆNES, COMPARED WITH THE MEDITERRANEAN COAST OF EGYPT; WITH INDIAN TANKS AND CINGALESE ARTIFICIAL LAKES. — HERODOTUS' MEASUREMENT OF THE EUXINE FROM THE BOSPHORUS TO PHASIS; — OF EXISTING OBELISKS. — DIODORUS' DIMENSIONS OF THE CEDAR SHIP OF SESOSTRIS COMPARED WITH MODERN SHIPS AND STEAM VESSELS. — THE CANAL OF SESOSTRIS FROM THE MEDITERRANEAN TO THE RED SEA. — THE EGYPTIAN OBELISKS AT ROME, PARIS, ALEXANDRIA, HELIOPOLIS, FIOUM, THEBES. — COLOSSAL STATUES AT MEMPHIS AND HELIOPOLIS. — MONOLITHS AT BUTOS, SAIS, MEMPHIS, THMOUIS, MAHABALIPURAM. — CELTIC MONUMENTS IN BRITTANY.

Tower of Belus.

RICH, along with Rennell and Porter, concurs in the opinion that the temple of Belus was built upon the site of the tower of Babel, but is at variance as to which of the two ruins, the Mujelibè or Birs Nimroud, is best entitled to the distinction: Rennell decides in favour of the Mujelibè, Rich and Porter incline to the Birs.

The brief notice of the extraordinary event which we find in Genesis serves little other purpose than to assure us of its actual occurrence. The first act of society that we find recorded subsequently to the destruction of the whole human race, except the family of Noah, was an attempt to rally its forces round a common centre, and to organise and cement the new community by some bond of union, indispensable not only to the progress of civilisation, but to the existence of society. We are informed that the place selected for this great experiment was the plain of Shinaar, and that there men proceeded to found a city, with a tower, whose top, in the language of scripture, "should reach to heaven." The real intentions of the founders of this gigantic structure have been the subject of much controversy, which has not hitherto led to any very satisfactory solution.

Herodotus, in describing the tower of Belus as he saw it, says, the Euphrates divides Babylon into two parts; in one part is a square enclosure, with brazen gates, the wall on each side being two stadii, and consecrated to Jupiter Belus. In the middle of this holy place is a solid tower, having the length and depth of a stadium; upon which there is another tower placed, and upon that another, and thus successively to the number of eight. On the outside of these towers are steps winding about, by which they go up to each tower. In the middle of this staircase is a lodge and seats, where those who mount up may rest themselves. In the last tower is a chapel, in the chapel an elegant bed, and near the bed a golden table. Herodotus does not state the height of the tower; but Strabo says that the tomb of Belus was a pyramid, one stadium in height, by a stadium in length and breadth at its base.

Fig. 56, A. Taking the 8 terraces to equal $\frac{8}{9}$ of a stade in height, the height of each terrace will equal $\frac{1}{9}$ of a stade;

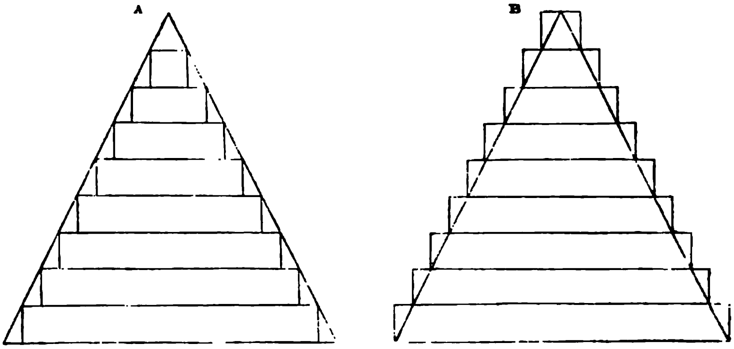


Fig. 56.

and as the side of the base, or the lowest platform on which the lowest tower stands, equals 1 stade = the height from the base to the apex of the teocalli or tower; thus the height of the apex will $= \frac{1}{9}$ stade above the highest platform, or the 8th tower.

Let $\overline{684}^2$ = circumference of the earth in stades; then $\overline{684}^2 \times 243$ will = circumference in units.

These formulæ are obtained by transposing the Babylonian numbers 243, so that the last 3 when placed the first, and the first 2 last, make 342, which multiplied by 2 =684, and 684 raised to the power of $2 = \overline{684}^2 = 467856$ = circumference in stades, and $\overline{684}^2 \times 243 = 113689008$ = circumference in units.

Next let us ascertain the value of the stade and unit in terms of English measurement.

Since 24899 miles, or 131466720 feet, equal the equatorial circumference of the earth (Herschel), then $131466720 \div 467856 = 280{\cdot}99825$, &c. feet = 1 stade.

Hence a Babylonian stade, which = 243 units, may be said to equal 281 feet English; then a Babylonian unit will $= \frac{281}{243}$ or 1·156378, &c. of an English foot, or =13·876, &c. inches.

The content of the tower, if made equal to $\frac{1}{3}\,\overline{243}^3$, would exceed $\frac{1}{24}$ of the earth's circumference, if the cubes of unity were placed in one continuous line.

So would $\overline{486}^3$, or the cube of the side of the square inclosure, exceed in cubes of unity the whole circumference of the earth.

The circumference, measured by cubes of unity, would lie between $\overline{484}^3$ and $\overline{485}^3$; and the content of the tower, to equal $\frac{1}{24}$ the circumference in cubes of unity, would lie between $\frac{1}{3}\,\overline{242}^3$ and $\frac{1}{3}\,\overline{243}^3$.

The way of correcting these differences will be seen in the construction of the Egyptian pyramids.

The sides of the 8 square terraces will be $\frac{1}{9}$, $\frac{2}{9}$, $\frac{3}{9}$, $\frac{4}{9}$, $\frac{5}{9}$, $\frac{6}{9}$, $\frac{7}{9}$, $\frac{8}{9}$ of a stade, so that the top of each of the 8 terraces will touch the sides of the circumscribing triangle, having the base = the height = 1 stade.

Thus the content of the teocalli or terraced pyramid will $= \frac{1}{3}$ of a cubic stade, $= \frac{1}{3}\,\overline{243}^3$ cubic units, by taking the stade to equal 243 units, or 3^5. 9 27

So the height of each terrace will $=3^3=27$ units; the height of the 8 terraces will $=8\times 3^3$. The sides of the terraces will $=1\times 3^3$, 2×3^3, 3×3^3, 4×3^3, 5×3^3, 6×3^3, 7×3^3, 8×3^3. The base of the circumscribing triangle = the height $=9\times 3^3=3^2\times 3^3=3^5=243$ units.

$$\begin{aligned} \text{The base of the pyramid will } &= \overline{243}^2 \\ \text{height } &= 243 \\ \text{and content } &= \tfrac{1}{3}\,\overline{243}^3. \end{aligned}$$

This we suppose to have been the construction of the tower of Belus, for reasons which will be seen when we come to the formation and measurement of the teocallis, or truncated pyramids of America.

Perhaps the lowest platform on which the lowest terrace stood might have been raised; for what is called the great pagoda at Tangore is built of hewn stone, in the form of a truncated pyramid, and consists of 12 perpendicular stories or terraces, the lowest being built on huge blocks of stone, forming the pedestal, rising by 4 steps from the ground. On the top is a temple or chapel.

The content of the 8 terraces will be to the content of the pyramid having the side of base and height equal the base and height of the circumscribing triangle,
$:: 1\,(1^2+2^2+3^2+4^2+5^2+6^2+7^2+8^2) : \frac{1}{3}9\times 9^2 :: 204 : 243.$
For $(1^2+2^2+3^2\ldots\ldots+8^2)=\frac{1}{3}\overline{n+1}\,.\,n\,.\,\overline{n+\frac{1}{2}}=\frac{1}{3}9\times 8\times 8{\cdot}5$ $=204.$

But we only want the content of the complete pyramid having the height and side of base $=1$ stade. So hereafter we shall only ascertain the content of the pyramid having the height and side of base equal the height and base of the triangle that circumscribes the sides and base of the teocalli.

In this calculation of the terraced tower of Belus the sides of the terraces are supposed to be perpendicular; but possibly this was not the case, for all the American teocallis, as far as we know, have the sides of the terraces inclined, excepting one in Peru, where the sides are perpendicular.

But whether the sides were perpendicular or inclined does not affect the content of the pyramid made by this calculation, that the base of the circumscribing triangle equalled the height, equalled 1 stade.

The content of the 9 terraces, B, *fig.* 56., will be to the content of the rectilinear pyramid having the side of the base and height = the base and height of the inclined triangle $:: \frac{1}{3}\overline{n+1} \,.\, n \,.\, \overline{n+\frac{1}{2}} : \frac{1}{3}n^3 :: 285 : 243$, and

$$\tfrac{1}{2}(204+285)=244{\cdot}5$$
$$244{\cdot}5-243=1\tfrac{1}{2}$$

= the difference between the mean of the 8 inscribed and the 9 circumscribing terraces and the rectilinear pyramid $\frac{1}{3}\,9^3$.

Next double the height and side of the base of this pyramid. Then such a rectilinear pyramid will $=\frac{1}{3}\,18^3=1944$. The 17 inscribed terraces will

$$=\tfrac{1}{3}\,\overline{n+1} \,.\, n \,.\, \overline{n+\tfrac{1}{2}}=\tfrac{1}{3}18\times17\times17\tfrac{1}{2}=1785.$$

The 18 circumscribing terraces will

$$=\tfrac{1}{3}19\times18\times18\tfrac{1}{2}=2109$$
$$\tfrac{1}{2}(1785+2109)=1947,$$

and $1947-1944=3=$ the difference between the mean of the two stratified pyramids of 17 and 18 terraces and the rectilinear pyramid $\frac{1}{3}\,18^3$.

Thus the rectilinear pyramid $\frac{1}{3}\,18^3$ is less than the mean of the content of 17 and 18 terraces by 3.

When the content of the rectilinear pyramid $=\frac{1}{3}\,9^3$, the rectilinear pyramid was less than the mean of the two stratified pyramids of 8 and 9 terraces by $1\frac{1}{2}$.

Thus the rectilinear pyramid having the height and side of the base $=n$, will be less than the mean of the content of the two stratified pyramids, the one being within and the other without the triangle $=\frac{1}{2}\,n^2$, by a cubic unit in every 6 terraces, or $\frac{1}{6}$ of a cubic unit in every terrace.

$$1 \text{ cubit} = \frac{33{\cdot}72}{4} = 8{\cdot}43$$

$$1 \text{ foot} = \frac{33{\cdot}72}{6} = 5{\cdot}62$$

$$1 \text{ palm} = \frac{5{\cdot}62}{4} = 1{\cdot}405$$

If an orgye be called b,

then a cubit $= \frac{b}{4}$

a foot $= \frac{b}{6}$

a palm $= \frac{b}{4 \times 6} = \frac{b}{24}$

and a stade $= 100\,b$.

English money is subdivided in the same relative proportion:

Let b = a silver two-shilling piece,

then $\frac{b}{4}$ = a silver sixpence,

$\frac{b}{6}$ = a silver fourpence,

$\frac{b}{4 \times 6} = \frac{b}{24}$ = a copper penny,

and $100\,b$ = a ten-pound note, or ten gold sovereigns.

To express in a popular way the proximate value of the terms in the table of Herodotus, in proportions of a man about 5 feet 7½ inches or 2 orgyes in height.

When the hand is placed flat, the fingers straight and touching each other; then the breadth across the four fingers, in a straight line from the top of the nail of the last or least finger to a little above the nail joint of the first or fore finger, will = 2·81 inches, the half of which will = 1·405 inches = a palm.

Twice the breadth of the four fingers will = 5·62 inches = 4 palms = 1 foot;

And three such breadths will = 8·43 inches = 6 palms = 1 cubit.

If a line be held between the thumb and fore finger of both hands, and the arms stretched horizontally to their full extent, the span, or length of the line, so intercepted, will = 67·44 inches = 2 orgyes.

If the distance so spanned by the arms be called two arms' length, then half the distance may be called one arm's length.

Thus half a span, or an arm's length, will = $\frac{1}{2}$67·44 = 33·72 inches = 1 orgye.

And a span, or two arms' length, will = 67·44 inches, or 5 feet 7$\frac{44}{100}$ inches = 2 orgyes = the height of a man.

Hence 100 arms, or the extended arms of 50 men, will = 1 stade.

And the height of 50 men will = 1 stade.

Also 100 orgyes = 6 plethrons = 1 stade.

By comparing the table of measurement of Herodotus with the corresponding value of each measure expressed in English feet and inches, and then by representing each portion of a stade by a part of the stature of man as its proximate equivalent, we shall have

	E. Inches.					
1	1·405	Palm, παλαιστη.				
2	5·62	4	Foot, πους.			
3	8·43	6	1½	Cubit, πηχυς.		
4	33·72	24	6	4	Orgye, οργυια.	
5	67·44	48	12	8	2	Man's height.

1 = half the breadth of the four fingers = 1 palm.
2 = twice the breadth = 1 foot.
3 = thrice the breadth = 1 cubit.
4 = the length of an arm = 1 orgye.
5 = the height of a man = 2 orgyes.

Possibly such a division of the stade into portions of the stature of man might originally have been given by the hierarchy to the people, as it would greatly assist the memory, and might have aided in establishing the Babylonian stade as the universal standard, since it combines the stature of man and the circumference of the earth.

Better method: —

The distance from the first joint of the thumb to the end of the nail = a palm = 1·405 inches.

When the first and little fingers are spanned, the nearest distance between the ends of the nails = a foot = 5·62 inches.

When the thumb and little finger are spanned, the distance between the ends of the nails = a cubit = 8·43 inches.

The measurement of seventeen mummies has been given by Pettigrew, from which it appears that the Egyptians were short in stature, as the average height of the male is 5 feet 3 inches, and of the female 5 feet.

But the mummies which have been examined seem all to belong to the more modern times of Egyptian history, when the Egyptians were no longer an unmixed Coptic race, as they had been conquered successively by the Arabs of Ethiopia, by the Persians, and by the Greeks.

Thus 100 arms would reach the height of the tower = 1 stade.

The pyramidal tower, which represents the law of gravitation, is supposed to reach from earth to heaven.

Hence the probable origin of the giant Briareus, with his 100 arms, who strove with heaven and made war against Jove. So fifty men of the stature of 2 × 33·72 inches, or 5·62 feet, would equal a stade. The giants also warred against heaven. The heroes or kings of the Assyrians and Egyptians are represented as gigantic in stature when engaged in battle.

Thus we find how the giants of antiquity might have been figuratively great, without supposing their stature to have exceeded that of an ordinary man.

The present Moorish race, inhabiting the vast archipelago of oases in the great Sahara describe a depth as equal to the height of 100 men.

In several of the oases in the Sahara of Algeria, and especially among the Rouara, according to Dumas, the whole irrigation is artificial, and all the water is derived from artesian wells, which have existed time out of mind in those remote regions. The Marabouts relate that an immense subterranean lake lies under the whole tract of the Sahara, at a depth of 25 to 200 fathoms; and the Arabs all declare that, in many of the villages, these artesian wells are 100 men's height in depth. They are square, and supported by beams of the palm tree. When the workman taps the spring below, the water sometimes rushes up with such force as to throw him senseless to the surface of the earth. The public use of these waters is regulated by strict principles of equity, and an injury done to a well is the greatest of crimes. The Sheikh of each village is the recognised protector of the source.

Richard I. caused several standard yards to be made in 1197; and it is said that the term yard was first applied to a measure exactly equalling in length the arm of a preceding monarch, Henry I.

It appears that a wheat-corn was the first standard of weight in England; and it is supposed that the metallic weight called a grain became used as a representative of the wheat-corn, and that the modern troy grain is nearly the same. After a time the pennyweight or "sterling" was reduced from 32 to 24 grains; 20 pennyweights made an ounce, and 12 ounces one pound: this was called the troy pound, and became the standard of English weight, consisting of 5760 grains. But still the legislature could not ensure uniformity in the weights; for there was the moneyer's pound of 5400 grains, the avoirdupois pound of 7000 grains, and the old commercial pound of 7600 grains.

The French weights and measures, until the last sixty years, were in principle but little better. Soon after the Revolution, the French mathematicians turned their attention to the introduction of a decimal system of notation on as extensive a scale as might be practicable. It was proposed to introduce the decimal mode of division into weights and measures, but it was deemed expedient first to obtain a rigorous standard of weight, of length, and of bulk, in lieu of the imperfect ones then in use. For this purpose they sought for a standard among the unchangeable works of nature, as being of more constant application than any of the productions of man. The circumference of the globe was fixed upon; for we have no reason to believe that this circumference increases or diminishes.

The distance of either pole from the equator is mathematically equal to one quarter of the circumference passing through both poles, and is, therefore, called a quadrant; and it was determined to make the ten-millionth part of this quadrant a standard of measure from which a standard of weight might be deduced. The next point, therefore, was to determine the exact number of toises (or any other known measure of length) equal to a quadrant of the earth's circumference. This was a very delicate operation, requiring the resources of the astronomer and the mathematician. The result arrived at was, that the distance from north pole to the equator was equal to 5,130,470 French toises, or 10,936,578 English yards. The ten-millionth part of this quantity was taken as the standard of length, and called a *mètre,* being equal to about 39·371 English inches. From this standard were obtained not only other measures of length, but also measures of weight and of capacity, the decimal mode of subdivision being employed throughout.

Compare the measurements given by Herodotus with the Babylonian standard.

The circuit of the lake Mœris, says Herodotus, equals 3600 stades, or 60 schænes, which is equal to the length of the sea coast of Egypt.

In describing the three mouths of the Nile, he remarks that "one on the east opens to the sea at Pelusium, another on the west at Canopus; the third runs straight through the Delta to the sea." Then he mentions the canals supplied with water from these branches, and proceeds: — "Besides the opinion I have of Egypt is confirmed by the testimony of an oracle, which was delivered by Jupiter Ammon, and which I did not hear till after I was persuaded of what I believe of Egypt. It appears that the inhabitants of the cities of Mœreotis and Apis, which are on the frontiers of Egypt, towards Libya, imagined that they were Libyans and not Egyptians, and as they began to be more negligent of their ceremonies, they would no longer abstain from sacrificing cows, and sent to the temple of Jupiter Ammon, asserting that they had nothing in common with the Egyptians; that they dwelled beyond the province of the Delta; that they spoke not the same language, and, therefore, they pretended that it was allowable for them to eat of everything. But the god would not grant the permission they asked, and answered them that Egypt included all the country that was watered by the Nile; and that all who drank of these waters below the city of Elephantis were Egyptians."

The distance between Lake Mœreotis and Pelusium equals about three degrees of longitude, corresponding to the sea coast of Egypt; so that a degree will equal about 60 miles: then 3 x 60=180 miles for the distance between Lake Mœreotis and Pelusium in a straight line; but the curved coast of the Delta will exceed 180 miles.

Again, 18·79 stades = 1 mile; 3600÷18·79=191 miles, for the circuit of Lake Mœris and the extent of the sea coast of Egypt.

In another place Herodotus says the Egyptian coast, extending from the bay of Plinthene to the lake Selbonis, under Mount Casius, is sixty schænes in length. This would exceed the distance from Lake Mœrotis to Pelusium.

The distance from the sea to Heliopolis (Herod.) equals 1500 stades.

By the map the distance is about 80 miles; then 80 x 18·79 =1500 stades.

The distance of Thebes from the sea is 6120 stades; and 6120÷18·79=325 miles.

By the map the distance is about 360 miles.

The distances by the map are measured in straight lines, and not by the road or river.

Herodotus calls the distance from the sea to Heliopolis

1500 stades; from Heliopolis to Thebes 4860 stades; together they equal 6360 stades.

But he states the distance from the sea to Thebes 6120 stades.

A parasange equals 30 stades.

Volney remarks, that the description Herodotus gives of the soil, climate, and of all the physical state of Egypt is such that our most learned travellers have found as little to add as so criticise in it.

Malte Brun thinks that the famous canal Joseph served to conduct the water of the Nile to the lake Mœris. It is probable that this canal called Joseph, like many other memorable objects, was excavated by order of the king Mœris: the water would then fill the basin of the lake Birket-el-Karoun, to which they might have given the name of the prince, who had caused such a great alteration. Thus may be reconciled the different situations given to the lake Mœris by Herodotus, Diodorus, and Strabo; and why the ancients said that the lake had been formed by the hand of man, since Birket-el-Karoun has no appearance of such a labour.

The canal Joseph, which is partly filled with sand in some places, is about forty leagues in length, and from fifty to three hundred feet in breadth.

The number of the principal canals in all Egypt is about ninety. Mallet, who has included in his calculation all the small canals of derivation, reckons six thousand for Upper Egypt alone.

The Birket-el-Karoun is now only 7 or 8 leagues long, 2 or 3 broad, and 30 in circuit.

Diodorus appears more correct than Herodotus, when he says that Mœris made the lake available for irrigation, not that he dug it.

The following extract is from the popular geographies:—"Westward to Benisuef is the entrance to the fertile valley of Faioum. The chain of mountains that bounds the Libyan side of the Valley of the Nile—elsewhere continuous—here have a narrow opening, which, with a great artificial cut that continues it, admit the waters of the river into the valley.

This tract was, it is thought, the basin of an immense lake, called by the ancients Indris, which formed the grand sluice of the country, that drew off the waters when they were superabundant, and supplied them to the land when deficient. Some considerable dykes, used alternately for retaining and letting off the waters, indicate an extent of human labour only to be credited in the land of the Pyramids. The whole of the plain is about forty miles from east to west, and thirty from north to south; but the lake is at present contracted in breadth to five miles, though it still runs the whole length of the valley; and we are assured, after a close examination of the surrounding land by Jomard and Martin, that the present lake merely occupies a portion of the bed of the former one. In fact, the whole surrounding country bears every evidence of having been abandoned by the waters."

"The entire valley is surrounded by hills, and forms the most compact province in Egypt, rivalling even the Delta both in soil and productions. The eye contemplates with delight its smiling fields, watered by a thousand canals, whose streams, besides giving fertility to the soil, add a picturesque freshness to the landscape. Plantations of roses, celebrated all over the East for their superior perfume, trees bearing the finest fruits, with fields of rice and flax, combine to give a charming diversity to the scene."

This plain, having an extent of 40 miles by 30, will have a circuit of $2 \times 40 + 2 \times 30 = 140$ miles, which is less than the length of the sea-coast of Egypt.

The circuit of Lake Mœris equalled 60 schænes = 3600 stades = 191 miles. The lake was oblong, extending from north to south.

At Symbrumacum, a small town in the Carnatic, is a remarkable large tank, about eight miles in length by three in breadth, which has not been formed by excavation, like those in Bengal, but by shutting up with an artificial bank an opening between two natural ridges of ground. In the dry season the water is let out in small streams for cultivation, and it is said to be sufficient to supply the lands of thirty-two villages (should the rain fail), in which 5000 persons are employed in agricultural pursuits.

Bopal, a town in the province of Malwah, is extensive, and surrounded with a stone wall. Outside of the town is a fort called Futteghghur, built on a solid rock. It has a stone wall with square towers, but no ditch. Under the walls of the fort is a very extensive tank or pond, formed by an embankment at the confluence of five streams issuing from the neighbouring hills. The tank is about six miles in length.—(*East India Gazetteer.*)

Like the pyramids rising out of the middle of Lake Mœris, we find a monument rising from the centre of an Indian lake or tank. Shere Khan, the Afghan, who expelled the emperor Humayoon (the father of Acbar) from Hindostan, was buried at Saseram, in the province of Bahar, in a magnificent mausoleum rising from the centre of a large square lake, which is about a mile in circuit and bounded on each side by masonry, the descent to the water being by a flight of steps, now in ruins. The dome and the rest of the building are of a fine grey stone, at present greatly discoloured by age and neglect.

"The Candelay Lake, about thirty miles from Trincomalee in Ceylon," says De Butt, "is situate in an extensive and broad valley, around which the ground gradually ascends towards the distant hills that envelope it. In the centre of the valley, a causeway, two miles long, principally made of masses of rock, has been constructed to retain the waters that from every side pour into the space enclosed within the circumjacent hills and the artificial dam thus formed. During the rainy season, when the lake attains its greatest elevation, the area of ground over which the inundation extends may be computed at fifteen square miles. This work of art, and others of equally gigantic proportions in the island, sufficiently indicate that at some remote period Ceylon was a densely-populated country, and under a government sufficiently enlightened to appreciate and firm to enforce the execution of an undertaking which, to men ignorant of mechanical powers, must have been an Herculean labour; for such is the capricious nature of the mountain streams in this tropical island, where heavy rain frequently falls without intermission for several successive days, that no common barrier would suffice to resist the great and sudden pressure that must be sustained on such occasions. Aware of this peculiarity in the character of their rivers, the Cingalese built the retaining wall that supports the waters of the Lake of Candelay with such solidity and massiveness as to defy the utmost fury of the mountain torrents. Nearly the whole of its extent is formed with vast masses of hewn rock, to move which by sheer physical force must have required the united labour of thousands. The Cingalese have, from the earliest periods, been attentive to the formation of artificial reservoirs, wherever they could be advantageously constructed; and the Lakes of Candelay, Minere, Bawaly, and many others of less note, attest the energy and perseverance of the ancient islanders in such constructions."

"In Ceylon," observes Campbell, "there are many traces of an early civilisation, remains which show a great advancement in the arts, and that the country was well cultivated and thickly inhabited. There are extensive tracks of ruined canals, one of which was in some parts 15 feet deep and 100 wide. There are stone bridges; in one the stones are from 8 to 14 feet long, jointed into one another, the upright pillars being grooved into the rocks below. The tanks are of an immense extent, with gigantic embankments, and the remains of a canal are seen, which brought the water from one of these tanks sixty miles to Anarajahpoora, the ancient capital. This city was surrounded by a wall sixteen miles square; and there are the ruins of some great pagodas there, two of them 270 feet high, of solid brick-work, and which has been covered over with chunam, a lime cement which takes a polish like marble."

No monuments of antiquity in the island of Ceylon are calculated to impress the traveller with such a conception of the former power and civilisation of the island, as the gigantic ruins of the tanks and reservoirs, in which the water, during the rains, was collected and preserved for the irrigation of their rice lands.

"The number of these structures throughout vast districts now comparatively solitary is quite incredible," says Tennant, and their individual extent far surpasses any works of the kind with which he was acquainted elsewhere. Some of these enormous reservoirs constructed across the gorges of valleys, in order to throw back the streams that thence issue from the hills, cover an area equal to fifteen miles in length by four or five in breadth, and there are hundreds of a minor construction.

These are mostly in ruins. A visit to one is described: it was that of Pathariecaloru, in the Wanny, about seventy miles to the north of Trincomalee, and about twenty-five miles distant from the sea. It is a prodigious work, nearly seven miles in length, at least 300 feet broad at the base, upwards of 60 feet high, and faced throughout its whole extent by layers of square stone. About the centre of the great embankment advantage has been taken of a rock about 200 feet high, which has been built on to give strength to the work. Some wild buffaloes and a deer came to drink from the water-course; these were the only living animals to be seen in any direction. The embankment, estimated at the length of six miles, height 60 feet, breadth at base 200 feet, tapering to 20 at the top, would contain 7,744,000 cubic yards, and at 1*s.* 6*d.* a yard, with the addition of one-half that sum for facing it with stone, and constructing the sluices and other works, it would cost 870,000*l.* sterling to construct the front embankment alone, according to the estimate of the government engineer.

The existing sluice is a very remarkable work, not merely from its dimensions, but from its ingenuity and excellent workmanship. It is built of layers of hewn stones, varying from 6 to 12 feet in length, and still exhibiting a sharp edge, and every mark of the chisel. The ends of the retaining stones are carved with elephants' heads and other devices, like the extremities of Gothic corbels.

As to human habitation, the nearest was the village, where we had passed the preceding night; but we were told that a troop of unsettled Veddahs had lately sown some rice on the verge of the reservoir, and taken their departure after securing their little crop. And this is now the only use to which this gigantic undertaking is subservient; it feeds a few wandering outcasts; and yet, such is its prodigious capabilities, that it might be made to fertilise a district equal in extent to an English county.

Some thirty others, of nearly similar magnitude, are still in existence, but more or less in ruin, throughout a district of 150 miles in length from north to south, and about 90 from sea to sea.

It is said that some one of the sacred books of Ceylon records the name of the king who built this reservoir. It may be remarked that the length of this embankment = 6 miles = one side of the square that enclosed Babylon.

The height of the embankment =60 feet.
„ of the walls of Babylon=70 feet.

The distance from the mouth of the Euxine Sea to the river Phasis is estimated by Herodotus at 11,100 stades. Taking Phasis as the extreme eastern part of the Euxine, as laid down by D'Anville, the latitude of Phasis is 42° north, and a degree of longitude corresponding to latitude 42°= 51·42 miles English, and 18·79 stades=1 mile.

So that 11,100 stades will=11½ degrees of longitude corresponding to latitude 42°. The parallel of longitude between Phasis and the west side of the Euxine includes 13° by the map; but the distance from Phasis to the Bosphorus will be somewhat less than 13°. So that 11,100 stades will very nearly correspond to the distance from the Bosphorus to Phasis, according to modern geography; and this is the distance assigned by Herodotus for the length of the Euxine.

Herodotus makes his calculation by taking the average sailing of a vessel by day and by night, and the time occupied in sailing from the Bosphorus to Phasis he calls nine days and eight nights.

Next, try how this cubit of 8·43 inches English accords with the measurement of any monument, still existing, given by Herodotus in cubits. Now Herodotus states that "Pheron, having recovered his sight, presented to all the temples magnificent offerings; but he made especially to the temple of the Sun what are certainly remarkable and worthy the admiration of man; there he erected two obelisks, each of a single stone, in height 100 cubits, in breadth 8."

The temple of the Sun stood at Heliopolis. Now it appears, according to Ammianus Marcellinus, that three of the Roman obelisks were brought from Heliopolis, two by Augustus, and one conjointly by Constantine and Constantius. The latter is the great Lateran obelisk that formerly stood in the Circus Maximus. It appears that one of the two brought by Augustus was first placed in the Campus Martius; afterwards it was removed to where it now stands on the Monte Citorio. The whole height of the Citorio obelisk from the base to the apex measures 71 feet $5\frac{1}{3}$ inches.

Base ordinate = 8 feet $\frac{4}{1000}$ inch.

Top ordinate = 5 feet $1\frac{74}{1000}$ inch.

The other base ordinate is defective. Now compare the dimensions of this with one of the two obelisks erected at the Temple of the Sun. Taking the height given by Herodotus at 100 cubits, then $8 \cdot 43 \times 100 \div 12 = 70 \cdot 25$ feet, and the whole height of the Citorio obelisk = 71 ft. $5\frac{1}{3}$ inches. Herodotus gives the breadth at 8 cubits. Now $8 \cdot 43 \times 8 \div 12 = 5 \cdot 62$ feet only, a little more than the top ordinate.

Diodorus informs us that Sesoosis erected two obelisks of very hard stone 120 cubits high.

It appears from the inscriptions that the two obelisks which stood in front of the Luxor were erected by Ramses III. One of the Ramses was the Sesoosis of Diodorus and the Sesostris of Herodotus. One of these obelisks has been removed to Paris, which measures 74 French feet, or nearly 81 English feet, in height. The remaining obelisk is 3 French feet higher, which will make the height nearly equal 84·3 English feet.

$$120 \text{ cubits} = 120 \times 8 \cdot 43 \text{ inches}$$
$$= 84 \cdot 3 \text{ English feet.}$$

Ramses II., or the Great (says Sharpe), added to the temple of the Luxor, and set up two obelisks in front of it, one of which is now in Paris.

Ramses III., who is said in the legends chiselled on the face of one of these obelisks, " made these works (the propyla of the palace of the Luxor) for his father, Amun-Ra, and that he had erected these two great obelisks in hard stone before the Ramsesseion of the city of Amun."

Rosellini attributes the rock-cut temple of Abousambel to Ramses III., whom he calls the Great. Wilkinson attributes the same temple to Ramses II., whom he calls Ramses the Great.

In Rosellini's chronology the death of Ramses III. dates 1499 B. C.

Several sovereigns were named Ramses, all belonging to the brilliant era when the great monuments were erected. The name of Ramses is inscribed at Ipsambul, and on numerous monuments of Nubia; on the two obelisks at Alexandria; on three lying on the ground at San, the ancient city of Tanis, the Zoan of Scriptures. The name is perpetuated on durable stone from the northern extremity of Egypt to the southern of Nubia.

Sesoosis, the seventh from Mœris, was greater than any of his predecessors. According to Diodorus, he conquered Arabia and Libya. His army consisted of 600,000 foot, 24,000 horse, 28,000 chariots. He afterwards conquered Ethiopia, India beyond the Ganges, Scythia, and Thrace, and fixed the yearly tribute which the conquered nations should pay. He made two obelisks of hard stone, each 120 cubits high, on which he described the greatness of the kingdom, and the tributes of the subject states.

Sesoosis II., his son, assumed the name of Sesoosis. The son was struck blind, but recovered his sight.

Diodorus mentions that the wall erected by Sesoosis, between Pelusium and Heliopolis, to prevent the plundering excursions of the Arabs, was 1500 stades long, which is the number of stades assigned by Herodotus for the distance from the sea to Heliopolis.

We make the distance from Heliopolis to the nearest coast less than the distance from Heliopolis to Pelusium.

We have also a bulletin of Rameses III. or IV., almost as successful a conqueror as his great ancestor Sesostris. Beneath a painting which depicts his return to Egypt, the following address to his troops is put in his mouth: — "Give yourselves up to joy; let it rise to heaven; the strangers are overthrown. The terror of my name is come over them, and has petrified their hearts. Like a lion I have opposed them, pursued them like a hawk, and have annihilated their guilty souls. I have passed over their rivers, and burned down their fortresses. I am a wall of brass for Egypt. Thou, my father, Ammon Ra, hast so commanded me, and I have pursued the barbarians; I have passed victoriously through all parts of the earth, till at length the world itself withdrew from my steps. My arm subdued the kings of the earth, and my foot trampled on the nations."

This reminds one of another affiliated child of Ammon, who, after having subdued the kings of the earth and trampled on the nations, cried for more worlds to conquer.

The oriental bulletin of Buonaparte reminded his troops that the ages of 4000 years were regarding them from the summit of the great pyramid.

The ages at different periods had also looked down from those pyramids on the armies of Rameses, Cambyses, Alexander, and many other triumphant kings, fluttering in the sunshine of glory.

The obelisk in front of St. Peter's at Rome formerly stood in the Vatican Circus. Pliny says it was cut by Nuncoreus, the son of Sesostris, who corresponds to the Pheros of Herodotus. It seems to have been broken, and to have lost part of its length; yet it is still 83 feet 2 inches, or 120 cubits high.

Diodorus mentions that Sesoosis placed in the temple of Vulcan his own and his wife's statue, 30 cubits in height. Herodotus states that Sesostris erected several statues at the entrance of Vulcan's temple. Two of these, representing himself and wife, are 30 cubits in height; and four other statues, representing his four sons, are 20 cubits each.

So it appears that Diodorus and Herodotus made use of the same cubit in measuring these statues; hence we may infer that they used the same cubit, that of Babylon, 8·43 inches, in their measurements of obelisks.

If the obelisk at St. Peter's be 120 cubits high, it cannot be one of the two obelisks erected by Pheros. Neither can the Lateran obelisk, which is said to have been brought from Heliopolis, have been one of Pheros' obelisks; for this is said to be the largest obelisk in the world, measuring from the base to the apex 105 feet 7 inches, or 150 cubits. The sole remaining obelisk at Heliopolis is 67½ feet high, according to Pocock; so this may be one of Pheros' obelisks, the companion to the Citorio obelisk. If so, one of the obelisks of Pheros, erected at Heliopolis, will be 100 cubits high, and the other rather less in height. So will one of the obelisks erected by Sesostris at the Luxor equal 120 cubits in height, and the other rather less.

If the cubit of Diodorus be considered equal to the cubit of Herodotus, or of Babylon, we can measure the length of the ship of cedar wood built by Sesostris.

Diodorus informs us that Sesostris having constructed a ship of cedar-wood, 280 cubits long, lined the inside with silver, and the outside with gold, made an offering of it to the god whom they adore at Thebes.

280 × 8·43 inches = 196·7 feet English for the length of Sesostris' ship.

Now the Gipsy Queen, an iron steamer built on the banks of the Thames, measures in length from the figure-head to the taffrail, 197 feet 6 inches, and between the perpendiculars 175 feet. Breadth between the paddle-boxes, 24 feet. Burden 496 tons. Engines 240 horse power.

What is generally considered as constituting a horse power is a power to raise 130 pounds 100 feet in one minute.

The priests told Herodotus that Sesostris was the first king who, passing through the Arabian Gulf with a fleet of long ships, subdued those nations that inhabit the Red Sea.

The materials for ships were formerly transported overland from Gaza to the Red Sea, having been originally brought from Mount Lebanon. This is a common occurrence at the present day on the shores of the Red Sea, where no tree grows. Laborde mentions that scarcely a year elapses in which the timbers of vessels may not be seen passing in single pieces, through the streets of Suez, on their way to the shore, in order to be put together and launched.

In this manner, the cedar ship of Sesostris might have been built on the shores of the Red Sea with the cedars of Lebanon.

Necus, the son of Psammitichus, was the first, according to Herodotus, who attempted to dig a canal from the Nile to the Red Sea, which was afterwards completed by Darius, the Persian; so broad that two vessels could easily sail on it together. It extended from a little above Bubastis, not far from the modern Grand Cairo, on the Nile, to Patumos, a city of Arabia on the Red Sea, near the present Suez, about four days' sail. Strabo says this canal was first cut by Sesostris, before the Trojan war, and that it terminated at the city Arsinoë, or Cleopatris. He makes it 100 cubits broad. Pliny makes it 100 feet broad, and 30 deep. Both these authors say that Darius was prevented from finishing the canal, from an apprehension that the Red Sea, being higher than the land of Egypt, if let in would inundate the country and spoil the waters of the Nile. This canal was finished or renewed by the Ptolemies. It was cleaned by Trajan, and afterwards restored by the Arabs in the time of Omar. It is now choked up; and the trade between Cairo and Suez is carried on by caravans.

Herodotus says 120,000 men perished in digging this canal under Necus. The king being hindered from finishing it by an oracle, built a number of ships, partly on the Mediterranean, which Herodotus calls the North Sea, and partly on the Arabian Gulf. Some of these he ordered to sail round Africa, which voyage they performed.

Napoleon, accompanied by the French engineers in 1799, made a survey of the Suez canal. He was the first to discover the undoubted traces of the canal of Sesostris, which he followed from the northern point of the Gulf of Suez for several leagues, and found that they were lost in the dry basin of the Bitter Lakes. This ancient work extends in a direct line north, through the trough or valley, for 13½ English miles. The walls of the canal are of solid masonry, from 6 to 16 feet deep, and the space between them is 146 English feet. Strabo states it at 150 feet. The breadth at the bottom of the canal, according to the plan, is not given; but as the banks are inclined, this breadth may have been about half a stade, 200 cubits, or 140½ English feet.

The bed of the canal has been raised by sand and earth, washed into it by the torrents; and a new and higher bed has been curiously consolidated by natural means from the effect of calcareous filtrations. The French engineers dug through the fictitious bed, and found the real bed four or five feet beneath it. They then detected the artificial composition employed by the ancient engineers for retaining the waters of the canal, which they found to consist of moist saline sand, earthy clay, and gypsum.

The French line, resulting from Jacotin's survey, passes through the bed of the Bitter Lakes, the lake El Timseh, thence to the marshy grounds of El Karesh (nearly on a level with the Red Sea), thence to Dar El Casseh, afterwards to El Dowade; thence the line follows the traces of the old canal, and the ruins of the wall of defence of Sesostris, in a direct line, the ground being sandy, and lower than the Red Sea; hence to the occasionally flooded strip of land by Lake Menzaleh, where the excavation of the ancient canal reappears in a sandy valley; thence to the entrance of Tineh, passing between Faramah and Pelusium, where the land (having gradually declined, unobstructedly, the whole way from El Karesh) is 29 feet 6 inches lower than the Red Sea.

The length of this line is 85 miles (being prolonged to save expense).

Linant, an engineer who surveyed the Isthmus in 1841–2, confirms the report and survey of Jacotin and the French engineers of 1799; and recommends the same line, both on account of its practicability and economy.

An iron steam yacht for the Pacha of Egypt was launched from the banks of the Thames at Blackwall, in 1851. Burthen 2200 tons. Dimensions, — length between the perpendiculars 282 feet; length of keel for tonnage 258 feet; breadth for tonnage 40 feet; depth in hold 39 feet; draught of water 18 feet. Machinery 800 horse-power. She is pierced for the following number of guns: — Spar deck, twelve 10-inch 84-pounders broadside, 56 cwt.; spar deck, twelve 10-inch 84-pounders pivot guns, 85 cwt.; main deck, fourteen 10-inch 32-pounders broadside, 56 cwt. Constructed ostensibly for a yacht, she can be turned into the most powerful steamer afloat for war purposes.

Length between perpendiculars = 282 feet,
281 feet = 1 stade
= 400 cubits.

Length of the cedar ship of Sesostris = 280 cubits.

The Great Britain steam-ship is built entirely of iron, with the exception of the flooring of her decks and the flooring and ornamental parts of her cabins. She is 322 feet in length from figure-head to taffrail, and 50 feet 6 inches in breadth. She is registered at 3500 tons, so that her bulk was at the time she was launched nearly equal to any two steamers in the world. She has four decks, the lowest of which is of iron. The upper deck is flush from stem to stern, measuring 308 feet. She has four engines of 250 horse power each, and is fitted with the Archimedian screw propeller.

The American ocean steam-ship Arctic is 3000 tons measurement; length of keel 275 feet, of main deck 284 feet. Draught on her trial trip 18 feet, when fully loaded 19. The diameter of the wheels 35½ feet. The engines weigh 750 tons; their boilers contain 250 tons of water, of which they evaporate 8000 gallons an hour, with a consumption of 2¾ tons of anthracite coal in the same time. It takes ten engineers and assistants, 24 firemen, and 24 coal heavers, working in three gangs, with relays of 8 hours each, to direct, feed, and operate them.

The length of the main deck exceeds 1 stade by 3 feet. The diameters of the wheels exceed $\frac{1}{8}$ stade by $\frac{3}{8}$ of a foot.

The Himalaya, built of iron, at Blackwall, on the banks of the Thames, is the largest ocean steam-ship in the world. She is 3550 tons register, equal to 4000 tons burden, and is of the extraordinary length of 372 feet 9 inches. The length of the keel is 311 feet; breadth for tonnage 46 feet 2 inches; depth of hold 24 feet 9 inches. These proportions, when contrasted with the dimensions of other ships, give a great advantage, particularly in length, to the Himalaya; for example, the Duke of Wellington, a screw line of battle ship, of 131 guns, although of a greater beam and depth, is inferior in length by 92 feet to the Himalaya. The iron screw steamer Great Britain is 40 feet shorter than the Himalaya, while the American clipper ship Great Republic, recently destroyed by fire in New York, was 47 feet less in length than the Himalaya. Although the Himalaya exceeds in so large a degree the length of the Duke of Wellington, yet she is inferior in tonnage to that ship by 209 tons.

The spar deck of the Himalaya is flush from stem to stern. An uninterrupted promenade of 375 feet, or 125 yards, is here provided. To walk round the spar deck precisely one-seventh of a mile has to be traversed. The engines are 700 horse power. The saloon, nearly 100 feet in length, will dine 170 persons. The bed cabins are the largest ever yet appropriated to marine travellers.

The Chinese Junk, lately arrived in London from China by the Cape of Good Hope, measures in length 165 feet; height of stern, 40 feet; burthen about 700 tons. This is the first Chinese junk that has been seen in England; hitherto it has been supposed that Chinese vessels were unable to make extensive voyages, and therefore precluded from making discoveries. It is now proved that they are capable of circumnavigating the globe.

This junk sailed from Canton, rounded the Cape of Good

Hope, anchored at St. Helena, thence visited New York, North America, and ultimately arrived at London.

The largest Chinese junks are about 1000 tons burden. The Chinese rarely make long voyages, for though they have been for many centuries acquainted with the use of the compass, they seldom lose sight of the coast. In their trading to Singapore, Batavia, and New Holland, they employ a foreign master, who is generally a Portuguese. The Chinese think that the magnetic attraction is to the south, and therefore have that end of the needle coloured red. They have only twenty-four points in their compass. On the bows are placed two large eyes. There is, neither in the building nor in the rigging and fitting up of a Chinese junk, one single thing which is similar to what we see on board a European vessel. From her peculiar form, her measurement has not been ascertained, but it is supposed that she may measure about 400 tons, and carry 700. The figure of a cock is one of the zodiacal constellations of the Chinese. It is represented on the stern with expanded wings.

Athenæus thus describes a ship given to Philopater by Hiero, King of Syracuse. It was built under the care of Archimedes, and its timbers would have made sixty triremes. Besides baths and rooms for pleasures of all kinds, it had a library, and astronomical instruments, not for navigation, as in modern ships, but for study, as in an observatory. It was a ship of war, and had eight towers, from each of which stones were thrown at the enemy by six men. Its machines, like modern cannons, could throw stones of 300 lbs. weight, and arrows of 18 feet in length. It had four anchors of wood and eight of iron. It was called the ship of Syracuse, but after it had been given to Philopater, it was known by the name of the ship of Alexandria.

The royal barge, in which the king and court moved on the quiet waters of the Nile, was 330 feet long, and 45 feet wide. It was fitted up with state rooms and private rooms, and was nearly 60 feet high to the top of the royal awning.

According to Plutarch, Ptolemy Philopater built a vessel of forty benches of oars, which was 420 feet long, and 72 from the keel to the top of the poop, and carried 400 sailors, besides 4000 rowers, and near 3000 soldiers. Pliny says that it had fifty benches; and he mentions another of Ptolemy Philadelphus with forty.

Trajan selected Lake Aricinus (now the Lake of Nemi) as the scene of his retreat from the care of government. This lake is at the distance of about fifteen miles from Rome, in the vicinity of the Appian Way, and is surrounded with hills covered with trees, and always verdant. The atmosphere is salubrious and temperate, the soil fertile, and the scenery most beautiful, boasting, among other attractions, of the grotto and fountain of Egeria, so celebrated in the time of Numa Pompilius. The lake itself is very deep, and the water clear as crystal. It was here Trajan caused to be constructed a ship or bark of an immense size, composed of the most durable and expensive timber, on which a palace, decorated and adorned in a magnificent manner, was erected. The roof was supported and ornamented with massive beams of brass; the pavement was inland with stones of the most varied and beautiful colours; and the Egerian water was conducted by leaden pipes into the vessel, where it formed a refreshing fountain. The shores of the lake were laid out in gardens, planted with a diversity of trees and shrubs, and intersected with serpentine walks. Everything that imagination could suggest was effected to improve and assist the natural beauties of the place. The bark was moored in the centre of the lake, and was built with the greatest strength and solidity; the planks were of extraordinary thickness, and fastened not only with nails, of which great quantities were used, but also by smaller planks inserted in grooves, and secured in the most effectual manner. The outside was sheathed with plates of lead of a double thickness where exposed to the action of the water, and between the planks and sheathing were placed woollen cloths saturated with oil and pitch, in order to preserve the timbers from the water. The whole structure was most magnificent, and well fitted for the retirement of a prince. It was, however, in succeeding ages, and during the tyranny and misgovernment, the wars and troubles, the barbarian inroads, and the factious dissensions that ravaged Italy and the tributary states, and which caused the fall of the Roman Empire, neglected and suffered to fall into decay. Time and storms gradually reduced it to ruins, and it eventually sunk to the bottom of the lake, where it still remains imbedded and almost forgotten.

Marchi, in his account of his descent in a diving-machine, states that it was then (A. D. 1535) 1340 years or more since the bark was submersed at the spot where it then remained sunk, at a great depth, by the eastern edge of the lake. He contrived to measure the bark, which he found to be, in English measure, about 500 feet in length by 270 in breadth, and 60 in depth. If we compare these dimensions with a British man-of-war, we shall have some idea of the immense size of the floating vessel, and of the importance of the building erected on it. The length of a first-rate ship of war of 120 guns is about 205 feet (or two fifths of that of Trajan's floating palace), and the breadth 53 feet, being less than one-fifth the dimensions of the bark.

This floating palace has recently been raised up; the timbers, which were of cypress and larch, were found sound after 1400 years' immersion.

Ordinates of the Obelisk.

Fig. 57C. Let ABCD represent the four sides of an obelisk, having the two greater sides AD, BC, equal, and the two less sides, AB, DC, also equal. The greater square equals the

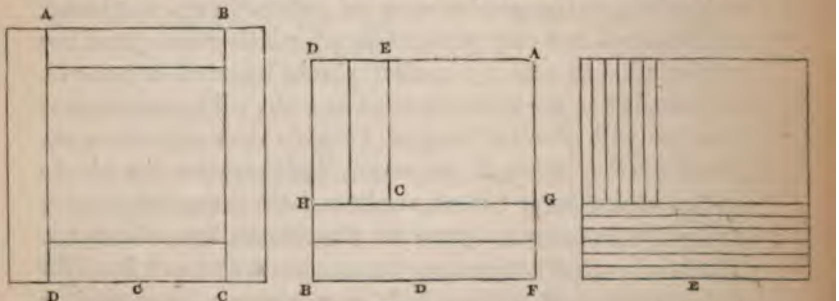


Fig. 57.

square of the greater side, and the less square the square of the less side.

The following calculations are made for two square obelisks, one having the square of the greater side greater than the base of the obelisk; the other having the square of the less side less than the base of the obelisk ABCD.

The difference of the squares
$= \frac{1}{2}$ the perimeters of the 2 squares × $\frac{1}{2}$ their difference,
$= \frac{1}{4}$ the perimeters × their difference
= the rectangle by the sum of the two sides of the squares and their difference.

The rectangle of the sum and difference of the sides of two squares, or the rectangle of the sum and difference of the two ordinates, = the difference of their squares, or sectional axis of the obelisk.

Fig. D. AB, AC, are two squares,
rectangled parallelogram FH + rectangled parallelogram HE = their difference,
FG or DE the difference of their sides,
$FG \times FB + DE \times EC = FH + DC$
or $\overline{FB + EC} \times FG$ or $DE = FH + DC$.

Let $AF = 6$, $AG = 4$,
then $AF^2 - AG^2 = 6^2 - 4^2 =$ area $HF + HE = 6 \times 2 + 4 \times 2 = 12 + 8 = 20$ square units.

Thus the ordinates $6^2 - 4^2 =$ a line of 20 square units = a line of the length of 20 linear units = the axis intercepted by the two ordinates 6 and 4.

Fig. E. Let the sides of the square ordinates be 18 and 12; $18^2 - 12^2 = 324 - 144 = 180$, and $6 \times \overline{18 + 12} = 180 =$ axis intercepted by the two ordinates.

When the difference of the two ordinates = 1, the sum of the two ordinates = the difference of their squares:

$$\text{As } \begin{array}{r} 10^2 = 100 \\ 9^2 = 81 \\ \hline 19 = 19 \end{array} \qquad \begin{array}{r} 23^2 = 529 \\ 22^2 = 484 \\ \hline 45 = 45 \end{array}$$

When the difference of the two ordinates = 2, twice the sum of the two ordinates = the difference of their squares.

$$\begin{array}{rr} \text{As, } 10^2=100 & 23^2=529 \\ 8^2=\ \ 64 & 21^2=441 \\ \hline 2\times 18 = 36 & 2\times 44 = 88. \end{array}$$

When the difference of the two ordinates = 3, three times the sum of the two ordinates = the difference of their squares :

$$\begin{array}{rr} \text{As, } 23^2=529 & 232^2=53824 \\ 20^2=400 & 229^2=52441 \\ \hline 3\times 43 = 129 & 3\times 461 = 1383. \end{array}$$

When the difference of the two ordinates = n, then n times the sum of the two ordinates = the difference of their squares; or the difference of the two ordinates = 2 n times the greater ordinate less n^2, or = $2n$ times the less ordinate $+n^2$:

$$\begin{array}{l} \text{As, } 23^2=529 \\ \quad\ \ 17^2=289 \\ \hline 6\times 40 = 240 = 12\times 23-6^2 = 12\times 17+6^2 \\ \quad\ \ 23^2=529 \\ \quad\ \ 15^2=225 \\ \hline 8\times 38 = 304 = 16\times 23-8^2 = 16\times 15+8^2. \end{array}$$

Obelisks.

We shall quote from the "Library of Entertaining Knowledge" some extracts and the dimensions of the Egyptian obelisks now at Rome.

"Of all the works of Egyptian art," says the writer, "which, by the simplicity of their form, their colossal size and unity, and the beauty of their sculptured decorations, excite our wonder and admiration, none can be put in comparison with the obelisks. As lasting records of those ancient monarchs, whose names and titles are sculptured on them, they possess a high historical value, which is increased by the fact that some of the most remarkable of these venerable monuments now adorn the Roman capital. The Cæsars seem to have vied with one another in transporting these enormous blocks from their native soil; and since the revival of the study of antiquities in Rome, the most enlightened of her pontiffs have erected those which had fallen down and were lying on the ground in fragments.

"An obelisk is a single block of granite, cut into a quadrilateral form. The horizontal width of each side diminishes gradually, but almost imperceptibly, from the base to the top of the shaft, which is crowned by a small pyramid. Most obelisks, of which any accurate dimensions have been given, have only the opposite pairs of sides equal; one pair often exceeding the other in the horizontal breadth by 6 or 7 inches, or even more than a foot. As an obelisk rises from its base in one continuous unbroken line, the eye, as it measures its height by following the clearly defined edges, meets with no interruption, while the absence of all small lines of division allows the mind to be fully impressed with the colossal unity of the mass.

"It would appear, from the inspection of the great gateway of the Luxor, from the remains of Heliopolis, and the two obelisks at Alexandria, that they were principally used in pairs, and placed on each side of the propyla, or great entrance of a temple. But they were also placed occasionally within the interior of the temples, but still in front of gateways, as at Carnak; just as small obelisks are said to be found within the rock-cut temples of Ellora.

"Of the two obelisks at Alexandria only one is standing. But they must have been both standing when Abd-el-Latif wrote, about the close of the twelfth century; for he says he saw two obelisks near the sea, without making any mention of one of them being on the ground; though when he speaks of the two obelisks of Heliopolis he takes care to say that one of them had fallen.

"The Lateran obelisk now stands before the north portico of the Lateran church at Rome, where it was placed in 1588 A. D. This is the largest of all the Roman obelisks, and perhaps the largest in the world. It is the same which the Emperor Constantius erected in the Circus Maximus. Mercati, who carefully measured it when lying on the ground, says it was broken into three pieces. The whole length of the three parts was 148 Roman palms; but the base of the lowest part was so much damaged that it was necessary to take off four palms before it could be safely set on its pedestal. This reduces the length of the shaft to 144 palms, or 105 feet 7 inches English. The whole height, with the pedestal and ornaments at the top, is about 150 feet. The sides of the obelisk are not all of equal breadth. The width of the north and south sides (as they now stand) at the base is 9 feet 8⅗ inches; the width of the same sides below the pyramidal top is 6 feet 9⅓ inches. The two other sides at the base and top are respectively 9 feet and 5 feet 8 inches. The obelisk is of Syene granite. The whole surface from the base to the very pointed top is covered with exquisite sculptures, superior to those of the other obelisks at Rome."

Let us find a unit such that the difference between the squares of the base and top ordinates shall equal the height of the shaft intercepted by these ordinates. Such a unit for the Lateran obelisk will = 6 inches English.

Elsewhere it is said the pyramidal top of the Lateran obelisk surpasses the width of the base by about one-third.

	F.	I.
So, from the entire height	105	7
Deduct for the pyramid, say ...	8	1
Then the height of the shaft =	97	6

	F.	I.	
1st base ordinate =	9	8⅗	= 19·43 units
1st top ordinate =	6	9⅓	= 13·55 ,,

and $\overline{19\cdot43}^2 = 377$

$\overline{13\cdot55}^2 = 183$

$2\overline{)194}$

Height = 97 feet

Measured height = 97½ feet.

The ordinates on the two other sides are,

	F.	I.	
2nd base ordinate =	9	0	= 18 units,
2nd top ordinate =	5	8	= 11·33 ,,

and $\overline{18}^2 = 324$

$\overline{11 \cdot 33}^2 = \underline{128}$

$2\overline{)196}$

Height $= \overline{98}$ feet

Measured height $=$ 97½ feet.

The whole height of the obelisk at present
$=$ 105 feet 7 inches $=$ 144 Roman palms,
and 105⅜ feet $=$ 150 cubits $=$ ⅜ stade.

The whole height, with pedestal and ornaments, $=$ 150 feet.

The addition of 4 palms, for the part cut off, would make the original height of this obelisk $=$ 154 cubits. The unit $=$ 6 inches, or nearly so, for the two different sides of the obelisk.

The height of the apex of the obelisk above the top of the shaft, or base of the pyramid, corresponding to the two greater sides will $=$ 183 units. The height of the apex for the other two sides above the shaft will $=$ 128 units. Difference $=$ 55 units, or 27½ feet.

Pliny, speaking of the two large obelisks in his time, one of which stood in the Campus Martius, and the other in the Circus Maximus, the latter being the Lateran obelisk, says, "The inscriptions on them contain the interpretation of the laws of nature, the results of the philosophy of the Egyptians."

On first beholding these obelisks, with their unbroken outlines, their forms appeared as mysterious to us as their hieroglyphics still continue to be. So we must leave others to ascertain whether any of the inscriptions admit of the interpretation mentioned by Pliny. But should that not be the case, still the obelisk itself, without any inscription, contains the interpretation of the laws of nature. Champollion remarks that the Lateran obelisk belongs to Thouthmosis.

If the height, from the base of the obelisk to the apex of the pyramid on the top, be made the height of a pyramid, similar to the top pyramid, then the content of the supposed pyramid may be found. Thus the supposed pyramid will be similar to the pyramid on the top of the obelisk, and their contents will be as the cube of their heights.

The part cut off the truncated obelisk is wanting; but the truncated part is seen.

The part cut off the truncated pyramid is seen; but the truncated part is wanting.

The second obelisk in size is that which C. Cæsar erected in the Vatican circus; it was removed in the time of Sextus V. to its present position in front of St. Peter's, and was the first of the four which this pontiff restored. There are no hieroglyphics upon it. Pliny says it was cut by Nuncoreus, the son of Sesostris, who corresponds to the Pheros of Herodotus. It seems to have been broken, and to have lost part of its length; yet it is still 83 feet 2 inches high (without the modern ornament at the top), of which six feet belong to the pyramidal apex. Each side is said to be of equal width, being at the base 8 feet 10 inches, and under the pyramid about 5 feet 11 inches.

The height of the shaft will = 83 feet 2 inches, less 6 feet = 77 feet 2 inches.

Let the unit = 6·66 inches,

then base ordinate = 8 feet 10 inches = 15·92 units
top ordinate = 5 feet 11 inches = 10·66
and $\overline{15{\cdot}92}^2 = 253$
$\overline{10{\cdot}66}^2 = 113{\cdot}6$
height = $\overline{139{\cdot}4}$ units
= 77·3 feet

measured height = 77 feet 2 inches.

It appears, however, that there are great discrepancies about the dimensions of this obelisk, which induced Zoëga to conclude that a more exact measurement was necessary, in order to determine if this were one of the obelisks of Pheros or not. It is, however, not easy to measure the obelisk at present. The whole height, with the pedestal and cross at the summit, is about 132 feet.

The two obelisks of Pheros each equalled 100 cubits in height.

$\therefore 100 \times 8{\cdot}43$ inches = $70\frac{1}{4}$ feet English,

which is less than the obelisk at St. Peter's.

St. Peter's obelisk is said to have lost part of its length, yet its present height is 83 feet 2 inches.

120 cubits = 84·3 feet.

Richardson says, near the centre of the great temple of Carnac there are three noble obelisks, about 70 feet high, and 9 square at the base; a fourth obelisk is lying on the ground, cut into two pieces.

In the vicinity of Syene, now Assouan, are those extensive quarries which furnished the ancient Egyptians with materials for their colossal statues and obelisks. Here is still to be seen a half-formed obelisk, between 70 and 80 feet long.

The Flaminian obelisk (Flaminio del Popolo) is the next in size to the Vatican. This was one of the two obelisks that Augustus transported to Rome and erected in the Great Circus. It consists of three parts, which altogether, according to Mercati's measurements, made up 110 Roman palms; but three palms were cut off from the lower part before it was put up in its present position, which will reduce the height to about 78 feet 5 inches. The sides are of unequal width; those on the north and south, which correspond, are 7 feet 10 inches at the base and 4 feet 10 inches at the top. The other two, at the same positions respectively, are, at the base, 6 feet 11 inches and 4 feet 1 inch. The northern face of this obelisk shows marks of damage from fire, but the other sides are uninjured.

No mention is made of the pyramidal top. In an engraving of this obelisk the height of the pyramid exceeds the side of the base.

Call the height 5 feet 5 inches:

Then the height of the shaft will = 78 feet 5 inches less 5 feet 5 inches = 73 feet.

Let the 1st unit = 6·164 inches:

Then, 1 . base ordinate = 7 feet 10 inches = 15·25 units,
1 . top ordinate = 4 feet 10 inches = 9·408 „

and $\overline{15{\cdot}25}^{2} = 232{\cdot}5$
$\overline{9{\cdot}408}^{2} = \underline{88{\cdot}5}$
height = 144 units,
= 73·96 feet.

Measured height = 73 feet.

The unit for the less side of this obelisk will = $\frac{5}{6}$ the unit of the greater side = $\frac{5}{6}$ 6·164 = 5·12 inches.

Then 2nd base ordinate = 6 feet 11 inches = 16·21 units,
2nd top ordinate = 4 feet 1 inch = 9·55 „

and
$$\begin{aligned} \overline{16{\cdot}21}^2 &= 262{\cdot}5 \\ \overline{9{\cdot}55}^2 &= \underline{91} \\ \text{height} &= 171{\cdot}5 \text{ units} \\ &= 73 \text{ feet.} \\ \text{Measured height} &= 73 \text{ feet.} \end{aligned}$$

Or, let the unit of the greater sides = 6·2 inches:

Then, 1. base ordinate = 7 feet 10 inches = 15·15 units,
1. top ordinate = 4 feet 10 inches = 9·35 „

and
$$\begin{aligned} \overline{15{\cdot}15}^2 &= 229{\cdot}5 \\ \overline{9{\cdot}35}^2 &= \underline{87{\cdot}5} \\ \text{axis or height} &= 142 \text{ units,} \\ &= 73{\cdot}3 \text{ feet.} \\ \text{Measured height} &= 73 \text{ feet.} \end{aligned}$$

Let the unit of less sides = 5·12 inches.

Then, 2nd base ordinate = 6 feet 11 inches = 16·12 units,
2nd top ordinate = 4 feet 1 inch = 9·55 „

and
$$\begin{aligned} \overline{16{\cdot}12}^2 &= 262{\cdot}5 \\ \overline{9{\cdot}55}^2 &= \underline{91} \\ \text{axis or height} &= 171{\cdot}5 \text{ units,} \\ &= 73 \text{ feet.} \\ \text{Measured height} &= 73 \text{ feet.} \end{aligned}$$

The mean of the two different units

$$\begin{aligned} &= \tfrac{1}{2}(6{\cdot}2 + 5{\cdot}12) = 5{\cdot}66 \text{ inches,} \\ \text{a Babylonian foot} &= 5{\cdot}62 \quad \text{„} \end{aligned}$$

Height from base to apex = 78 feet 5 inches,
110 cubits = 110 × 8·43 inches = 77·27 feet,
110 Roman palms was the original height.

If to the present height, 78 feet 5 inches, there be added 2 feet 5 inches for the part cut off, we shall have for the original height of the obelisk, from the base to the pyramidal top, 80 feet 6 inches, which = 115 cubits.

The Citorio obelisk is the fourth in size. Augustus placed this obelisk in the Campus Martius as a sun-dial. It was erected on the Monte Citorio in 1792 by Pius VI. It is about 71 feet $5\frac{1}{2}$ inches English in length. The height of the pyramidal top is 5 feet $\frac{578}{1000}$ inch. The south and north bases of the pyramid measure respectively 4 feet $11\frac{4}{5}$ inches; the east and west, 5 feet $1\frac{74}{1000}$ inch. The eastern and western sides of the base of the shaft measure each 8 feet $\frac{4}{1000}$ inch. The bases on the north and south sides could not be measured, on account of the corrosion of the granite. The whole height of this obelisk, with its pedestal, is about 110 feet. This obelisk of the Campus was found broken in four pieces, the lowest of which was so injured by fire that it was necessary to substitute in its place another block of the same size; the sculptures are also damaged on the remaining parts.

	F.	I.
Height of the obelisk =	71	5·578,
" " pyramid =	5	5·578, say
Height of shaft =	66 feet.	

Let the unit of the Citorio

$= \frac{1}{2}$ a Babylonian unit

$= \frac{1}{2} \times \frac{281}{2{\cdot}43}$ of a foot = 6·9382, &c. inches.

	F.	I.	
Base ordinate =	8	$\frac{4}{1000}$	= 13·83 units,
Top ordinate =	5	$1\frac{74}{1000}$	= 8·8 "

and $\overline{13{\cdot}83}^2 = 191$

$\overline{8{\cdot}\ 8}^2 = 77$

Height = 114 units = 57 Babylonian units,

= 66 feet.

Measured height = 66 feet.

The height of the Citorio obelisk, from its base to the apex of the pyramid = 71½ feet, which corresponds with the height of one of the obelisks of Pheros = 100 cubits = 70¼ feet.

Pliny says this obelisk came from Heliopolis, and was the work of King Sesostris.

The measure of the ordinates of the four largest obelisks only are given; but, including the false obelisks, there are altogether twelve at Rome.

There are two obelisks at Alexandria; but only one of them is standing, which is called Cleopatra's Needle. Its dimensions are:—

	F.	I.
Width of one base - - - - - - -	8	2
Width of same face of the obelisk at the base of the pyramidal top - - - - - -	5	1⅞
Width of the adjacent base (the two opposite ones, as usual, being equal) - - - -	7	$8\frac{1}{10}$
Width of base of pyramidal top - - -	4	8⅛
Height of obelisk from base of shaft to base of pyramidal top - - - - - - -	57	$6\frac{2}{8}$
Height of pyramidal top - - - - -	6	6⅞
Whole height of obelisk - - - - -	64	1⅛

These dimensions of the base are not taken quite at the bottom of the shaft, but on one side 3 feet and ¼ inch above the bottom, and on the other side somewhat less.

Let the unit of the greater sides = 8·43 inches = a Babylonian cubit.

	F.	I.	
1st base ordinate =	8	2	= 11·62 units,
1st top ordinate =	5	1⅞	= 7·35 ,,

and

$$\overline{11\cdot62}^2 = 135$$

$$\overline{7\cdot35}^2 = \underline{54}$$

$$\text{Height} = 81 \text{ units,}$$

$$= 57 \text{ feet.}$$

Measured height = 57½ feet.

Let the unit for the less sides = $1-\frac{1}{20}$ cubit = 8 inches.

F. I.
2nd base ordinate = 7 8·7 = 11·6 units,
2nd top ordinate = 4 8·5 = 7·06 „

and $\overline{11\ 6}^2 = 134{\cdot}56$

$\overline{7{\cdot}06}^2 = \underline{\ 49{\cdot}84}$

Height = 84·72 units,
= 56·48 feet.
Measured height = $57\frac{1}{2}$ feet.

The whole height of the obelisk, from the base to the pyramidal top = 64 feet $1\frac{1}{5}$ inch, and $63\frac{1}{4}$ feet = 90 cubits.

	F.	I.
Height of the pedestal on which the obelisk rests	6	11
Respective height of the three plinths on which the base stands, 1 foot 7 inches, 1 foot $9\frac{1}{4}$ inches, 2 feet $1\frac{3}{5}$ inch, making altogether - - -	5	$5\frac{11}{20}$
Whole height of the obelisk and its supports -	76	$6\frac{1}{20}$

108 cubits = 76·9 feet.

The whole height from the base of the pedestal to the pyramidal top of the obelisk - -	71	$0\frac{1}{5}$
100 cubits = $70\frac{1}{4}$ feet = $\frac{1}{4}$ stade = the height of one of the obelisks of Pheros.		

The standing obelisk contains three different cartouches; two of which are titles, and the third is the name of Ramses.

That which lies on the ground contains five different cartouches; three of which, with some slight variations, are the same as on the other obelisk. The name of Ramses is found here also, together with another name.

In these calculations we have only made use of the height of the shaft; but the height of the obelisk may be regarded as the height of the single block of granite, which includes the shaft and pyramidal top.

So the height of Cleopatra's Needle = 64 feet $1\frac{1}{5}$ inch;
and 63·22 feet = 90 cubits;
height of pedestal = 10 cubits.

Denon makes the height of the cubical kind of base = 6 feet 6 inches, French. Taking the Paris foot = $1\frac{1}{11}$ English feet, the side of the cube will =7 feet English =10 cubits.

The cubical base is no part of the obelisk, being a separate block, like the base of the obelisk which Belzoni removed from Philæ. While the French army was at Alexandria, the earth was removed from the base of Cleopatra's Needle, and it was laid bare to the lowest foundation stone, when the French measures were obtained, which are somewhat different from those given on English authority.

Not having Denon's nor Belzoni's works to refer to, we cannot say what may be the precise meaning of the cubical kind of base. Nor do we know the dimensions of the cubical base of the Philæ obelisk. The length was 22 feet, and width at the base 2 feet.

30 cubits = 21·075 feet.
3 „ = 2·1075.

Pliny states that Ptolemæus Philadelphus erected at Alexandria an obelisk 80 cubits high, which King Nectanebus had cut out; but it took much more labour to take the stone to its destination and set it up than it did to cut it out. This obelisk, being inconvenient to the naval station, was brought to the Forum at Rome by a certain Maximus, a prefect of Egypt, who cut off the top, intending to add a gilded one; but this was never done.

We do not know the measure of Pliny's cubit; but 80 cubits of 8·43 inches each = 56 feet.

The obelisk now standing in the Piazza Navona (at Rome), called the Pamphilian obelisk, is said to be 54 feet high; but it is ranked among the pseudo-obelisks at Rome.

Besides the obelisks now standing at Rome, others which cannot be found are mentioned by writers of the 16th and 17th centuries; while various fragments which still exist, or lately existed, in different parts of the city, attest the number of works of this kind which once adorned the imperial capital, and the devastations of barbarians, both foreign and domestic.

The only obelisk now standing at Heliopolis is supposed to be one of the most venerable monuments of antiquity that the land of Mizraim possesses; but one about which there is considerable discrepancy in the accounts of travellers. Pococke states that he found by the quadrant it was 67½ feet high. This obelisk is 6 feet wide to the north and south, and 6 feet 4 inches to the east and west; and it is discoloured by the water (the annual inundation) to the height of nearly 7 feet. It is well preserved; except that on the west side it is scaled away for about 15 feet high.

The pedestal on which this obelisk stands is said by some writers to be entirely covered with earth. If so, the whole height would exceed that taken by the quadrant. If the height were 70¼ instead of 67½ feet, it would = 100 cubits.

"It was during the reign of Osirtasen," remarks Wilkinson, "that the temple of Heliopolis was either founded or received additions, and one of the obelisks bearing his name attests the skill to which they had attained in the difficult art of sculpturing granite. Another, of the same materials, indicates the existence of a temple erected or embellished by this monarch in the province of Crocodilopolis. The accession of the first Osirtasen, I conceive to date about the year 1740 B. C."

Rawlinson, in his "Assyrian Researches," says that the city of Ra-bek, in the land of Misr, or Egypt, which was always spoken of as the chief place in the country, was the Biblical "On" and the Greek Heliopolis; the name being formed from "Ra," the sun, and "bek" (Coptic *baki*), a city.

"Nothing remains of the celebrated city of Heliopolis," says Lepsius, "which prided itself of possessing the most learned priesthood next to Thebes, but the walls, which resemble great banks of earth, and an obelisk standing upright, and perhaps in its proper position. This obelisk possesses the peculiar charm of being by far the most ancient of all known obelisks; for it was erected during the old empire by King Sesurtesen I., about 2300 B. C., — the broken obelisk in the Faium near Crocodilopolis, bearing the name of the same king, being rather an obelisk-like long-drawn stele. Boghos Bey has obtained the ground on which the obelisk stands as a present, and has made a garden round it. The flowers of the garden have attracted a quantity of bees, and these could find no more commodious lodging than in the deep and sharply-cut hieroglyphics of the obelisk. Within the year they have so covered the inscriptions of the four sides that a great part has become quite illegible. It had, however, already been published; and our comparison presented few difficulties, as three sides bear the same inscription, and the fourth is only slightly varied."

Afterwards Lepsius found, standing in its original place in a grave of the beginning of the seventh dynasty, an obelisk, of but a few feet in height, but well preserved, and bearing the name of the person to whom the tomb was erected. "This form of monument," remarks Lepsius, "which plays so conspicuous a part in the New Empire, is thus thrown some dynasties farther back into the Old Empire than even the obelisk at Heliopolis."

Abd-al-Latif spent some years in Egypt, and saw two obelisks at Ain-schems (Heliopolis), one standing and the other fallen.

"Among the monuments of Egypt we must reckon those of Ain-schems (the Fountain of the Sun), a small town which was surrounded by a wall, now easily recognised, though in ruins. These ruins belong to a temple, where we see surprising colossal figures cut in stone, which are more than 30 cubits in height, with all their limbs in proportion. Of these figures some were standing on pedestals, others seated in different positions in perfect regularity. In this town are the two famous obelisks called Pharaoh's Needles. They have a square base, each side of which is 10 cubits long, and about as much in height, fixed on a solid foundation in the earth. On this base stands a quadrangular column of pyramidal form, 100 cubits high, which has a side of about 5 cubits at the base, and terminates in a point. The top is covered with a kind of copper cap, of a funnel shape, which descends to the distance of 3 cubits from the top. This copper, through the rain and length of time, has grown rusty and assumed a green colour, part of which has run down along the shaft of the obelisk. I saw one of these obelisks that had fallen, and was broken in two, owing to the enormity of the weight. The copper which had covered its head was taken away. Around these obelisks are many others, too numerous to count, which are more than a third or one-half as high as the large ones."

The breadth of the base is here said to be 5 cubits only which is evidently too small to be proportionate to the height Pocock's measurement is 6 feet 4 inches, which = 9 cubits for the greater sides.

Herodotus tells us that Pheros erected two obelisks in the temple of the Sun, each of a single stone, 100 cubits in height and 8 cubits in breadth.

Hence it would appear that the two obelisks called Pharaoh's Needles, at Heliopolis (the City of the Sun), were the two which Pheros erected at the temple of the Sun on the recovery of his sight.

The Citorio obelisk, pronounced to be one of the most beautiful of all now existing at Rome, both for the proportion of its parts and the colour of the material, corresponds in height to one of Pharaoh's Needles and to one of Pheros' obelisks.

On the pedestal of the Citorio obelisk is the following inscription: — "This obelisk of King Sesostris, once erected as a sun-dial in the Campus by C. Cæsar Augustus, after suffering much, both from time and the action of fire, was taken out of the rubbish by Pope Benedict XIV. Pius VI., after repairing and beautifying the obelisk, removed it from the place where Benedict had left it, and again placed it on a pedestal, in the year 1792, and the eighteenth of his pontificate."

The son of Sesostris corresponds to the Pheros of Herodotus.

Of the obelisk at Heliopolis Hasselquist says, "At Matarie (Heliopolis) is an obelisk, the finest in Egypt. I could not have believed that natural history could be so useful in matters of antiquity as I found it here. An ornithologist can determine at the first glance to what genus those birds belong which the ancient Egyptians have sculptured."

According to Norden, the hieroglyphics, though inferior to those of the obelisks of Luxor, are still well executed. Hasselquist pronounces the sculptured birds to be so well cut that it is very easy to point out the originals in nature. He recognises the screech-owl, a kind of snipe, a duck or goose, and none more readily than the stork, in the very attitude in which he may now be seen on the plains of Egypt—with upraised neck and drooping tail.

The obelisk now standing a few miles from Medinet-el-Faioum is described by Pococke as being of red granite, and 43 feet high, measuring 4 feet 2 inches on the north side, and 6 feet 6 inches on the east. The hieroglyphics are divided by lines into three columns on each side. The obelisk is much decayed all round for 10 feet high; the whole is very foul, from the birds sitting on the top, so that it would have been difficult to have taken off the hieroglyphics. This obelisk has the top rounded in Burton's drawings.

The height of this obelisk = 43 feet, and 60 cubits = 42·15 feet.

The less breadth = 4 feet 2 inches,
= 6 cubits.

The golden image erected by Nebuchadnezzar in the plains of Dura was 60 cubits high and 6 cubits in breadth.

At Axum in Abyssinia (lat. 14° 6′) there is an obelisk of a single block of granite. The height has been stated to equal 80 feet; it has also been called equal to 60 feet.

Several other obelisks lie broken on the ground, one of which is of still larger dimensions.

Among other antiquities discovered at Nimroud by Layard is an obelisk in basalt, six feet high, in a perfect state of preservation, and ornamented with twenty-four bassi-relievos, representing battles, camels of Bactriana, and monkeys; which, it is said, involuntarily recalls to mind the expedition of Semiramis to India.

Pliny records an incident which strikingly illustrates the importance the ancients attached to obelisks. An obelisk being hewn and brought to its destination, was about to be erected: so anxious was the monarch that it should meet with no accident in this difficult operation, that, to oblige his engineers to exert all their prudence and skill, he bound his own son to the apex.

"The far Syene" was renowned for its granite quarries, and the well into which the sun is said to shine without a shadow, though the town is in fact north of the tropic. It stands immediately before the cataract opposite to the isle of Elephantine.

The chisel-marks in the quarries of Syene are still sharp. In one place is seen an obelisk half severed from the rock, but broken and abandoned.

That Abd-al-Latif made use of the same cubit as Herodotus would appear probable from the dimensions both give of the colossal statues at Memphis.

Abd-al-Latif describes what Memphis was, even in the twelfth century. He says, "Its ruins offer to the spectator a union of things which confound him, and which the most eloquent man in vain would attempt to describe. As to the figures of idols found among these ruins, whether we consider their number or their prodigious size, the thing is beyond description. But the accuracy of their forms, the justness of their proportions, and their resemblance to nature, are most worthy of admiration. I measured one which, without its pedestal, was more than thirty cubits, its breadth from right to left about ten cubits, and from front to back it was thick in proportion. This statue was formed of a single block of red granite, and was covered with a red varnish, to which its antiquity seemed only to give a new freshness."

Both Herodotus and Diodorus mention the height of each of the statues of Sesostris and his wife at the temple of Vulcan to be thirty cubits.

Lying among the ruins of Memphis there is a noble specimen of Egyptian sculpture, said (in the "Athenæum") to be a colossal statue of Ramses the Second, the Sesostris of the Greeks—one of the two statues mentioned by Herodotus as having been in front of the temple of Vulcan. This statue is almost entire, wanting only the top of the royal head-dress and the lower part of the legs; and in its present state it measures 36 feet 6 inches in length.

30 cubits of Herodotus = 21 feet English; but the height of the discovered statue = $36\frac{1}{2}$ feet.

By reference again to that authority, we find it mentioned that among the many magnificent donations which Amasis presented in the most famous temples, he caused a colossus, lying with the face upwards, 75 feet in length, to be placed before the temple of Vulcan at Memphis; and on the same basis erected two statues, of 20 feet each, wrought out of the same stone, and standing on each side of the colossus. Like to this another is seen at Sais, lying in the same posture, cut in stone, and of equal dimensions.

Now 75 feet of Herodotus = 35 feet English;
for 600 feet = 1 stade = 281 English feet,
and 75 feet = $\frac{1}{8}$ stade = 50 cubits.

An obelisk stands in the public place at Arles in France, where it was erected in 1676, having been found in some gardens near the Rhone. There is no record of the time when it was brought to France, but it would appear a probable conjecture that it had lain up to 1676 just in the position in which it was landed from the ship. It consists of a single piece of granite: the height is 52 feet French; the base has 7 feet diameter.

Taking the Paris foot to = $1\frac{1}{11}$ of an English foot, the 52 French feet will be between 56 and 57 feet English, and 56·2 feet English = 80 cubits.

Pliny mentioning the obelisk, 80 cubits high, which was brought from Alexandria to Rome, states that six such obelisks were cut out of the same mountain, and the architect received a present of fifty talents. The obelisk sent to Rome is said to have been clean cut out. Should that be understood as having been cut and left without sculptures? If the obelisk at Arles be a true one, which can now be determined, since the geometrical construction of ancient obelisks is known, it may possibly have been one of the six mentioned by Pliny, as it is 80 cubits high, and has no hieroglyphics inscribed upon it. Bouchaz says "The obelisk at Arles came from Egypt, like those at Rome. There are no hieroglyphics upon it, and probably the Romans brought it from Egypt, intending to erect it in honour of some of their emperors."

Pompey's Pillar stands on a small eminence between the walls of Alexandria and the shores of Lake Maræotis, about three quarters of a mile from either, and quite detached from any other building. It is of red granite; but the shaft, which is highly polished, appears to be of earlier date than the capital or pedestal, which have been made to correspond. It is of the Corinthian order. The column consists only of three pieces — the capital, the shaft, and the base — and is poised on a centre stone of breccia, with hieroglyphics on it, less than a fourth of the dimensions of the pedestal of the column, and with the smaller end downwards; from which circumstance the Arabs believe it to have been placed there by God. The earth about the foundation has been examined, probably in the hopes of finding treasures. It is owing, probably, to this disturbance that the pillar has an inclination of about seven inches to the north-west. The centre part of the cap-stone has been hollowed out, forming a basin on the top; and pieces of iron still remaining in four holes prove that this pillar was once ornamented with a figure, or some other trophy.

Various dimensions of Pompey's Pillar have been given; the following, however, were taken by one of the party who assisted in making the ascent by means of a rope-ladder: —

	F.	I.
Top of the capital to the astragal (one stone)	10	4
Astragal to first plinth (one stone)	67	7
Plinth to the ground	20	11
Whole height	98	10
Measured by a line from the top	99	4
It is to be remembered, however, that the pedestal of the column does not rest on the ground, its elevation being	4	6
The height of the column itself is therefore	94	10
Diagonal of the capital	16	11
Circumference of the shaft (upper part)	24	2
„ „ (lower part)	27	2
Length of side of the pedestal	16	6

Here the height of the shaft = the assigned height of the obelisk at Heliopolis, according to Pococke's measurement: according to another measurement by British officers, who found the Greek inscription dedicating the pillar to the Roman Emperor Diocletian, the height of the shaft = 64 feet, which = the height of Cleopatra's Needle from the base of the shaft to the pyramidal top.

Neither Strabo nor Diodorus make mention of this pillar. Denon supposes it to have been erected about the time of the Greek emperors or of the caliphs of Egypt. With regard to the inscription, some have remarked that it might have been added after the erection of the column. Few monuments of antiquity have afforded so wide a field for conjecture and speculation as Pompey's Pillar. Its erection has been assigned to Pompey, Vespasian, Hadrian, and Diocletian.

As Alexandria was embellished by the Ptolemies with works of art collected from the ancient cities of Egypt, the shaft may have originally been a circular obelisk, which, on being removed to Alexandria, was placed on a pedestal and crowned with a capital.

When the difference of 2 ordinates = 5, then as both ordinates increase by $\frac{1}{10}$, the difference of their squares will increase by 1, and the difference of the two ordinates will always = 5.

Or when each of the ordinates has increased by 1, as from 15 and 10 to 16 and 11, the difference of the squares of the last set of ordinates will exceed the difference of the squares of the first set by 10:

$$\begin{array}{rl} \text{since} & 16^2-11^2=135 \\ \text{and} & 15^2-10^2=\underline{125} \\ & \text{Difference} = \ \ 10 \end{array}$$

The height of the shaft of an obelisk = the sum × difference of the two ordinates = the difference of their squares.

If an obelisk have the lowest ordinate = 6,
and highest ordinate = 1,
the height of the shaft = $6^2-1^2=35$;

then, by adding $\frac{1}{10}$ to each of these two ordinates, they become

6·1 and 1·1; the difference of their squares will = 36, and so on.

ORD.	SQUARE,	DIFF.
6	36	
1	1	
	—	35
6·1	37·21	
1·1	1·21	
	——	36
6·2	38·44	
1·2	1·44	
	——	37
6·3	39·69	
1·3	1·69	
	——	38
6·4	40·96	
1·4	1·96	
	——	39
6·5	42·25	
1·5	2·25	
	——	40

Thus the difference of the squares of

6 and 1 =35
6·1 and 1·1=36
6·2 and 1·2=37
6·3 and 1·3=38
6·4 and 1·4=39
6·5 and 1·5=40

7 and 2 =45
8 and 3 =55
9 and 4 —65
10 and 5 =75
11 and 6 =85

15 and 10 =125
15·1 and 10·1=126
15·2 and 10·2=127
15·3 and 10·3=128
15·4 and 10·4=129
15·5 and 10·5=130

16 and 11 =135
17 and 12 =145
18 and 13 =155
19 and 14 =165
20 and 15 =175

Diff. of sq. 6·1 and 1·1 exceeds diff. of sq. 6 and 1 by 1
6·5 and 1·5 - - - - - 5
7 and 2 - - - - - 10
11 and 6 - - - - 50

Diff. of sq. 15·1 and 10·1 exceeds diff. of sq. 15 and 10 by 1
15·5 and 10·5 - - - - - 5
16 and 11 - - - - - 10
20 and 15 - - - - - 50
25 and 20 - - - - - 100
30 and 25 - - - - - 150

Thus 15 exceeds 10 by 5, and 30 exceeds 25 by 5
30 15 15,
25 10 15,
$30^2 - 25^2$ exceeds $15^2 - 10^2$ by 150.

When ·1 is added both to 15 and 10, their sum is increased by ·2. The increase of their sum × their difference = ·2 × 5 = 1.

When ·5 is added to both, increase × difference = 1 × 5 = 5.

When 1 is added, increase × difference = 2 × 5 = 10.
When 5 is added, increase × difference = 10 × 5 = 50.
When 10 is added, increase × difference = 20 × 5 = 100.

If the difference between two ordinates = 6, then, when both ordinates are increased by 1, the difference of their squares will be increased by 2 × 6, or 12.

ORD.	SQUARE.	DIFF.
7	49	
1	1	
	—	48
8	64	
2	4	
	—	60
9	81	
3	9	
	—	72

Hence, when the difference between two ordinates = n, then, as each ordinate increases by 1, the difference of their squares will increase by $2n$.

Or, when the difference between two ordinates = n, then when both ordinates are increased by m, the difference of their squares will be increased by $2mn$.

The rude Druidical quadrilateral, monolithic, obeliscal monuments descend many feet below the surface. The three monolithic obelisks called the "Devil's Arrows," near Boroughbridge, in Yorkshire, are nearly all of the same height. The base of the central one has been traced to 6 feet below the surface; its height above the surface is 22½ feet. At Rudston, in the same county, stands a similar obelisk, upwards of 29 feet high; its depth in the ground has been traced to 12 feet, without coming to the bottom. It stands 40 miles from any quarry where the same sort of stone is found; and, like all similar monuments, it remains without either historical or traditional record. The *men-hir* or *stone-long*, in Brittany, is 40 French feet high above the surface; and not less than 10 feet of the same obeliscal monolith is supposed to descend below the surface.

Along the coast of Carnac (Morbiham), a bay in Brittany, rude Druidical stones, ranged in many lines over a surface of half a league, may be counted by hundreds; they present the appearance of an army in battle.

An obelisk, now fallen and broken, measuring 64 feet English in length, and computed to weigh upwards of 300 tons, is described among the remarkable monuments, usually called Druidical, at the Bourg of Carnac, in the Department of Morbiham (the country of the ancient Veneti), on the south coast of Brittany.

Jablonski's Lexicon gives a derivation of the word Osiris, which he deduces from Osh Iri, that is, he who makes time.

Osiris holds in one hand a kind of key, with a circular handle, which from its having some resemblance to the letter T, is often called the Sacred Tau, or crux ansata.

The serpent of the Egyptians may have been held sacred from its form resembling the circular obelisk, the emblem of eternity.

Burton found sculptured, on the obelisk at San, the crux ansata, or tau, with the circle attached to the top, suspended from the middle part of the serpent.

The tau formed by the double ordinate, and the sectional axis of the obelisk, may be regarded as symbolical of time, velocity, and distance, or the generator of lines, areas, and solids. This sacred tau or key, as represented in the hand of Osiris, unfolds to view the long concealed type of the law of gravitation embodied by the geometrical and mechanical skill of the ancients in a single block of granite. Some obelisks remain perfect after having endured the revolution of three or four thousand years.

The Egyptian obelisk being truncated, the part wanting above the top might be supposed to denote the legendary period elapsed before history commenced. The visible part of the obelisk from the truncated top to the surface of the earth might indicate the historic period. A future indefinite period might be symbolised by the supposed continued descent of the obelisk below the earth's surface.

The Jains say that time has neither beginning nor end.

"The temple of Latona, at Butos, near the mouth of the Nile, where oracles are given, is a magnificent structure adorned with a portico 10 orgyes in height. But of all things I saw there, nothing astonished me so much as a quadrangular chapel in this temple, cut out of one single stone, and containing a square of 40 cubits on every side, entirely covered with a roof of one stone, having a border 4 cubits thick. This chapel, I confess, appeared to me the most prodigious thing I saw in that place.—(Herodotus.)

The height of the portico = 10 orgyes = 40 cubits = $\frac{1}{10}$ stade.

The exterior of the stone chapel is a cube of 40 cubits.

40 cubits = $\frac{1}{10}$ stade = 28·1 feet Eng.

Taking the thickness of the sides of the cubic chapel = the thickness of the stone that formed the roof = 4 cubits.

$$\text{Then } 40 - 8 = 32$$
$$40^3 = 64000$$
$$32^3 = 32768 = \tfrac{1}{2}\,40^3.$$

Thus the external cube is double the internal cube.

The content of the walls = the internal cube = $\frac{1}{2}$ the external cube = 32000 cubic cubits.

The height of the granite pedestal on which is placed the equestrian bronze statue of Wellington, in the front of the London Exchange = 14 feet, and the height of the statue = 14 feet; together they = 28 feet = $\frac{1}{10}$ stade = the height of the cubic stone chapel at Butos.

The content of the Lateran obelisk may be compared with the content of the chapel formed out of one stone.

Taking the shaft of the obelisk = 200, and height from base to apex = 400 units; here unity = 6 inches.

The height × ordinates of the greater side

$$\begin{aligned} &= 400 \times 20^2 = 160000 \\ \text{and } & 200 \times 14{\cdot}4^2 = \underline{40000} \\ &\text{Difference} = 120000 \\ &\text{half} = 60000 = \text{content in units} \\ &= 7500 = \text{in feet} \end{aligned}$$

Thus the content of obelisk when estimated by the greater ordinates = 60000 cubic units. When estimated by the lesser ordinates = 54200; the mean = $\frac{1}{2}(60000 + 54200)$ = 57100 cubic units = 7137 cubic feet

= 525 tons

by taking a cubic foot of granite to equal 165 pounds avoirdupois.

Wood makes by measurement a stone at Balbec = 14128 cubic feet, which will equal twice the content of the Lateran obelisk.

The thickness of the sides of the chapel = 4 cubits = 33·72 inches.

then 2 × 33·72 = 5·62 feet, is to be deducted from 28 feet, the side of the external cube.

28 − 5·62 = 22·38 feet for the side of the internal cube.

$$\begin{aligned} 28^3 &= 21972 \text{ cubic feet} \\ 22{\cdot}2^3, \&\text{c.} &= 10986 = \tfrac{1}{2}\, 28^3 \\ &= \text{the internal cube} \\ &= \text{the content of the walls of the chapel,} \end{aligned}$$

from which deduct the top part, or roof, = $28^2 \times 2{\cdot}81$ = 2203 cubic feet, and 10986 − 2203 = 8783 cubic feet for the content of the 5 sides of the cubic chapel that would have to be transported to Butos in one piece. The weight, if granite, would be about 646 tons. Thus the content of the

Lateran obelisk : the content of the cubic chapel :: 7137 : 8783 in cubic feet. Or weights as 525 : 646 tons.

Herodotus says that Psammitichus, having sent to Butos to consult the oracle of Latona, which is the truest of all oracles in Egypt, was answered that he would be avenged by men of copper coming from the sea.

The same oracle announced that Mycerinus would live only 6 years, and die in the 7th.

It was at Butos the oracle answered Cambyses : — "It is destined that Cambyses, the son of Cyrus, shall end his days at Ecbatan."

Probably the oracle might be given in this cubic chapel.

When the Athenians were afflicted with the plague, an oracle ordered the cubic altar of Apollo to be doubled.

There were also temples at Butos dedicated to Apollo and Diana.

Stonehenge, on Salisbury Plain, is supposed by Davis to have been the round temple dedicated to Apollo, according to this substantive description given by Diodorus : — "Among the writers of antiquity, Hecateus and some others relate that there is an island in the ocean, opposite to Celtic Gaul, and not inferior in size to Sicily, lying towards the north, and inhabited by Hyperborei, who are so called because they live more remote from the north wind. The soil is excellent and fertile, and the harvest is made twice in the same year. Tradition says that Latona was born there, and therefore Apollo is worshipped before any other deity; to him is dedicated a remarkable temple of a round form."

Latona, the daughter of Titan, had an oracular temple at Butos, formed of one gigantic stone. These oracles were celebrated for their truth, and for the decisive answers given. The oracles at the temple of her son Apollo, at Delphi, delivered by the priestess Pythia, were celebrated in every country.

It is said Neptune, moved with compassion towards Latona, when driven from heaven and wandering from place to place, because Terra, influenced by Juno, refused to give her a place where she might rest and bring forth, struck with his trident Delos, one of the Cyclades, and so made immoveable that island, which before wandered in the Ægean, and appeared sometimes above and sometimes below the sea. There Apollo was born, to whom the island became sacred. One of the altars consecrated to Apollo at Delos was reckoned among the seven wonders of the world.

Here we find a striking similarity between the temple of Latona in Egypt, and those of her son Apollo in Greece.

The temple at Delos stood on a once floating island. The temple at Butos stood near the great lake, on which floated the island of Chemmis. The oracles delivered at the temples of Latona and Apollo were greatly celebrated. The altars at both were reckoned among the wonders of the world, and at both were cubic altars; at one the external cube was double the central cube, at the other, the cubic altar was required to be doubled.

No wonder then that Herodotus recognised in Egypt the gods of his country, — as the Sepoys in the British army that came from India during the Egyptian campaign recognised the gods of their country, and worshipped them in the colossal temples of Egypt.

Burckhardt says the excavated temples of Nubia, from their strong resemblance, recalled to his mind those of India. Here are the links of the mythological chain, like those of learning and science, connecting Asia, Africa, and Europe.

The following extract, descriptive of a visit to the Temple of Dendera, is from "Scenes and Impressions in Egypt." The author traverses Egypt in the Overland route from India.

"To one who has just quitted a country where the priest still officiates, and the worshipper bows down and prostrates himself in the temples of idolatry, who is familiar with the aspect, the habits and customs, the rites and ceremonies of the Hindoo, this temple is an object of no common interest; for here the Indian soldier fancied he recognised the very gods he worshipped, and with sadness and indignation complained to his officers, that the sanctuary of his god was neglected and profaned. He saw a square and massive building, a colossal head on the capitals of huge columns; on the walls, the serpent; the lingam, in the priapus; the bull of Iswara, in the form of Apis; Garuda, in Arueris; Hanuman, in the *round headed* cynocephalus; a crown very similar to that of Siva, on the head of Osiris; and in the swelling bosom of Isis, that of the goddess Parvati: while on the staircase, the priest and the sacred ark must have reminded him, and strongly, of the Brahmins, and the palanquin litter of his native country. Many, many forms he must have missed, many too have observed, to which he was an entire stranger."

Again, speaking of the low tombs near the great pyramid, two of which have their walls covered with paintings. "There is the birth and story of Apis, the cow calving; there are sacrifices, feasting, dancing; there is an antelope in a small wood; and there is a figure (though a mere trifle) called and fixed my attention, a man carrying two square boxes across the shoulder on a broad flat bending piece of wood; exactly similar to this is the manner in which burdens are borne in India, by what we there call bangy-coolies. It suggests to me, what I had forgotten before to remark, the peculiar way in which you see, in paintings at Thebes, the end of the girdle or loin cloth gathered, plaited, as it were, and hanging down before their middles; this is *exactly* Indian; nor in my eye is either the complexion or feature, either in the paintings or statues, very different from some tribes of Brahmin."

For the following mythological details history is indebted to Herodotus:—"The Pelasgians, the most ancient people of Greece, honoured their gods without knowing them, and even without giving them names. They were called gods, and regarded masters of all things. It was not till a period far distant from their origin that they knew the names of their gods came from Egypt. Then they went to consult the oracle of Dodona, the most ancient in Greece, and inquired if they ought to receive the names of the gods given by barbarians. Upon the oracle answering that they ought to receive them, they sacrificed to the gods, and invoked them by names. It was from the Pelasgians the Greeks received these names. One remains still ignorant whence each god came,—if he had always existed,—what was his form? For myself, I believe they came from Egypt; and if I should be told that the Egyptians knew not Neptune, Castor, Vesta, Themis, the Graces and Nereids,—I should answer, that the Pelasgians learned these names from the Samothracians with whom they associated. As to all the other gods, their names came from Egypt."

Thus it appears that at a remote period an intercourse had been established from India to the west of Asia; thence to Egypt and the Mediterranean, through the agency of commerce, migratory masons, wandering philosophers, or magi. So that India had long been enlightened before the first ray of science had pierced the last European darkness.

Though India may appear to stand the first, and Europe the last in the scale of antiquity of science and learning, yet perhaps China may contend with India, and America with Europe for priority. These remote epochs call to mind the exclamation which Plato, in the "Timæus," puts into the mouth of the priests of Sais—"O Solon, O Solon! ye Greeks still remain ever children; nowhere in Hellas is there an aged man. Your souls are ever youthful. Ye have no knowledge of antiquity, no ancient belief, no wisdom grown venerable by age."

Herodotus, describing Sais, says, "What I admire above all other things is a house made out of one stone, which was brought by Amasis from Elephantis. Two thousand men were employed during three whole years in transporting this house, which has in front 21 cubits, in depth 14, and 8 in height; this is the measure of the outside.

"The inside is 18 cubits in length, 12 in depth, and 5 in height. This wonderful edifice is placed by the entrance of the temple of Minerva."

External measurement = 21, 14, 8 cubits
Internal " = 18, 12, 5
Difference = 3, 2, 3

Let the common difference $= 2{\cdot}5$;

then 21, 14, 8
less 2·5, 2·5, 2·5
equals 18·5, 11·5, 5·5 for internal sides.
and $18{\cdot}5 \times 11{\cdot}5 \times 5{\cdot}5 = 1170$ internal content,
$2 \times 1170 = 2340$;
but $21 \times 14 \times 8 = 2352$ external content.

Thus the external content = double the internal content.

The chamber, according to this calculation, would not exceed a 15½ feet sectional length of a London sewer. By placing the chamber on one side, a man might walk upright on a floor about 15½ feet by 3 feet 10 inches. These dimensions are too insignificant for a monolith which took 2000 men, for three whole years, to transport from Elephantis.

If the dimensions had originally been written orgyes instead of cubits, then, by this supposition, the orgye being = 4 cubits, the content of the mass to be moved, which = the sides of the chamber = ½ the external content, will = about ½ 52266, or 26133 cubic feet = 1925 tons, if the stone were granite.

The content would = nearly twice the content of the Balbec stone, and the Balbec stone = twice the content of the Lateran obelisk.

The granite block which composes the pedestal of the bronze equestrian statue of Peter the Great, at St. Petersburgh, was estimated at the weight of 1500 tons.

Then, according to the preceding calculation, the weight of the monolithic temple transported from Elephantis to Sais would be to the weight of the monolithic block of granite transported from the Gulf of Finland to St. Petersburgh :: 1925 : 1500.

The St. Petersburgh block formed the remnant of a huge rock which lay in a morass about four miles from the shore of the Gulf of Finland, and at the distance of about fourteen miles by water from St. Petersburgh.

The means adopted in conveying this block, both by land and water, are also stated.

"I found the rock," says the engineer employed, "covered with moss. Its length was 42 feet, its breadth 27, and its height 21 feet."

"The expense and difficulties of transporting it," says Coxe, "were no obstacles to Catherine the Second. The morass was drained, the forest cleared, and a road formed to the Gulf of Finland. It was set in motion on huge friction-balls and grooves of metal by means of pulleys and windlasses, worked by 500 men. In this manner it was conveyed, with 40 men seated on the top, 1200 feet a day, to the shore; then embarked on a nautical machine, transported by water to St. Petersburgh, and landed near the spot where it is now erected. Six months were consumed in this undertaking, which was certainly laborious in the extreme; for the rock weighed 1500 tons. In its natural state the stone would have been a magnificent support for the statue; but the artist, in his attempts to improve it, deprived it of half its grandeur."

The height of the figure of the emperor is 11 feet; that of the horse, 17 feet. The weight of both together is 36,636 pounds English.

Since 500 Russians conveyed a monolith weighing 1500 tons, in six months, to St. Petersburgh, the conveying a monolith, weighing 1873 tons, by water, in three years, by 2000 Egyptians, from Elephantis to Sais, does not seem an impossibility.

Wilkinson thus describes the broken statue in the Memnonium, which was formerly in a sitting attitude:—"To say that this is the largest statue in Egypt will convey no idea of the gigantic size or enormous weight of a mass which, from an approximate calculation, exceeded, when entire, nearly three times the solid contents of the great obelisk at Karnak, and weighed about 887 tons. The obelisk weighs about 297 tons, allowing 2650 ounces to a cubic foot."

The smaller of two Luxor obelisks, lately removed to Paris, was calculated by Lebas to weigh 246 tons English.

Montverrand, a French architect, has raised a granite column at St. Petersburgh, which is a single block, about

96 feet high, and weighs three times as much as the obelisk of Luxor.

The monolithic granite temple, called the "Green Tabernacle, or Chamber," at Memphis, was, according to Arab writers, formed of one single stone, 9 cubits high, 8 long, and 7 broad. In the middle of the stone a niche or hole is hollowed out, which leaves 2 cubits of thickness for the sides, as well as for the top and bottom.

$$\begin{array}{lrrrl} \text{Exterior} & 9, & 8, & 7 & \text{cubits} \\ \text{Deduct} & \underline{4,} & \underline{4,} & \underline{4} & \\ \text{Interior} & 5, & 4, & 3 & \end{array}$$

Then $9 \times 8 \times 7 = 504$ exterior content.
and $5 \times 4 \times 3 = 60$ interior content.
$8 \times 60 = 480.$

In order that the interior content should $= \frac{1}{8}$ exterior content, the internal dimensions should $= \frac{1}{2}$ the external dimensions; or internal $= 4{\cdot}5 \times 4 \times 3{\cdot}5 = 63$,

and $8 \times 63 = 504$:

so 8 times the internal content will = the external content.

Makrizi, speaking of the same monolith, adds, "There was at Memphis a house (chamber) of that hard granite which iron cannot cut. It was formed of a single stone, and on it there was sculpture and writing. On the front, over the entrance, there were figures of serpents presenting their breasts. This stone was of such a weight, that several thousand men together could not move it. The Emir S. S. Omari, broke this green chamber about the year 750 of the Hegira (A. D. 1349), and you may see pieces of it in the *jamy* (mosque) which he caused to be built in the quarter of the Sabæans, outside of Cairo."

A monolith at Tel e' Tmai, the ancient Thmouis, in the Delta, still remains; it is of polished granite, and rectangular. According to Burton, it is 21 feet 9 inches high, 13 feet broad, and 11 feet 7 inches deep; the thickness of the walls being about 2½ feet.

This will make the height, breadth, and depth of the chamber, each 5 feet less than the external height, breadth, and depth. If instead of 5 feet, 5·2 feet be deducted from each external measure, this will give the interior content, $\frac{1}{4}$ of the exterior content.

Exterior	21·8	13	11·6
deduct	5·2	5·2	5·2
Interior	16·6	7·8	6·4

then $21{\cdot}8 \times 13 \times 11{\cdot}6 = 3287$
and $16{\cdot}6 \times 7{\cdot}8 \times 6{\cdot}4 = \frac{1}{4}\ 3287$
or the interior content = $\frac{1}{4}$ the exterior content.

The height, 21 feet 9 inches, will be about 31 cubits. The Butos monolith being a cube of 40 cubits, or the height of 40 cubits.

In the vicinity of Mahabalipuram, on the sea-coast of the Carnatic, are the celebrated ruins of ancient Hindoo temples, dedicated to Vishnu. Facing the sea there is a pagoda of one single stone, about 16 or 18 feet high, which seems to have been cut on the spot out of a detached rock. On the outside surface of the rock are bas-relief sculptures, representing the most remarkable persons whose actions are celebrated in the Mahabharat. Another part of the rock is hollowed out into a spacious room.

On ascending the hill, there is a temple cut out of the solid rock, with some figures of idols in alto relievo upon the walls, very well finished: at another part of the hill, there is a gigantic figure of Vishnu, asleep on a bed, with a huge snake wound round in many coils as a pillow, which figures are all of one piece, hewn out of the rock. A mile and a half to the southward of the hill are two pagodas, about 30 feet long by 20 wide, and the same in height, cut out of the solid rock, and each consisting originally of one single stone. Near to these is the figure of an elephant, as large as life, and a lion much larger than the natural size; but otherwise a just representation of a real lion, which is, however, an animal unknown in this neighbourhood, or in the south of India. The whole of these sculptures appear to have been rent by some convulsion of nature, before they were finished. The great rock above described is about 100 yards from the sea; but on the rocks washed by the sea are sculptures indicating that they once were cut out of it. East of the village, and washed by the sea, is a pagoda of stone, containing the Lingam, and dedicated to Mahadeva. The surf here breaks far out, and (as the Brahmins assert) over the ruins of the city of Mahabalipuram, which was once large and magnificent; and there is reason to believe, from the traditional records of the natives, that the sea, on this part of the Coromandel coast, has been encroaching on the land. All the most ancient buildings and monuments at this place are consecrated to Vishnu, whose worship appears to have predominated on this coast; while, on the opposite coast, in the neighbourhood of Bombay, that of Mahadeva, or Siva, prevailed to a greater extent. (*East India Gazetteer.*)

We do not know that any such rectangular Druidical monolith monuments exist; but we find a description of a large dolmen formed by 17 or 18 blocks of stone.

The finest Celtic monument, the largest and most regular, within the limits of Brittany or Anjou, is seen near the village of Bagneux, about a mile from Saumur. This monument is a dolmen of a rectangular form, raised on the side of a hill, and composed of enormous blocks of sandstone. It is 58 feet long, 21 wide, and about 7 feet high from the ground. The disposition of the stones is perfectly uniform, four at each side for the walls, four for the roof, one on the left side near the entrance, one at the west, closing up the dolmen at that end; two smaller ones standing up near the entrance, and a single isolated block at the bottom, like a pillar, helping to sustain the weight of the roof. There are altogether seventeen of these immense blocks, and from some rough masonry, which may be seen supplying a vacancy on the right of the entrance, it is inferred that there were originally eighteen. Scattered about in disorder outside the entrance are some flat stones, which it is conjectured may have once stood upright in continuation of the northern wall.

The great blocks which form this singular structure are all unhewn, yet of such equal dimensions that, with a single exception, the result apparently of an accident, they lie almost as closely together as if they had been carefully smoothed for the places they occupy. They vary in thickness from 18 inches to 2½ feet, and are all of extraordinary magnitude; the largest, that which closes the west end, presenting a square surface of twenty-one feet to the side. It is said, that upon digging round the monument, the walls are found to be buried nearly 9 feet in the earth, which would give the upright blocks a height of almost 16 feet. The fact is remarkable, as Celtic stones in general are seldom sunk to such a depth. But in this instance there appears to have been a necessity for it, as the blocks, instead of being vertical in the usual way, incline so far towards the centre, that a plummet dropped from the top would fall more than a foot from the base. It is impossible to visit these prodigious masses of stone without renewed astonishment at the marvellous mechanical power by which they were raised from their quarries, transported to their destination, and arranged in symmetrical order. In the vineyards, about 40 or 50 yards distant, is a solitary *peulven*, about 6 or 7 feet high, out of the line of the dolmen, and apparently having no connection with it; and on the top of a hill not far from the neighbouring village of Riau is a smaller dolmen, consisting of six great stones, also set towards the east, equally regular in form, but considerably dilapidated by the action of the weather. This dolmen presents the additional peculiarity of a flooring of flag stones. The blocks of which these monuments are built are composed of sandstone, found in the environs of Saumur; but at such a distance from the place selected for the mystical purposes to which the Celts applied them, that they must have been carried at least half a league over a difficult country, intersected with ravines and valleys. The work of cutting these prodigious blocks out of the quarry, and raising them from their beds, is intelligible to a people who understand the use of the wedge and the lever; but the mechanical power by which they were conveyed across rivers and hills, and placed in this regular order of walling and roofing, is utterly incomprehensible.

A glance into the dolmen of Bagneux, this vague damp hall, fills the mind with a sort of dreary wonder not very easy to describe. What could have been the object of this rude, stony temple, mausoleum, or whatever else it was? The twilight within is by no means impressive, except in the same way, but with a sort of palpable horror in it, as a great subterranean sepulchre can be felt to be impressive. When you creep in, rather shudderingly, you have an instinctive conviction of the tremendous solidity of the masses of stone around and above you, which have stood there for centuries heaped upon centuries; yet it is of so dismal a kind, that you can hardly overcome a certain sense of terror, lest the whole mass should fall and crush you to atoms. It is probably the consciousness of your own weakness and insignificance in the presence of so ponderous a mystery that produces this feeling.

Formerly the neighbourhood of Saumur was scattered over with Celtic ruins, of which few are now remaining, and of these which are still described in the local books some have already disappeared. They have been broken up for materials to mend the roads.

The sides of this dolmen would seem from the description to resemble the sides of an Egyptian propylon, the sides of both being inclined, and both structures colossal.

Perhaps rectangular structures formed of several large stones to resemble a rectangular monolith may be found among Druidical remains.

In Gaul, the power of the Druid priesthood was so directly inimical to the Roman domination, that, as Gibbon remarks, under the specious pretext of abolishing human sacrifices, the emperors Tiberius and Claudius suppressed the dangerous power of the Druids; next the priests themselves; their gods and their altars subsisted in peaceful obscurity until the final destruction of paganism.

PART IV.

PYRAMID OF CHEOPS. — ITS VARIOUS MEASUREMENTS. — CONTENT EQUAL THE SEMI-CIRCUMFERENCE OF EARTH. — CUBE OF SIDE OF BASE EQUAL $\frac{1}{4}$ DISTANCE OF MOON. — NUMBER OF STEPS. — ENTRANCE. — CONTENT OF CASED PYRAMID EQUAL $\frac{1}{18}$ DISTANCE OF MOON. — KING'S CHAMBER. — WINGED GLOBE DENOTES THE THIRD POWER OR CUBE. — THREE WINGED GLOBES THE POWER OF 3 TIMES 3, THE 9TH POWER, OR THE CUBE CUBED. — SARCOPHAGUS. — CAUSEWAY. — HEIGHT OF PLANE ON WHICH THE PYRAMIDS STAND. — FIRST PYRAMIDS ERECTED BY THE SABÆANS AND CONSECRATED TO RELIGION. — MYTHOLOGY. — AGE OF THE PYRAMID. — ITS SUPPOSED ARCHITECT. — SABÆANISM OF THE ASSYRIANS AND PERSIANS. — ALL SCIENCE CENTRED IN THE HIERARCHY. — TRADITIONS ABOUT THE PYRAMIDS. — THEY WERE FORMERLY WORSHIPPED, AND STILL CONTINUE TO BE WORSHIPPED, BY THE CALMUCS. — WERE REGARDED AS SYMBOLS OF THE DEITY. — RELATIVE MAGNITUDE OF THE SUN, MOON, AND PLANETS. — HOW THE STEPS OF THE PYRAMID WERE MADE TO DIMINISH IN HEIGHT FROM THE BASE TO THE APEX. — DUPLICATION OF THE CUBE. — CUBE OF HYPOTHENUSE IN TERMS OF THE CUBES OF THE TWO SIDES. — DIFFERENCE BETWEEN TWO CUBES. — SQUARES DESCRIBED ON TWO SIDES OF TRIANGLES HAVING A COMMON HYPOTHENUSE. — PEAR-LIKE CURVE. — SHIELDS OF KINGS OF EGYPT TRACED BACK TO THE FOURTH MANETHONIC DYNASTY. — EARLY WRITING. — LIBRARIANS OF RAMSES MIAMUM, 1400 B.C. — DIVISION OF TIME. — SOURCES OF THE NILE.

Pyramid of Cheops.

HAVING made repeated attempts, and as many failures, to ascertain the magnitude of the Pyramid of Cheops from stated measurements which differed so greatly from each other, we at last abandoned all hopes of arriving at any satisfactory conclusion.

Herodotus only says, "The Pyramid of Cheops is quadrilateral; each side being 8 plethrons in length, and height the same." These statements we found to be inaccurate; for we had already ascertained the value of the plethron of Herodotus.

Savary gives the dimensions of the Great Pyramid from the following authors:—

	HEIGHT. Feet.	BASE. Feet.
Herodotus	800	800
Strabo	625	600
Diodorus	600	700
Pliny	—	708
Le Brun	616	704
Prosper Alpinus	625	750
Thevenot	520	682
Niebuhr	440	710
Greaves	444	648

To these might be added a list more numerous, with discrepancies not less.

The number of sides of the pyramid - = 4
Suppose each side - - - - - = 4^2
(linear plethra)
then the perimeter will - - - - = 4^3
and the area of the base - - - - = 4^4
(square plethra).

Let the sum of the indices of 4, or $1+2+3+4=10$, be the height in plethra:

Since 1 phlethrum - - - - = 40·5 units,
10 plethra will - - - - = 405 "
= $468\frac{1}{3}$ feet.
The side of the base will = $16 \times 40{\cdot}5$ = 648 units,
= $749\frac{1}{3}$ feet.

By the addition of somewhat more than unity to the height, we have

Content of the pyramid = $\frac{1}{3}$ height × base area,
= $\frac{1}{3}$ 406, &c. × $\overline{648}^2$,
= $\frac{1}{2}$ 113689008 units,
= $\frac{1}{2}$ circumference of the earth

which may also be expressed by $\frac{1}{3}(\overline{324 \times 2 - 243} \times \overline{324 \times 2}^2)$; 324 being the Babylonian numbers 243 transposed.

Height : base :: 10 : 16
Height $= \frac{10}{16}$ or $\frac{5}{8}$ base.

Herodotus makes the height the same as the base:

Height = 405 units,
Base = 648 units,

from which take 243, or 1 stade, and there will be left 405 units for the height, which makes the height = the side of the base, less 1 stade.

The cube of the side of the base $= 648^3 = 272097792$
4 cubes = 1088390065 units.

The distance of the moon from the earth = 60 semi-diameters of the earth = 9·55 circumference, say = 9·57 circumference,

then 9·57 × 113689008 = 1088003806 units,
and 9·55 circumference = 1085730026 „

Hence the distance of the moon from the earth = 4 times the cube of Cheops = the cubes of the four sides.

Diameter of the earth = 7926, and circumference = 24,899 miles.

Distance of Mercury from the Sun = about 150 times the distance of the moon from the earth.

Distance of moon = 4 cubes,
∴ distance of Mercury = 4 × 150 = 600 cubes,
= 10 × 60 cubes of Cheops.

Distance of the moon = 9·57 × circumference = 4 cubes,
= 9·57 × 24899,
= 238283·43 miles,
∴ 150 × 4 = 600 cubes = 150 × 238283·43
or, distance of Mercury = 35742514 miles.

By the tables, the distance of Mercury = about 36 or 37 millions of miles. So the distance of Mercury from the sun will somewhat exceed 150 times the distance of the moon from the earth, or 600 cubes of Cheops.

The distance of the moon from the earth, by the tables, = 60 and 61 semi-diameters of the earth.

According to Herschel, the mean distance of the centre of the moon from that of the earth is 59·9643 of the earth's equatorial radii, or about 237,000 miles.

The mean distance of Mercury from the Sun is about 36,000,000 miles.

Thus 152×237000 miles $= 36,024,000$ miles for the distance of Mercury, which is nearly 150 times the distance of the moon.

It will be seen hereafter, that the distance of Mercury : distance of Belus :: 1 : 150 nearly, and distance of Mercury
$= 150 \times$ distance of moon
$= 150 \times 4$ cubes.

Hence the distance of Belus will
$= 150^2 \times$ distance of moon
$= 150^2 \times 4$ cubes
$= 22500 \times 4 = 90000 = 300^2 = (5 \times 60)^2$ cubes of Cheops.

The distance of Saturn $= 25$ times the distance of Mercury $= 25 \times 150 \times 4 = 15000 = \frac{3}{2}\,100^2$ cubes

or Mercury $= 600$
Saturn $= 15000$
Belus $= 90000$.

Cube of side of base $= \frac{1}{4}$ distance of moon
2 sides $= 2$
4 $= 16$

The cube of twice the side $= (2 \times 648)^3 =$ twice the distance of the moon. Distance of Mercury $= 150$ times the distance of moon $= 75$ times the cube of twice the side.

Distance of Belus $= 150 \times 75$ times the cube of twice the side.

2 pyramids = circumference.
Twice pyramid : cube of perimeter of base
:: circumference : 16 distance of moon
:: 1 : 152
:: distance moon from earth : distance Mercury from sun.

The side of the base does not $= 8$ plethrons, but it $= (\frac{1}{2}\,8)^2$ $= \frac{1}{4}$ the square of 8 plethrons, $= \frac{1}{4}8^2 = 16 =$ twice 8 plethrons, or 1600 feet of Herodotus, and the height, 10 plethrons, will equal 1000 feet.

The perimeter will $=8^2 = 64 = 70 - 6 = 70$ plethrons less 1 stade, and side $= \frac{1}{4}\ 64 = 16$ plethrons. Height $= 10 = 16 - 6$ plethrons = side of base less 1 stade. Thus the side = twice 8 plethrons, and height equals the side less 1 stade.

Herodotus says the side equals 8 plethrons, and the height equals the side.

Hence dimensions of the pyramid of Cheops, which represents the $\frac{1}{2}$ circumference of the earth, might easily be impressed on the memory by saying the perimeter of the base equals 70 plethrons less one stade, and the height equals the side of the base less 1 stade.

The number 1600, which indicates the side of the base in feet of Herodotus, corresponds with the number of talents of silver which the interpreter told Herodotus was inscribed on the side of the pyramid as having been expended in furnishing the workmen with radishes, onions, and garlic.

A pyramid having the same base as that of Cheops, and height = side of base would $= \frac{1}{3}$ the cube of Cheops, $= \frac{1}{3}$ of $\frac{1}{4} = \frac{1}{12}$ distance of moon.

The contents of all the pyramids were assigned without reference to the cube of the sides of the bases, for we did not discover that these cubes were measures of the distance of the moon and planets till after the estimates of the pyramids were made.

Since pyramid of Cheops $= \frac{1}{2}$ circumference and cube of Cheops $= \frac{1}{4}$ distance of moon. Height : side of base of pyramid of Cheops :: 406 : 648 :: pyramid of Cheops : a pyramid having same base and height = side of base. Distance of moon = 4 cubes = 12 pyramids each having height = side of base.

So $406 : 648 \times 12 :: \frac{1}{2}$ circumference : 9·57 circumference.

Thus distance of moon = 9·57 circumference.

The cube has 12 edges, each = height or side of base of cube.

So $\frac{1}{2}$ circumference : distance of moon :: height of pyramid : the 12 edges of the cube :: height of pyramid : 3 times the perimeter of the base.

The pyramid of Belus $= \frac{1}{3}$ cube of 1 stade $= \frac{1}{24}$ circumference and height = side of base.

So 24 pyramids = 8 cubes = circumference; pyramid : circumference :: 1 : 24 :: height : twice 12 edges of the cube :: height : 6 times the perimeter of base.

The mean distance of the moon from the earth = 237000 miles

$$400 \times 237000 = 94800000 \text{ miles}$$

and 95000000 miles =

the distance of the earth from the sun. Hence the distance of the earth from the sun = 400 times the distance of the moon from the earth = 400 × 4 = 1600 cubes = 1600 times the cube of the side of the base of the pyramid of Cheops.

But the side of the base of the pyramid of Cheops = $2\frac{2}{3}$ stades = 648 units = 1600 Babylonian feet.

Hence the distance of the earth from the sun = as many cubes of the side of the base of the pyramid of Cheops as the side of the base contains Babylonian feet.

The distance of the moon from the earth = as many cubes of Cheops as the pyramid has sides.

or $3^5 = 243$
transposed = 324
doubled = 648

four times the cube of 648, or half the cube of twice 648 = distance of moon, or the cube of twice the side of the base = twice the distance of the moon from earth, = diameter of the orbit of moon.

The cube of the perimeter of the base = 8 × 2 = 16 times the distance of moon. Distance of earth from sun = 400 times the distance of moon from earth = $\frac{400}{16}$ = 25 times the cube of the perimeter of the base.

Side of base : height :: 648 : 406, &c., and $648^3 : 407^3$, &c. :: $\frac{1}{4} : \frac{1}{16}$ distance of moon.

Thus cube of side of base = $\frac{1}{4}$ distance of moon and cube of height = $\frac{1}{4}$ cube of side of base, or cubes are as 1 : 4.

Cube of twice side of base = $(2 \times 648)^3$
$= \frac{1}{4} \times 2^3 = \frac{8}{4} = 2$ distance of moon
= diameter orbit of moon

Cube of twice height = $(2 \times 407 \text{ \&c.})^3$
$= \frac{1}{16} \times 2^3 = \frac{8}{16} = \frac{1}{2}$ distance of moon

Cubes are as $\frac{1}{2}$: 2 :: 1 : 4
Cube of 4 times height = $(4 \times 407$ &c.$)^3$
$= \frac{1}{16} \times 4^3 = \frac{64}{16} = 4$ distance of moon
= 2 diameter orbit of moon.
Thus cube of 4 times height
= twice cube of twice side
= twice diameter orbit of moon
= twice 30 diameters of earth.

If 30 radii divided the circumference of earth into 30 equal parts, then pyramid would = 15 of these parts, and cube of side of base = 15 radii.

The inclined side of pyramid will = 521 units, and 521^3, &c. = $\frac{5}{4}$ circumference. Thus the cube of the inclined side of pyramid of Cheops will = height × area base of pyramid of Cephrenes = $\frac{5}{4}$ circumference. Cube of twice inclined side of pyramid of Cheops = $\frac{5}{4} \times 2^3 = \frac{40}{4}$ = 10 circumference.

Cube of twice side of base = twice distance of moon.
Cube of perimeter = 16 distance of moon
2 = 128
4 = 1024

2 cubes of 4 times perimeter of base = 2048 distance of moon
and 2045 = distance of Jupiter

Side of base − height = 648 − 406 = 242
and 242^3, &c = $\frac{1}{8}$ circumference
$(2 \times 242$, &c.$)^3 = \frac{1}{8} \times 2^3 = \frac{8}{8} = 1$
Cube of twice difference = circumference.

The celestial distances are expressed in terms of the distance of the moon from the earth, and in terms of the circumference, which means the circumference of the earth.

Twice side of base = 2 × 648 = 1296 units.

Cylinder having height = diameter of base = 1296 units, will = $1296^3 \times \cdot 7854$
= 15 circumference,
Inscribed sphere - = 10 „
„ cone - = 5 „
Perimeter of base - = 4 × 648
Cylinder = 15×2^3 = 120 circumference,
Sphere = $\frac{2}{3}$ = 80 „
Cone = $\frac{1}{3}$ = 40 „

12 cylinders = 120 × 12 = 1440 circumference,
= distance of Mercury.

Twice perimeter of base = 8 × 648
Cylinder = 120 × 2^3 = 960 circumference,
Sphere = $\frac{2}{3}$ = 640 „
Cone = $\frac{1}{3}$ = 320 „
4 cylinders = 4 × 960 = 3840 „
= distance of earth.

Distances in terms of the cube of side of base:

Moon - - -	= 4 cubes,
Mercury - -	= 600 „
Earth - - -	= 1600 „
Saturn - - -	= 15000 „
Belus - - -	= 90000 „

Distances in terms of the cube of twice side of base:

Moon - - -	= $\frac{1}{2}$ cube,
Mercury - -	= 75 „
Earth - - -	= 200 „
Saturn - - -	= 1875 „
Belus - - -	= 11250 „

Distance of the earth = 1600 cubes of side
= 200 cubes of two sides
= 25 cubes of perimeter.

Cylinder, diameter = twice perimeter base
= 960 circumference

$1\frac{1}{2}$ cylinder = distance of Mercury.
4 „ = „ Earth.
75 „ = „ Uranus.
225 „ = „ Belus.
$\frac{1}{100}$ „ = „ Moon.

For distance of moon = $\frac{1}{150}$ distance of Mercury
$= \frac{1}{150} \times 1\frac{1}{2} = \frac{1}{100}$ cylinder.

So the distance of Belus from the sun will = 225 × 100 = 22,500 times the distance of the moon from the earth.

Or the distance of Belus = 15^2 cylinders = $15^2 \times 100$ times distance of the moon.

Thus 4 cubes of side of base = distance of the moon.
4 cylinders, diameter = 2 perimeter = distance of the earth.
Or 1 cube of 2 sides = diameter of the orbit of the moon.
1 cylinder, diameter 4 perimeters = diameter of orbit of the earth.
2 cubes of 4 perimeters = distance of Jupiter.

Pyramid : cube of side of base,
:: $\frac{1}{3}$ circumference : $\frac{1}{4}$ distance of the moon,
:: $\frac{1}{2}$ circumference : $\frac{60}{4}$ radii of the earth,
:: $\frac{1}{3}$ circumference : 15 „ „
:: arc of 12 degrees : radius of the earth.

Cube of side of base = $648^3 = \frac{1}{4}$ distance of the moon.
Cube of twice side = $1296^3 = 2$
= diameter of orbit of the moon;
but $1296 = 6^4$
$\therefore (6^4)^3 = 6^{12}$ = diameter of the orbit of the moon.
$3^5 = 243$.

Place the last numeral the first of the series, and 243 becomes 324; then 324 doubled and cubed = $648^3 = \frac{1}{4}$ distance of the moon in units.

Again: Transpose the first and last numerals, and 243 becomes 342; then 342 doubled and squared = 684^2 = circumference of the earth in stades.

$243 \times 684^2 = 3^5 \times 684^2$ = circumfer. of the earth in units.
$(2 \times 648)^3$ = diameter of the orbit of the moon in units,
$= 6^{12} = (2 \times 3)^{4 \times 3}$
= twice 3 to the power of 4 times 3.

Cube of twice side of base = $6^{12} = 1296^3$ = diameter of the orbit of the moon.

Cube of perimeter = $2^3 \times 6^{12}$.

50 cubes = $50 \times 2^3 \times 6^{12}$
= 400×6^{12}
= 400 times diameter of the orbit of the moon,
= diameter of the orbit of the earth.

Cube of twice side = diameter of the orbit of the moon.
150 cubes = „ „ „ Mercury.
150^2 cubes = „ „ „ Belus.

Cylinder having height = diameter of base = 1296 = 6^4
will = 15 circumference
Sphere = 10 „
Cone = 5 „

Sphere, diameter = side of base of pyramid = 648 will = $\frac{10}{8}$ = $\frac{5}{4}$ circumference = height × area base of Cephrenes' pyramid.

The dimensions of Cheops' pyramid will be, side of base = $\frac{1}{2}(6)^4 = \frac{4}{8}(6)^4$, and height = $\frac{5}{8}$ side of base.

Or, height = side of base less 1 stade,
= $\frac{1}{2}(6)^4 - 243$ units,
= $\frac{1}{2}(6)^4 - 3^5$.

$3^5 = 243$
$(3 \times 342)^3$ = distance of the moon;
$(3 \times 432)^3$ = diameter of orbit of the moon.

2, 3, 4 are Babylonian numbers derived from 3^5.
$3^5 = 243$
read backwards = 342
$(3 \times 342, \&c.)^3 = 1028^3$ = distance of the moon.
$3^5 = 243$
first figure placed last = 432
$(3 \times 432)^3 = 1296^3 = 6^{12}$ = diameter of orbit of the moon.

3^5 read backwards, tripled, and cubed = distance of the moon.

3^5 first figure being placed last, tripled, and cubed = diameter of the orbit of the moon.

$3^5 = 243$
$(2 \times 243)^3$ = circumference of the earth.
3^5 doubled and cubed = circumference of the earth.

In English measures we make the height of Cheops' pyramid - - - - - -	468 feet
and side of base - - - - -	749
Davidson makes the height - -	461
and side of base - - - - -	746.

It appears that Davidson in 1763 took the height of this pyramid, first, by measuring the steps or ranges of stone, and subsequently with a theodolite, and both accounts agreed. He found the number of ranges to be 206, and the platform on the top composed of six stones.

Colonel Coutelle was with the army of Napoleon in 1801, and officially employed with M. Le Père, an architect, at the pyramids of Gizeh.

By measuring the height of each step, and including the two ruined tiers at the top, they made the whole height to the platform = 139·117 metres, which = 456·4 feet English.

$1\frac{5}{8}$ stade = 456·625 feet English.

The trigonometrical survey agreed with this measured height.

This will make the height to the platform = $1\frac{5}{8}$ stade, and height to apex = 10 plethrons = $1\frac{2}{3}$ stade. Side of base = 16 plethrons = $2\frac{2}{3}$ stades. So height : side of base :: 10 : 16 :: 5 : 8.

Height = $\frac{5}{8}$ side of base.

So we shall have 10 plethrons less $1\frac{5}{8}$ stade, or 468·33 feet less 456·625 feet = 11·7 feet for the completion of the pyramid to its apex, which, according to Greaves, in 1638, wanted about 9 feet.

Height to platform = $1\frac{5}{8}$ stade.

Height to apex = $\frac{5}{8}$ side of base.

Height to platform of the teocalli of Cholula = $\frac{5}{8}$ stade.

Pyramid of Cheops = $\frac{1}{2}$ circumference.

Teocalli of Cholula = 1 circumference.

The pyramid of Cheops is terraced, and has a platform at the top like the Mexican teocalli.

The estimate of the teocalli of Cholula has since been modified, and the external pyramid made = $\frac{1}{10}$ distance of the moon.

Herodotus says that all the stones composing the pyramid of Cheops are 30 feet long, well squared, and joined with the greatest exactness, rising on the outside by a gradual ascent, which some call stairs, others little altars.

No mention is here made of the breadth or depth of these stones. Now if we take 30 feet as the average of the greatest perimeter of these squared stones, these 30 feet will equal 14·05 feet English, which will allow 4 feet for the length of each of the two greatest sides, and 3 feet for each of the two less sides; since a Babylonian foot equals 5·62 inches, which is less than half a foot English.

Again, if 30 feet be taken for half the greatest perimeter, or for the length of the two adjacent sides of the largest stones, this will allow 9 by 5 feet English for these two sides.

Coutelle says the stones of the Great Pyramid and those of the second, belonging to the outer covering, rarely exceed 9 feet in length and 6½ in breadth. The height of the steps do not decrease regularly, as we ascend the pyramid, but steps of greater height are sometimes interposed between steps of less height; but, he adds, the same level and the same perfectly horizontal lines appear in all the faces. The height of the steps decreases from the lowest to the highest; the greatest height being 4·628 feet, and the least 1·686 feet. The mean width of the steps is a little more than 1 foot 9 inches, which is deduced from the length of the base, and the side of the platform at the top, which in its present state is 32 feet 8 inches.

Greaves makes the side of the platform 13·28 feet, and says it is not covered with one or three massy stones, but with nine, besides two that are wanting at the angles. Pliny makes the breadth at the top to be 25 feet. Diodorus makes it but 9 feet.

The measurement of one of the larger stones of the pyramid by Coutelle = 9 feet by 6½ feet. Herodotus makes the length of one of these stones = 30 feet, which = 14 feet English. If that represented the length and breadth, and were written equal to 9 + 5 or 9 by 5 feet, then the dimensions of Herodotus and Coutelle would agree. For 18 by 12 feet of Herodotus would nearly = 9 by 6½ of Coutelle's feet.

The number of steps assigned to this pyramid by different authorities vary, according to Greaves, from 260 to 210; who says, that which by experience and by a diligent calculation I and two others found is this, that the number of degrees from the bottom to the top is 207, though one of them in descending reckoned 208.

The least and greatest distances of the sun from the earth has been estimated at 204 and 210 semi-diameters of the sun, the mean of which = 207 semi-diameters.

The entrance to the pyramid of Cheops is on the north side, and said to be about $47\frac{1}{2}$ feet above the base, and on a level with the fifteenth step, reckoning from the foundation; 1 plethron = $46\frac{5}{6}$ feet. Greaves says the entrance has exactly a breadth of $3\frac{463}{1000}$ English feet.

Entrance about $47\frac{1}{2}$ feet above the base = 41 units
$(10 \times 40{\cdot}8, \&c.)^3 = 408^3, \&c. = \frac{3}{5}$ circumference
$(10 \times 10 \times 40{\cdot}8, \&c.)^3 = \frac{3000}{5} = 600$
$4\frac{1}{2}$ cubes of 100 times height = 2700 circumference
= distance of Venus
60 cubes „ = „ Saturn
$(2 \times 10 \times 10 \times 40{\cdot}8, \&c.)^3 = 600 \times 2^3 =$ 4800 circumference
15 cubes of 200 times height = 72000
= distance of Uranus
45 cubes „ = „ Belus

or 30 cubes = diameter of the orbit of Uranus
90 cubes = „ „ Belus
9 cubes of 100 times height = diameter of the orbit of Venus.

Height to entrance = 40·8, &c. units
Height to apex = 407.

If height to entrance = $\frac{1}{10}$ height to apex, then cube of 10 times height to entrance = cube of height to apex = $\frac{1}{16}$ distance of the moon = $\frac{1}{4}$ cube of side of base.

Breadth of entrance = 3·463 feet = 2·993 = 3 units
$3^5 = 243$
$242, \&c.^3 = \frac{1}{8}$ circumference
$(2 \times 242, \&c.)^3 = 1$ „
or $(2 \times 3^5)^3 =$ „
cube of (2×3^5) = circumference in units

Twice breadth $=2 \times 3=6$ units
$6^4=1296$
$1296^3=$ diameter of the orbit of the moon
$(6^4)^3=6^{12}=$,, ,, ,,
or cube of $(2 \times 3)^4=$,, ,, ,,
and cube of $(2 \times 3^5)=$ circumference of the earth.

Again, 3 units $\times 243=3$ stades
$3^5=243$
$2 \times 3^5=2 \times 243=486$
$684^2=$ circumference in stades

or 3^5 doubled, transposed, and squared $=684^2=$ circumference of the earth in stades

$243 \times 684^2=$ circumference in units

or $3^5 \times$ (3^5 doubled, transposed, and squared) = circumference in units.

Cube of 6^4 = diameter of the orbit of the Moon
150 cubes of 6^4 = ,, ,, Mercury
400 cubes of 6^4 = ,, ,, Earth
150^2 cubes of 6^4 = ,, ,, Belus

Sphere diameter of 6^4 = 10 circumference.

Writers since Greaves, in 1638, make the number of steps as follow: —

1655.	Thevenot	208
1692.	Maillet	208
1711.	Pere Sicard	220
1743.	Pococke	212
1763.	Davidson	206
1799.	Denon	208

The Leaning Tower of Pisa is inclined more than 14 feet from the perpendicular. It is built of marble and granite, and has 8 stories, formed by arches, supported by 207 pillars, and divided by cornices. The different stated heights are from 150 to 187 feet.

Here are associated the 8 stories of the tower of Babylon. The 207 pillars, the same number as the terraces of the Great Pyramid, and the height of a teocalli $= \frac{5}{8}$ stade = 175 feet.

This tower was built A. D. 1174: so these associations have only been preserved by repeated copies, like the minarets, which are only imperfect copies of the circular obelisk, because they are devoid of the principle by which the obelisk is constructed.

The number 1600, which represents the side of the base of the Great Pyramid in feet, is also associated with the number of pillars in a Ceylon temple, said to have had 9 stories — none now exist — but 1600 stone pillars, upon which the building was erected, remain. They form a perfect square, each side about 200 feet, containing 40 pillars; around which temple are immense solid domes, having altitudes equal to their greatest diameter. They are for the most part surmounted by spiral cones, that, in some measure, relieve the vastness and massiveness of their gigantic proportions. Like the pyramids of Egypt, their simplicity and solidity of construction have defied the ravages of time.

The solid content of the largest of them has been estimated to exceed 450,000 cubic yards. Its greatest diameter and altitude are equal, and measure 270 feet.

From this description these large domes seem to correspond with the solid generated by the hyperbolic reciprocal curve of contrary flexure, which has an altitude equal the diameter of the base; and the dome terminates in a spiral curve of contrary flexure to the body of the dome.

1 stade = 281 feet.

Side of square = 200 feet = 173 units

175^3, &c. = $\frac{1}{50}$ distance of moon

= $\frac{1}{10}$ cube of Cephrenes

$(10 \times 175, \&c.)^3 = \frac{1000}{50} = 20$

20 cubes of 10 times side = 400 times distance of the moon

= distance of the earth

$(2 \times 10 \times 175, \&c.)^3 = 20 \times 2^3 = 1600$

Cube of 20 times side or of 5 times perimeter

= 1600 times distance of the moon

= twice the diameter of the orbit of the earth

10 cubes of 10 times side = 200 distance of the moon.
20 cubes = 400 „ „
= distance of the earth.

Vyse's Measurements of the Pyramid of Cheops.

	Feet.
Former base - - - -	764
Present base - - - -	746
Present perpendicular height - -	450·9
Present height inclined - -	568·3
Former height inclined - -	611
Perpendicular height by casing stones -	480·9

Having calculated the terraced pyramid of Cheops = $\frac{1}{2}$ circumference, a plain pyramid having the sides cased, and side of base and height = the former base and height by Vyse's measurement will = $\frac{1}{18}$ distance of moon.

Former base = 764 feet = 660 units
Former height = 480·9 = 416 „

Then height × area base
= 413, &c. × 662^2 = $\frac{1}{6}$ distance of moon
Pyramid = $\frac{1}{18}$ „

Thus it appears that the pyramid of Cheops in its present state may be regarded as a teocalli or terraced pyramid having the content = $\frac{1}{2}$ circumference of the earth.

But if the terraced pyramid were completely cased on all sides, the plain pyramid would = $\frac{1}{18}$ distance of the moon.

Vyse's former base = 764 feet
former height = 480·9
present base = 746
present height = 450·9

∴ former base : present base
:: former height : height to apex of present pyramid
or 764 : 746 :: 480·9 : 469·6 feet
469·6 feet = 406 units

According to our calculation

height to apex = 406 units
side of base = 648 „

So that the completely cased pyramid would be similar to the terraced pyramid if completed to the apex.

Height of each pyramid will = $\frac{5}{8}$ side of base.
Cube of height = 414^3 = $\frac{5}{8}$ circumference
Cube of 2 = 5
Cube of 4 = 40
Cube of 4 times height
= 40 times circumference

Cube of side of base = 662^3 = $\frac{8}{30}$ distance of the moon
Cube of side of base
of terraced pyramid = $\frac{1}{4}$ „
Cubes will be as $\frac{8}{30} : \frac{1}{4} :: 32 : 30$
Former inclined side = 611 feet
= 528 units
$528^3 = \frac{8}{50}$ distance of the moon
$(5 \times 528)^3 = \frac{8}{50} \times 5^3 = \frac{200}{10} = 20$
20 cubes of 5 times inclined side
= 400 times distance of the moon
= distance of the earth
$(10 \times 528)^3 = \frac{8000}{50} = \frac{800}{5}$ distance of the moon
5 cubes of 10 times inclined side
= 800 times the distance of the moon
= diameter of the orbit of the earth

Area of base of cased pyramid = 662^2 units
„ terraced „ = 648^2 „

Vyse makes the area of the

	A.	R.	P.
former base =	13	1	22
present base =	12	3	3

Terraced pyramid height = 405 units
side of base = 648 „
Cased pyramid height = 414 „
side of base = 662 „

In both pyramids, height = $\frac{5}{8}$ side of base.

Their contents are as $\frac{1}{2}$ circumference : $\frac{1}{18}$ distance of moon.

$$:: \tfrac{1}{2} \text{ circumference} : \frac{9{\cdot}55}{18} \text{ circumference}$$

$$:: 18 \quad : 19{\cdot}1$$

Greaves, in describing the interior of the Great Pyramid, says, this gallery or corridor, or whatever else I may call it, is built of white and polished marble, which is very evenly cut in spacious squares or tables. Of such materials as is the pavement, such is the roof, and such are the side walls that flank it; the knitting of the joints is so close, that they are scarcely discernible to a curious eye; and that which adds grace to the whole structure, though it makes the passage the more slippery and difficult, is the acclivity and rising to the ascent. The height of this gallery is 26 feet, the breadth is $6\frac{870}{1000}$ feet; of which $3\frac{435}{1000}$ feet are to be allowed for the way in the midst, which is set and bounded on both sides with two banks (like benches) of sleek and polished stone; each of these hath $1\frac{717}{1000}$ feet in breadth, and as much in depth.

Breadth of gallery = 6·87 feet = 5·94 units
if = 6

The way in the middle = $\frac{1}{2}$ breadth of gallery = $\frac{1}{2}6 = 3$ units,

$3^5 = 243$,

243 transposed, doubled and squared, $= 684^2$,

$$243 \times 684^2 = \text{circumference of earth.}$$

Thus $3^5 \times (3^5$ transposed, doubled and squared$)$ = circumference.

Or $(2 \times 3^5)^3 = (2 \times 243)^3$,
and $(2 \times 242, \text{\&c.})^3$ = circumference.

So $2^3 \times 3^{15}$ = circumference nearly.

Breadth of gallery = 6 units.

Breadth to the power of 4 times $3 = (6^4)^3 = 6^{12}$
$=$ diameter of the orbit of moon,
or $(2 \times 3)^{12} =$ twice distance of moon.

Height of gallery $= 26$ feet $= 22{\cdot}41$ units,
$(10 \times 22{\cdot}4, \&c.)^3 = 224^3, \&c. = \frac{1}{10}$ circumference
$(10 \times 10 \times 22{\cdot}4, \&c.)^3 = \frac{1000}{10} = 100.$

Cube of 100 times height
$= 100$ times circumference.

Sphere diameter $6^4 = 10$ circumference.

This gallery is of the hyperbolic order. See *fig.* 42, hyperbolic areas.

Greaves, describing what is now commonly called the King's Chamber, containing a granite sarcophagus, says the length of this chamber on the south side, most accurately taken at the joint where the first and second row of stones meet, is $34\frac{380}{1000}$ English feet. The breadth of the west side, at the joint or line where the first and second row of stones meet, is $17\frac{190}{1000}$ feet. The height is $19\frac{1}{2}$ feet.

"These proportions of the chamber, and those of the length and breadth of the hollow part of the tomb, were taken by me with as much exactness as it was possible to do; which I did so much the more diligently, as judging this to be the fittest place for fixing the measures for posterity; a thing which hath been much desired by learned men, but the manner how it might be exactly done hath been thought of by none."

Chamber.

Length $34{\cdot}38$ feet $= 29{\cdot}72$ units.
Breadth $17{\cdot}19$ „ $= 14{\cdot}86$
Height $19{\cdot}5$ „ $= 16{\cdot}85$
$(10 \times 29{\cdot}6, \&c.)^3 = 296^3, \&c. = \frac{24}{1000}$ distance of moon
$(5 \times 10 \times 29{\cdot}6, \&c.)^3 = \frac{24}{1000} \times 5^3 = \frac{3000}{1000} = 3$
$(5 \times 5 \times 10 \times 29{\cdot}6, \&c.)^3 = 3 \times 5^3 = 375$

10 cubes of 250 times length = 3750 distance of Moon
= distance of Saturn
20 cubes „ = Uranus
60 cubes „ = Belus.

Breadth = 14·86 units.

$(10 \times 14{\cdot}8, \text{\&c.})^3 = 148^3, \text{\&c.} = \frac{3}{1000}$ distance of moon
$(10 \times 10 \times 14{\cdot}8, \text{\&c.})^3 = \frac{3000}{1000} = 3$

Height = 16·85 units.

$(10 \times 16{\cdot}5, \text{\&c.})^3 = 165^3, \text{\&c.} = \frac{4}{100}$ circumference
$(10 \times 10 \times 16{\cdot}5, \text{\&c.})^3 = \frac{4000}{100} = 40$
$(2 \times 10 \times 10 \times 16{\cdot}5, \text{\&c.})^3 = 40 \times 2^3 = 320$

12 cubes of 200 times height = 320 × 12 = 3840 circumference = distance of Earth.

Cube of 100 times breadth = 3 distance of Moon.
Pyramid = $\frac{1}{3}$ cube = distance of Moon.
50 cubes = 150 distance of Moon.
= distance of Mercury.

Content = 29·6, &c. × 14·8, &c. × 16·5, &c. = 7290,
4 times content = 4 × 7290 = 29160,
and distance of Belus = about 29160^3
= the cube of 4 times content of chamber
= the cube of Babylon
= the cube of 120 stades.

In the "Library of Entertaining Knowledge," the

Height of this chamber = 19·214 feet
Length on south side = 34·348
Width on west side = 17·056

Cube of 4 times content : cube of 5 times content :: $4^3 : 5^3$ :: 64 : 125 :: 1 : 2 nearly.

Thus cube of 4 times content
= distance of Belus = cube of Babylon;
cube of 5 times content
= twice distance of Belus = twice cube of Babylon
= distance of Ninus = cube of Nineveh.

Length + breadth + height
$=29·6$, &c. $+ 14·8$, &c. $+ 16·5$, &c. $= 61$ units
$(600 \times 61)^3 = 36600^3 =$ diameter of orbit of Belus,
cube of 600 times (sum of 2 sides + height)
$= 36600^3 =$ diameter of orbit of Belus
= distance of Ninus
= cube of Nineveh.

There is a very small temple at Philæ, by some supposed to be Grecian. There is only a single chamber in it, about 11½ feet long by 8 wide, with a doorway at each end, opposite to one another.

11·5 by 8 feet
= 10 by 7 units.
$10·2^3 = 12$ seconds.
$7·1^3 = 4$ „
Cubes of the sides are as 1 : 3.

Hamilton found at Gau Kebir, at the furthest extremity of the temple, a monolith chamber of the same character. It had a pyramidal top, and measured 12 feet in height and 9 in width at the base. Within were sculptured hawks and foxes, with priests presenting offerings to them, and the same ornaments on the doorway as are seen on the entrances of the great temples.

12 feet = 10·3 units,
9 „ = 7·7 „

If the base be a square, content will $= 7·7^2 \times 10·3 = 610$ units, and $610^3 =$ twice circumference.

7 seconds = 613·9 units,
1″ „ = 87·7 „ = 101·4 feet English,
1‴ „ = 1·461 „ = 1·69 „
$5''' = 5 \times 1·69$ = 8·45 „
12 cubits = 8·43 „

113689008 units = circumference = 360 degrees.
315802 „ = 1 degree.
5263 „ = 1 minute.
87·7 „ = 1 second.
1·461 „ = 1‴

Denon found granite monoliths of small dimensions at Philæ, both of them in the great temple, and placed respectively at the extremity of the two adjoining sanctuaries. The dimensions of one of them are 6 feet 9 inches in height, 2 feet 8 inches in width, and 2 feet 5 inches deep, French measure.

Not knowing the exact proportion between the French and English foot, but taking the French to exceed the English by $\frac{1}{20}$ part,

Dimensions in English feet:—

$$\begin{array}{rlll} & =6{\cdot}75 & 2{\cdot}66 & 2{\cdot}41 \\ & =5{\cdot}84 & 2{\cdot}33 & 2{\cdot}1 \text{ units} \\ \frac{1}{20} & = \underline{\ {\cdot}29} & \underline{\ {\cdot}11} & \underline{\ {\cdot}1} \\ & =6{\cdot}13 & 2{\cdot}44 & 2{\cdot}2. \end{array}$$

Content $= 6{\cdot}13 \times 2{\cdot}44 \times 2{\cdot}2 = 33$,

and about $33{\cdot}2^9$ = diameter of the orbit of Belus

= distance of Ninus.

$30{\cdot}7^3$, &c. = 29160 units = 120 stades,

= side of Babylon.

$33{\cdot}2^3$, &c. = 36450 units = 150 stades,

= side of Nineveh.

Thus content raised to the power of 3 times 3

= cube of Nineveh

= distance of Ninus.

Three winged globes, one above another, decorate the architrave of the doorway. The frieze and cornice are ornamented with a series of serpents erect. The holes in which the hinges of the door were fastened are still visible.

The winged globe, flanked on each side by the erect serpent, usually ornaments the frieze of the doorway of an Egyptian temple. The cube of the dimensions of these temples denote celestial distances.

Hence the winged globe denotes the third power.

Three winged globes denote three times the third, or the ninth power.

" From the top to the bottom of this chamber (of Cheops) are six ranges of stone, all of which being respectively sized to an equal height, very gracefully in one and the same altitude run round the room. The stones which cover this place are of a strange and stupendous length, like so many huge beams lying flat and traversing the room, and withal supporting that infinite mass and weight of the pyramid above. Of these there are nine, which cover the roof; two of them are less by half in breadth than the rest; the one at the east, the other at the west."

"Within this glorious room," says Greaves, "as within some consecrated oratory, stands the monument of Cheops or Chemmis, of one piece of marble, hollow within and uncovered at the top, and sounding like a bell. This tomb is cut smooth and plain, without any sculpture or engraving. The exterior superficies of it contains in length 7 feet $3\frac{1}{2}$ inches; in depth it is 3 feet $3\frac{3}{4}$ inches, and the same in breadth. The hollow part within is in length, on the west side, $6\frac{488}{1000}$ feet. In breadth, at the north end, $2\frac{218}{1000}$ feet. The depth is $2\frac{860}{1000}$ feet."

Sarcophagus outside:—

length 7 ft. $3\frac{1}{2}$ in. = 6·3 units
depth 3 ft. $3\frac{3}{4}$ in. = 2·863
breadth = 2·863

Sum = 12·026 units
Content = 51·53 units

$(10 \times 51{\cdot}4)^3 = 514^3 = \frac{1}{8}$ distance of the Moon
$(2 \times 10 \times 51{\cdot}4)^3 = \frac{8}{8} = 1$ „ „
Cube of 20 times content = „ „
150 cubes „ = „ „ Mercury
150^2 cubes „ = „ „ Belus

depth = breadth = 2·863 units
$10 \times 2{\cdot}86 = 28{\cdot}6$
Distance of Neptune = $28{\cdot}6^9$

or 10 times breadth to the power of 3 times 3 = distance of Neptune.

Length = 6·3 units
$(100 \times 6{\cdot}32)^3 = 632^3 = \frac{20}{9}$ distance of the Moon
$(3 \times 100 \times 6{\cdot}32)^3 = \frac{20}{9} \times 3^3 = 60$

5 cubes of 300 times length
= 300 times distance of the Moon
= diameter of the orbit of Mercury.
$6{\cdot}4^2$, &c. = 41, &c.
$\frac{1}{2}(6{\cdot}4$, &c.$)^2 = 20{\cdot}5$, &c.
and $20{\cdot}5^9$, &c. = distance of Mars
$(\frac{1}{2}(6{\cdot}4$, &c.$)^2)^9 = 20{\cdot}5^9$, &c. = „

$\frac{1}{2}$ square of length to the power of 3 times 3 = distance of Mars.

Depth = 2·863 units
$2{\cdot}87^3 = 23{\cdot}5$, &c.
and $23{\cdot}5^9$, &c. = distance of Jupiter
$(2{\cdot}87^3)^9 = 2{\cdot}87^{27} = 2{\cdot}87^{3\times3\times3} = 23{\cdot}5^9$, &c.

Depth to the power of 3 times 3 = $23{\cdot}5^9$, &c. = distance of Jupiter.

Sum of length, depth, and breadth = 12·026 units
$(\frac{1}{2}\ 12)^{12} = 6^{12}$ = diameter of the orbit of the Moon.

Depth × breadth $= 2{\cdot}86 \times 2{\cdot}86 = 8{\cdot}17$, &c.
$100 \times 8{\cdot}17 = 817$
and $816^3 = \frac{1}{2}$ distance of the Moon

Sarcophagus inside: —
length 6·488 feet = 5·61 units
breadth 2·218 feet = 1·917
depth 2·86 feet = 2·473
content = 26·595 units
$(10 \times 26{\cdot}7)^3 = 267^3 = \frac{7}{400}$ distance of Moon
$(2 \times 10 \times 26{\cdot}7)^3 = \frac{7}{400} \times 2^3 = \frac{56}{400}$
$(10 \times 2 \times 10 \times 26{\cdot}7)^3 = \frac{56000}{400} = 140$
2 cubes of 200 times content = 280 distance of Moon
= distance of Venus

Length + breadth + depth $= 5{\cdot}61 + 1{\cdot}917 + 2{\cdot}473 = 10$ units
length = 5·61
$(100 \times 5{\cdot}65$, &c.$)^3 = 565^3$, &c. $= \frac{1}{6}$ distance of Moon
breadth = 1·917
$10 \times 1{\cdot}917 = 19{\cdot}17$
and 19^9 = distance of Venus

$100 \times 1{\cdot}917 = 191{\cdot}7$
and 189^3, &c. $= \frac{3}{50}$ circumference.
depth $= 2{\cdot}473$
$100 \times 2{\cdot}473 = 247{\cdot}3$
and 247^3, &c. $= \frac{4}{30}$ circumference.

Cube of 10 times external content : cube of 10 times internal content :: $\frac{1}{8}$: $\frac{7}{400}$ distance of Moon :: 400 : 56 :: 50 : 7.

Depth $= 2{\cdot}473$ units
$2{\cdot}48^3$, &c. $= 15{\cdot}35$
$2 \times (2{\cdot}48$, &c.$)^3 = 30{\cdot}7$
and $30{\cdot}7^9$ = distance of Belus
$(2 \times (2{\cdot}48$, &c.$)^3)^9 = 30{\cdot}7^9$.

Twice cube of depth to the power of 3 times 3 $= 30{\cdot}7^9 =$ distance of Belus.

The measurements of the sarcophagus made by Greaves differ from those lately made by Vyse. The latter makes the external

length 7 ft. $6\frac{1}{2}$ in. $= 6{\cdot}51$ units
breadth 3 ft. 3 in. $= 2{\cdot}81$ „
height 3 ft. 5 in. $= 2{\cdot}95$ „

Internal

length 6 ft. 6 in. $= 5{\cdot}62$ units
breadth 2 ft. $2\frac{1}{2}$ in. $= 1{\cdot}908$ „
depth 2 ft. $10\frac{1}{2}$ in. $= 2{\cdot}48$ „
external length $= 6{\cdot}51$
$3 \times 6{\cdot}51 = 19{\cdot}53$
distance of earth $= 19{\cdot}5^9$, &c.

3 times length to the power of 3 times 3 = distance of earth.

External content $= 6{\cdot}51 \times 2{\cdot}81 \times 2{\cdot}95 = 53{\cdot}96$
$\frac{1}{2} = 26{\cdot}98$
distance of Uranus $= 26{\cdot}9^9$, &c.

Half content to the power of 3 times 3 = distance of Uranus.

Davison has since discovered a chamber immediately over the king's chamber, which is now called Davison's chamber. It is reached by mounting, with the help of a ladder, to a hole at the top of the upper part of the high ascending gallery. The stones which form the ceiling of the king's chamber form also the floor of the upper chamber, but the room is four feet longer than that below.

More recently Caviglia has discovered a large chamber cut in the rock, and under the centre of the pyramid. The dimensions are not minutely given. The chamber is stated to be about 66 feet by 27, with a flat root and very irregular floor.

27 feet $=23{\cdot}34$ units
66 feet $=57{\cdot}$ „
$23{\cdot}5^{9}$, &c. = distance of Jupiter
$\frac{1}{2}57=28{\cdot}65$
and $28{\cdot}6^{9}=$ distance of Neptune.

Wilkinson observes, no doubt it was by the causeways that stories were carried on sledges to the pyramids; that of the Great Pyramid is described by Herodotus as 5 stades long, 10 orgyes broad, 8 orgyes high, of polished stones, adorned with figures of animals (hieroglyphics), and it took no less than ten years to complete it. Though the size of the stade is uncertain, we may take an average of 610 feet, which will require this causeway to have been 3050 feet in length (a measurement agreeing very well with the 1000 yards of Pococke, though we can now no longer trace it for more than 1424 feet, the rest being buried by the alluvial deposite of the inundation). Its present breadth is only 32 feet, the outer faces having fallen; but the height, 85, exceeds that given by Herodotus, and it is evident, from the actual height of the hill, from 80 to 85 feet, to whose surface the causeway actually reached, and from his allowing 100 feet from the plain to the top of the hill, that the expression 8 orgyes (48 feet) is an oversight either of the historian or his copyist.

It was repaired by the caliphs and Memlook kings, who made use of the same causeway to carry back to the

"Arabian shore" those blocks that had before cost so much time and labour to transport from the mountains; and several of the finest buildings of the capital were constructed with the stones of this quarried pyramid.

The length of the causeway of Herodotus
= 5 stades = 1405 feet
The breadth = 10 orgyes = 28·1
The height = 8 ,, = 22·48

The length of the causeway of Wilkinson
= 1424 feet
Breadth = 32 ,,
Height = 85 ,, or 30 orgyes.

The causeway, which formed the wonderful approach to the pyramidal temples, was 5 stades in length (the line of measure so frequently associated with the sacred structures in the four quarters of the world).

As 5 stades is so frequently mentioned, it may be as well to give an instance of a granite structure of nearly that length.

Waterloo bridge, over the Thames, has nine arches, is built entirely of granite, and is 1280 feet in length. The breadth of the carriage road or causeway is 28 feet. The parapet, or foot walk on each side of the carriage road, is 7 feet in breadth.

5 stades = 1215 units
$\frac{1}{2}$ = 607·5

601^3 = cube of Cephrenes = $\frac{1}{5}$ distance of Moon
1202^3 ,, = $\frac{8}{5}$,,

610^3 = 2 circumference
1220^3 = 16 ,,

5 stades = 1215 units
1424 feet = 1231
123^3, &c. = $\frac{1}{80}$ circumference
$(10 \times 123, \text{\&c.})^3 = \frac{1000}{60} = \frac{100}{6}$
$(6 \times 10 \times 123, \text{\&c.})^3 = \frac{100}{6} \times 6^3 = 3600$

10 cubes of 6 times length $=36000$ circumference
$=$ distance of Saturn
20 cubes ,, $=$,, Uranus
60 cubes ,, $=$,, Belus.

Should the length have equalled originally 1296 units $=5\frac{1}{3}$ stades.

Then cube of length $=1296^3=6^{12}$
$=$ diameter of orbit of Moon.

Sphere, diameter $1296=10$ circumference.

1 stade $=243$ units
and 242^3, &c. $=\frac{1}{8}$ circumference
$(2\times 242, \&c.)^3=1$
$(4\times 242, \&c.)^3=8$

5 stades $=1215$ units
5 stades $+$ 5 units $=1220$ units.

Cube of (5 stades $+$ 5 units) $=1220^3$
$=$ 16 circumference.

Cube of 5 times (5 stades $+$ 5 units)
$=16\times 5^3=2000$ circumference

$3\times 5\times$ (5 stades $+$ 5 units) $=2000\times 3^3$
$=54000$ circumference

4 cubes of 15 times (5 stades $+$ 5 units)
$=216000$ circumference $=$ distance of Belus

$2\times 3\times 5\times$ (5 stades $+$ 5 units) $=54000\times 2^3$
$=432000$ circumference $=$ diameter of orbit of Belus.

Cube of 30 times (5 stades $+$ 5 units)
$=$ diameter of orbit of Belus
$=$ distance of Ninus.

According to Ctesias, the bridge over the Euphrates at Babylon was 5 stades in length. Strabo says the Euphrates at Babylon was a stade in breadth.

It is stated in the "Athenæum" that the blocks of which the pyramid of Cheops is composed are roughly squared, but built in regular courses, varying from 2 feet 2 inches to 4 feet 10 inches in thickness, the joints being properly broken throughout. The stone used for casing the exterior, and for the lining of the chambers and passages, were obtained from the Gebel Mokattam, on the Arabian side of the valley of the Nile; it is a compact limestone, called by geologists swine-stone, or stink-stone, from emitting, when struck, a fetid odour, whereas the rocks on the Libyan side of the valley, where the pyramids stand, are of a loose granulated texture, abounding with marine fossils, and, consequently, unfit for fine work, and liable to decay. The mortar used for the casing and for lining the passages was composed entirely of lime; but that in the body of the pyramid was compounded of ground red brick, gravel, Nile-earth, and crushed granite, or of calcareous stone and lime, and in some places a grout, or liquid mortar, of desert sand and gravel only has been used. It is worthy of especial notice that the joints of the casing-stones, which were discovered at the base of the northern front, as also in the passages, are so fine as scarcely to be perceptible. The casing-stones, roughly cut out to the required angle, were built in horizontal layers, corresponding with the courses of the pyramid itself, and afterwards finished, as to their outer surface, according to the usual practice of the ancients. In order to insure the stability of the superstructure, the rock was levelled to a flat bed, and part of the rock was stopped up in horizontal beds, agreeing in thickness with the courses of the artificial work.

The plain on which the pyramids at Gizeh stand is a dry, barren, irregular surface. According to Jomard the elevation of the base of the foundation stone, let into the solid rock, at the north-east end of the Great Pyramid, is 140 feet above the superior cubit of the Nilometer at Rouda; nearly 130 feet above the valley, and the mean elevation of the floods (from the year 1798 to 1801); and nearly 164 above the mean level of the low state of the Nile for the same period.

140·5 feet English = $\frac{1}{2}$ stade = 300 feet of Herodotus, who states that the pyramids of Cheops and Cephrenes are of equal height, and stand on the same hill, which is about 100 feet high.

On this platform of rock stand the massive pyramids, — monuments of the skill of man and the antiquity of science,— temples of a remote epoch, where man adored the visible symbol of nature's universal law, and through that the invisible God of creation.

Here the pyramid of Cheops indicates the $\frac{1}{2}$ circumference of the earth, and the $\frac{1}{2}$ diameter of the earth's orbit. Its towering summit may be supposed to reach the heavens, and the pyramid itself to represent the law of the time of a body gravitating from the earth to the sun.

The solid hyperbolic temple — the Shoemadoo at Pegu — represents the law of velocity corresponding to this law of the time.

These two symbols of the laws of gravitation that pervade the universe resemble the close alliance of Osiris and Isis, — husband and wife, brother and sister, — the two ancient deities said to comprehend all nature. On the statue of the goddess were inscribed these words: — "I am all that has been, that shall be, and none among mortals has raised my veil."

The Brahmins say the gods are merely the reflecting mirrors of the divine powers, and finally of God himself.

The pyramid may be supposed to reach the heavens. So it was by building pyramids that the giants of old were said, figuratively, to have scaled the heavens.

L'Abbé de Binos (1777), in his letters addressed to Madame Elizabeth of France, mentions that the pyramids of Egypt are supposed by some to be the tombs of the ancient kings; that they are called by others the mountains of Pharaoh; that the poets have described them as rocks heaped one upon the other by the Titans, in order to scale Olympus.

The Abbé ascended the Great Pyramid, and found the top of it about twelve feet square; and upon it he observed six large stones, arranged in the form of an L, which he was told signified a hieroglyphic.

The pyramids may be regarded as scientific and religious monuments. The great pyramid of Cheops may have been both a temple and fortress, like the teocalli of Mexitli, or, like the great teocalli of Teotihuacan, a temple of the Sun, before which the glorious orb of day may have been worshipped as an emblem of God, when he rose above the eastern range of hills between the Nile and the Red Sea, then passing to the west till he set beyond the Libyan desert, a region of desolation and aridity, extending from the pyramids, through the Sahara, to the "Sea of Darkness,"—the distant Atlantic Ocean.

De Sacy has endeavoured to trace the origin of the word pyramid, not in the Greek language, but in the primitive Egyptian language. The radical term signifies something sacred, the approach to which is forbidden to the vulgar.

The worship of the planets, says Jablonski, formed a remarkable feature in the early religion of Egypt, but in process of time it fell into desuetude.

The Burmese hyperbolic temples, like the Egyptian and Mexican pyramidal temples, were most probably originally dedicated to the worship of the heavenly bodies.

The Persian poet, Firdausi, represents them as "pure in faith, who, while worshipping one supreme God, contemplate in sacred flame the symbol of divine light." The fire-worshippers abhorred alike the use of images and the worship of temples; they regarded fire as the symbol of God.

The Sabæans regarded the pyramidal and hyperbolic temples and the obelisk as the symbols of divinity.

Thus, a simple quadrilateral monument, without a cypher, has transmitted to the present age a proof of the scientific acquirements of an epoch that long preceded the earliest dawn of European civilisation. The pyramidal, like the obeliscal records of science, monuments combining the physical and intellectual power of man, have endured ages after all traditional and written records have perished.

The laws formed by the Creator for the government of the celestial bodies had become, by the uniformity of their action, known to man, after a lengthened series of astronomical observations. These laws, when symbolised in geometrical forms, became objects of reverence, and the invisible

Creator was worshipped through the visible type of his laws.

Such appears to have been the origin and mode of worship of the ancient Sabæans. Yet, however remote the period of its origin might have been, and however generally it might have been adopted at an early epoch, at the present time it embraces very few votaries in comparison with those it formerly numbered.

The obelisk and pyramid are symbolical of the laws that govern the heavens. Religion taught the people to kneel before these sublime monuments, — to look with reverential awe on heaven's law, and worship heaven's God.

The Egyptians of a later period also believed in the unity of the Deity; but when they spoke of his attributes they personified them separately, and, in process of time, fell into the natural course of idolatry. They mingled truth with error, and, as is usually the case, truth was obscured and error prevailed.

The Egyptians believed in the immortality of the soul. They adopted the doctrine of the transmigration of the souls of the wicked, through various animals, for a period of 3000 years, or "the circle of necessity," to expiate the sins of the flesh: whereas the souls of the just were absorbed into the Deity; they became part of Osiris, and their mummies were invested with the emblems of the gods, to signify that their soul had become a part of the divine essence.

Champollion Figeac thus expresses his views of the Egyptian theocracy: — "A theocracy, or a government of priests, was the first known to the Egyptians; and it is necessary to give this word priests the acceptation that it bore in remote times, when the ministers of religion were also the ministers of science and knowledge; so that they united in their own persons two of the noblest missions with which man can be invested, — the worship of the Deity, and the cultivation of intelligence."

"This theocracy was necessarily despotic. On the other hand, with regard to despotism (we add these reflections to reassure our readers, too ready to take alarm at the social condition of the early Egyptians), there are so many different kinds of despotism, that the Egyptians had to accept one of them, as an unavoidable condition. In fact, there is in a theocratic government the chance of religious despotism; in an aristocracy, or oligarchy, the chance of a feudal despotism; in a republic, the chance of a democratic despotism — everywhere a chance of oppression. The relative good will be where these several chances are most limited. And, with respect to the form of government best adapted to the social happiness of man, opinions are as varied as are the countries and human races on the earth. That institution which is admirably suited to Europeans may be odious and deleterious to Orientals."

The early mythology of the ancient nations would appear to have centred in the divine attributes and operations, which created, animated, and preserved the celestial and terrestrial systems, — this mythology being represented under an embodied form, which, not being generally understood, led eventually to the introduction of idolatrous practices. Thus superstition and darkness spread over these countries. The purity of the original faith being sullied, the whole mythology was misunderstood, and its tenets and symbols misrepresented and perverted.

The primeval theology peculiar to those early ages may be deemed the spiritual. The less refined system prevalent in later times, and from which most of the writers, both ancient and modern, have drawn their inferences, may be termed the physical. The spiritual, which may be regarded as arcanic, comprised the more abstruse stores of ancient wisdom, and was revealed to the initiated only. The physical, being rendered palpable to the senses, was adapted to the capacity of the unlearned and unreflecting.

Herodotus attributes the building of the three pyramids at Gizeh to Cheops, Cephrenes, and Mycerinus. He says,— "They informed me that Cephrenes reigned 56 years, and that the Egyptians, having been oppressed by building the pyramids, and all manner of calamities, for 160 years, during all which time the temples were never opened, had conceived so great an aversion to the memory of the two kings, that no Egyptian will mention their names, but they always attribute their pyramids to one Philition, a shepherd who kept his cattle in those parts. They said, also, that after the death of Cephrenes, Mycerinus, the son of Cheops, became king; and, disapproving the conduct of his father, opened the temples, and permitted the people, who were reduced to the last extremities, to apply themselves to their own affairs, and to sacrifice as in preceding times."

Since Mycerinus permitted the people to sacrifice as in preceding times, it follows that sacrifice was not practised during the two preceding reigns at least, since the Egyptians had been oppressed for 160 years. Cheops reigned 50 years.

The religious rites of Boodha are performed at this day before the solid hyperbolic temples, where sacrifice is never practised.

We shall not stop to inquire whether Cheops and Cephrenes were, as sovereign pontiffs, innovators or reformers of the national religion; or whether they wished by closing the temples to compel the people to worship before the pyramids, teocallis and obelisks;—or whether these pyramids were built by Cheops and Cephrenes,—for, according to Manetho, that of Cheops was built by Suphis, 1000 years before their reigns. The builders, however, adopted the same Babylonian standard of unity in the construction of these pyramids as that used in the sacred structures of the Brahmins, Boodhists, Chaldeans, Druids, Mexicans, and Peruvians.

Suphis was arrogant towards the gods; but, when penitent he wrote the sacred book which the Egyptians value so highly. From this account of Suphis he appears also to have been a reformer of the Egyptian religion.

These inquiries may be left to those conversant with hieroglyphics and Egyptian researches, whose recent labours have thrown so much light on the manners and customs of ancient Egyptians, that, it is said, Lepsius intends to write the Court Journal of the Fourth Memphite dynasty.

Wilkinson thinks that the oldest monuments of Egypt, and probably of the world, are the pyramids to the north of Memphis; but the absence of hieroglyphics and of every trace of sculpture precludes the possibility of ascertaining the exact period of erection, or the names of their founders. "From all that can be collected on this head it appears that Suphis and his brother Sensuphis erected them about the year 2120 B. C.; and the tombs in their vicinity have been built, or cut in the rock, shortly after their completion. These present the names of very ancient kings, whom we are still unable to refer to any certain epoch, or to place in the series of dynasties."

Sayuti and other Arabic writers conceive that the pyramids were erected before the Deluge, or more correct accounts of them would have existed.

Jomard says that the tradition that the pyramids were antediluvian buildings only proves their great antiquity, and that nothing certain was known about them. They have been attributed to Venephes, the fourth king of the first dynasty; and to Sensuphis, the second king of the fourth Memphite race.

According to Lepsius, the pyramids of Gizeh were built under the fourth dynasty of Manetho, 4000 B. C. Vyse found Shoopho, whom the Greeks called Suphis the First, in the quarriers' marks in the new chamber of the Great Pyramid, scored in red ochre, in hieroglyphics, on the rough stones.

"The tombs around the pyramids," remarks Gliddon, "afford us abundance of sculptural and pictorial illustrations of manners and customs, and attest the height to which civilisation had attained in the reign of Shoopho; while, in one of them, a hieroglyphical legend tells us that this is 'the sepulchre of Eimei, great priest of the habitations of King Shoopho.' This is probably that of the architect, according to whose plans and directions the mighty edifice—near the foot of which he once reposed—the largest, best-constructed, most ancient, and most durable of mausolea in the world, was built, and which, for 4000 to 5000 years after his decease, still stands an imperishable record of his skill."

Shoopho's name is also found in the Thebaid as the date of a tomb at Chenaboscion. In the peninsula of Mount Sinai his name and tablets show that the copper mines of the Arabian district were worked by him. Above his name the titles "Pure King and Sacred Priest" are in strict accordance with Asiatic institutions, wherein the chief generally combines in his person the attributes of temporal and spiritual dominion. His royal golden signet has recently been discovered. The sculptures of the Memphite necropolis inform us that Memphis once had a palace called "the abode of Shoopho."

Lepsius thinks the tomb to be that of Prince Merhet, who, as he was a priest of Chufu (Cheops), named one of his sons "Chufu-mer-nuteru," and possessed eight villages, the names of which are compounded with that of Chufu. And the position of the grave on the west side of the pyramid of Chufu, as well as the perfect identity of style in the sculptures, render it more than probable that Merhet was the son of Chufu, by which the whole representations are rendered more interesting. This prince was also "Superintendent-General of the Royal Buildings," and thus had the rank of high court architect, a great and important post in these times of magnificent architecture, and which we have often found under the direction of princes and members of the royal family. It is therefore to be conjectured that he also overlooked the building of the Great Pyramid.

If the pyramid be regarded as typical of Osiris,—"he who makes time,"—and the hyperbolic solid symbolic of Isis,—velocity $\propto$ inversely as time—then the Egyptian pyramid and Burmese hyperbolic temple, both being typical of gravity, may be supposed to represent Osiris and Isis, husband and wife, brother and sister, both of divine origin.

In the great hyperbolic temple, the Shoemadoo of Pegu, is a statue of Mahasumdera, the protectress of the world; but, when the time of general dissolution arrives, by her hand the world is to be destroyed.

The obelisk, combined in the same figure (49.) with the pyramid and hyperbolic solid, is symbolical of both time and velocity at a small distance only from the surface of the earth. So the obelisk may have been regarded as Horus, the son of Osiris and Isis.

Typhon assumed the form of a crocodile to avoid the vengeance of Horus. The crocodile we suppose to have been held sacred, from its round and tapering body resembling a circular obelisk.

As the pyramid is generated from the base to the apex, the obelisk, which $\propto D^2$ from the apex, decreases from the base to the apex; so that at the end of the descent the pyramid is completed and the obelisk consumed. So Osiris may be said to devour his own child. For Osiris substitute Saturn, who was also Kronos or time, and we have the myth of Saturn, the son of Cœlum and Terra, or Vesta devouring his own offspring. Cœlum married his own daughter Terra.

Saturn succeeded Cœlum, and married his own sister Ops, Rhea, or Cybele. The ancients dedicated the cube to Cybele.

The brothers of Saturn and Cybele were the Titans, Centimani, or hundred-handed giants with fifty heads.

The height of the tower of Belus equalled the height of 50 men, or the length of 100 arms. But the pyramidal tower, like the hyperbolic solid, would represent any supposed distance in the heavens. So the giants may be said to have scaled the heavens.

The tower contained as many cubes of unity as equalled in extent $\frac{1}{24}$ of the earth's circumference.

After Saturn was deposed by Jupiter, he ordained laws and civilised the people of Latium, as Osiris did the Egyptians. Both instructed the people in agriculture. The curve of Osiris resembles a crosier or sickle. Saturn received from his mother a scythe or sickle. The hour-glass, formed of two hollow cones or circular pyramids, is a symbol of Saturn.

The marriage of Cœlum and Terra is figurative of the laws of gravitation by which the earth and the heavenly bodies are mutually influenced, and the harmony of the solar system preserved during "all the time that has been and all the time that shall be."

Cœlum was the son of Ether and Dies, and the most ancient of all the gods.

Typhon, the evil genius, emasculated and murdered Osiris. Typhon is sculptured as an ugly and repulsive figure. Suppose the power to be repulsive, or the body to be repelled from instead of attracted to the centre, or the poles changed; then, instead of the pyramid Osiris being formed by the attractive power, the pyramid Typhon will be generated by the repulsive.

Great was the grief of Isis for the death of Osiris. Yet a Typhonium, or temple of the evil deity, is seen at Edfou at a short distance from the great temple. This is inferred from the ugly being that appears on the plinths of the quadrangular-topped pillars, just as he is seen on the capitals of the columns in a small temple at Denderah, which is near the large one.

It may here be remarked, that in the temple at Denderah the cubical block surmounts the capitals of some of the pillars. A Typhonium is found by the side of the temple of the good deity at Philæ. In Upper Nubia, at Naga, near the Nile, are the remains of a Typhonium, in which are seen pillars with a rude Isis' head on each side, and a figure of Typhon under it.

Diodorus mentions that the people above Meroe worship Isis and Pan, and, besides them, Hercules and Zeus (Ammon), considering these deities as the chief benefactors of the human race.

So great was the magnitude of Typhæus, or Typhon, the son of Juno, conceived by her without a father, that he touched the east with one hand, the west with the other, and the heavens with the crown of his head. A hundred dragons' heads grew from his shoulders. He was one of the defeated giants, and, lest he should rise again, the whole island of Sicily was laid upon him.

Jupiter struck his son Tityus, one of the Titans, with a thunderbolt, which sent him from heaven down to hell, where he covered nine acres of ground.

The four sides of the pyramid face the four cardinal points, the vertex reaches the heavens, and base covers acres of ground.

Thus we have an explanation of the myth of the Titans with hands extending from east to west, the crowns of their heads touching the heavens, and their bodies covering acres of ground.

The 100 arms of Briareus equalled 100 times 2·81 feet = 100 orgyes = 1 stade = the height of the tower of Belus, which reached the heavens.

The height of a man = 2 orgyes = 2 × 2·81 = 5·62 feet = the length of 2 arms.

So 50 men would equal the height of the tower.

Or, metaphorically, 50 giants would reach the heavens.

Whether the force in the centre be regarded as attractive or repulsive, as generating Osiris or Typhon, the same hyperbolic solid Isis would correspond to either the pyramid of Osiris or Typhon.

Both these forces, as positive and negative electricity, galvanism or magnetism, may have been regarded as agencies by means of which the perpetual motions of the planets round the sun are preserved.

The Grecian mythology calls Osiris the son of Jupiter by Niobe, the daughter of Phoroneus. He was king of the Argives many years, but was induced, by the desire of glory, to leave his kingdom to his brother Ægialus, whence he sailed to Egypt to seek a new name and new kingdoms. The Egyptians were not so much overcome by his arms as obliged by his courtesies and great kindness towards them. Afterwards he married Io, the daughter of Inachus, whom Jupiter formerly turned into a cow. When by her distraction she was driven into Egypt, her former shape was restored, and she married Osiris and instructed the Egyptians in letters; wherefore both her and her husband attained divine honours, and were both thought immortal by that people. But Osiris showed that he was mortal, for he was killed by his brother Typhon. Io (afterwards called Isis) sought him a great while, and when she found him in a chest, she laid him in a monument in an island near Memphis, which is encompassed by that sad and fatal lake, the Styx.

Herodotus informs us that the goddess principally worshipped by the Egyptians was called Isis, and they celebrated her festival with all imaginable solemnity.

Pausanias, when travelling in Greece in the second century, was not allowed to see the statue of Isis in the temple of Phlius, where the Isiac worship had been introduced.

The mysteries of Isis, according to Plutarch, were intended to preserve some valuable piece of history, or to represent some of the grand phenomena of nature.

Osiris is sometimes represented, as governor of the world, sailing with Isis in a boat round the world, which subsists and is held together by the pervading power of humidity. For humidity substitute gravity, which governs the universe, and by which the earth subsists and is held together, and by means of which the solar system, with its grand central luminary, dependent planets, and satellites are preserved.

According to Herodotus, the earliest kings mentioned in the Egyptian traditions were their gods, Osiris, Horus, and Typhon; these, however, they placed in a very remote antiquity, and showed 345 wooden statues of priests,—no doubt, observes Sharpe, royal priests, or kings,—who had descended from father to son, in a male line, through that number of generations, during which they considered that no gods had been upon earth. The expression of Herodotus, that "each was a Piromis born of a Piromis," may be quoted as a proof of the accuracy of his report; though the word "piromi," which he thought meant "of good birth," is, in the language of the Coptic version of the Scriptures, "a man;" and the meaning of his informer was, that each was born of a man.

Osiris was held sacred all over Egypt, and, to judge by the number of votive tablets which are found dedicated to him, he must have been the chief object of worship, although only an inferior god or deified hero. He was the Dionysus or Bacchus of the Greeks,—not the youthful god of wine, but the bearded Bacchus, the Egyptian conqueror of India beyond the Ganges, who first led an army into Asia; the son of Seb or Kronos, the husband of Isis, the father of Horus. Diodorus has preserved the following inscription to his honour: "Kronos, the last of the gods, is my father. I am Osiris the king, who led an army even to the uninhabited parts of India, and northward to the Danube, and on the other side of the ocean. I am the eldest son of Kronos, and the seed of beautiful and noble blood, and related to the day. I am everywhere and help everybody." He was the god of Amenti, in the regions of the dead, and hence called, in an inscription quoted by Letrone, "Petemp-amentes," and in that character presided at the trial of the deceased, as is seen on the papyri of the mummy-cases.

Isis, his queen and sister, generally accompanies him. Herodotus and Diodorus consider her the same as Ceres, or the earth. Her inscription, quoted by Diodorus, is as follows: "I am Isis, the queen of the whole earth. I was taught by Hermes. What I bind no one can unloose. I am Isis, the eldest daughter of Kronos, the last god. I am the wife and sister of King Osiris. I first taught men to use fruits. I am the mother of Horus the king. I am in the constellation of the dog. The city of Bubastis was built by me. Hail, Egypt, that nourished me!" She sometimes has cows' horns, but more often a throne on her head.

Horus, the son of Isis and Osiris, reigned on earth after his father. He was considered by Herodotus as the Apollo of the Greeks. He was also the Harpocrates of the Greek mythology, both in name and character. He frequently has his finger on his mouth, to represent that he is the god of silence. He is sometimes a child, forming with his father and mother a holy family; and when represented as a sitting figure, with his hand to his mouth, he is the hieroglyphic of a child. Sometimes he has a large lappet from his head-dress hanging over his ear: sometimes he is a crowned eagle.

Seb, the father of the gods, and Thore, the father of the gods, distinguished by the scarabæus, are probably the same persons, and also probably the god whom Diodorus calls Kronos, the father of Osiris, Isis, Typhon, Apollo, and Aphrodite. Here we find that the father of the gods is a much less important person than his son; a circumstance so peculiar, that we must suppose, in the case of Jupiter and Saturn, that the Greeks borrowed it from Egypt. With respect to this god marrying his sister, it was an event so common in Egypt, both with gods and kings, by the testimony of history and hieroglyphics, that the words wife and sister appear to be confused. Three of the Ptolemies styled their queens sister when they were not so; probably meaning to imply that they were more than queens consort, and were fellow sovereigns with them.

Neith, the great mother of the gods, is probably the goddess whom Diodorus calls Rhea, the mother of the gods. Plato says that Neith was worshipped at Sais, and called Minerva by the Greeks; but Plutarch says the Minerva of Sais was Isis. Cicero also mentions the Minerva of Sais.

The gods reigned in Egypt before men, according to Herodotus and Diodorus, both of whom conversed with the Egyptian priests.

The Egyptian deities, from Diodorus's account, appear to have been the powers of nature invested with forms and individual attributes. These gods reigned for 18,000 years, and the last of the race was Horus, the son of Osiris and Isis. Then began the reign of human kings, which comprised a period of nearly 5000 years from Men or Menes, the first mortal king, to the 180th Olympiad, or about 58 B. C., when Diodorus visited Egypt.

The outline of the area between the two asymptotes and the curve of an hyperbola or hyperbolic solid is typified by the horns of a cow, the distinctive emblem of Isis. Herodotus says that the figure of Isis is "that of a female with the horns of a cow, which is the form given by the Greeks to their Io."

Io was placed by Juno under the charge of Argus, who had 100 eyes.

Cybele, the Bona Dea, or Magna Deorum Mater (the great mother of the gods), wears a turreted tiara. Such a tiara is formed in the construction of the hyperbolic reciprocal curve. She carries a key, perhaps like the veil of Isis, indicative of concealed mysteries. She also holds the cornucopiæ,—the horn of abundance, typical of the horn of Isis,—of infinity.

A Sabæan dynasty might have preceded the reign of Menes. The Sabæans were the Titans who built pyramids that figuratively reached the heavens, and obelisks that represented the laws by which the universe was governed. Science was confined to their priesthood, who predicted eclipses, and so astonished the multitude that the people accorded to them a superiority so great as to render them sacred, and esteemed them as participating in the secrets of divinity.

Did these types of the laws that govern the heavens cease to be revered as objects of Sabæan worship when Menes began his reign, and were they succeeded by a less spiritual form of religion? However this might have been, we find the dynasties after Menes wearing these emblems of divinity as distinguishing characteristics of royalty,—for kings assumed the attributes of gods. Thus the pyramidal age might have been anterior to Menes, but the knowledge of gravitation might afterwards have become arcanic,—confined to the hierarchy, and adopted as emblems of royalty by the kings, who reigned both as sovereigns and pontiffs.

Diodorus mentions that the Egyptians worshipped the Sun under the name of Osiris, as they did the Moon by the name of the goddess Isis.

All history, sacred and profane, witnesses to the extreme antiquity of Sabæanism, or the worship of the heavenly host. Yet it is not to be supposed that when men began to adore the celestial orbs, they wished to forget or deny the existence of a Supreme Being; but, judging humanly, and seeing him not, they began to think he was too high or too distant to concern himself in directing the affairs of this world. They imagined he must have left these cares to powers which, although vastly inferior to himself, were incomparably superior to man in nature, and in the condition of their existence; and these they sought and found in the most glorious objects of the universe. Or, if the attributes of the Deity were to be typified—if the apprehensive faculties of man demanded more obvious symbols to convey ideas too abstract to be seized by the unassisted intellect, what more appropriate objects could have been chosen than those bright luminaries whose processions and influences were enveloped in mystery, although they were constantly present.

To the Sabæan worshipper, before his religion had become corrupted, the idea of representing God under a human form, or of ascribing to him human wants, or a human will, was abhorrent: when he worshipped he stood in the virtual presence of his God, and saw with his eyes the actual object of his adoration. He could not conceive that the sun, or the moon, or the planets which he daily saw dwelling in heaven, could reside on earth, in houses built by human hands, or that any spot of earth could be more sacred to them than another, for they shined alike everywhere, and on all. The Sabæan could worship everywhere; best, however, in the open air, and on the highest places, whence the heavenly orbs could be the most easily and longest seen. In the open country, it was the hill; in towns, the roof of every man's house was his praying place.

In its known astronomical character, the Assyrian religion was closely allied to that of Egypt; but while the sun was the chief object of worship on the banks of the Nile, the sun, moon, and stars — "the host of heaven," — were adored by the people of the Chaldæan and Assyrian plains. Herodotus says that the sun was the only god adored by the Massagetes.

According to Olrich, the Vedas assign four great periods (yags) to the development of the world; and to the Almighty the three great qualities, first, of creation (Brahma); secondly, of preservation (Vishnoo); and thirdly, of destruction (Shiva).

They say that the angels assembled before the throne of the Almighty, and humbly asked him what he himself was. He replied, "Were there another besides me I would describe myself through him. I have existed from eternity, and shall remain to eternity. I am the great cause of everything that exists in the east and in the west, in the north and in the south, above and below. I am everything, older than everything. I am the truth. I am the spirit of the creation, the Creator himself. I am knowledge, and holiness, and light. I am Almighty."

He adds, though this fundamental principle no longer prevails, though the objects of devotion are no longer the same, yet this religion still exercises as powerful an influence over the people as in the most remote ages; and, though the deism of the Vedas as the true faith, including in itself all other forms, has been displaced by a system of polytheism and idolatry, has been nearly forgotten, and is recollected only by a few priests and philosophers, yet the belief in a Being far exalted above all has not been obliterated.

Paterson expresses an opinion that the religion of India was at one time reformed on a philosophical model, to which the various superstitions now prevalent have been gradually superadded. Murray remarks, that whatever we may think upon this subject, it is certain that it contains a basis of very abstruse and lofty principles, so strikingly similar to those of the Grecian schools of Pythagoras and Plato as apparently to indicate a common origin. The foundation consists in the belief of one supreme mind, or Brahme, the attributes of which are described in the loftiest terms. Such are those employed in the Gayatri, or holiest texts of the Vedas, accounted the most sacred words that can pass the lips of a Hindoo. The following paraphrase of a text is of high authority: — "Perfect truth, perfect happiness, without equal, immortal, absolute unity, whom neither speech can describe, nor mind comprehend; all pervading, all transcending; delighted with his own boundless intelligence; not limited by space or time; without feet moving swiftly, without hands grasping all worlds; without eyes all surveying; without ears all hearing; without an intelligent guide, understanding all; without cause, the first of all causes; all ruling, all powerful; the creator, preserver, and transformer of all things; such is the Great One."

The Ghebers place the spring-head of fire in that globe of fire, the sun, by them called Mythras, or Mihir, to which they pay the highest reverence in gratitude for the manifold benefits flowing from its ministerial omniscience. But they are so far from confounding the subordination of the servant with the majesty of its Creator, that they not only attribute no sort of sense or reasoning to the sun or fire, in any of its operations, but consider it as a purely passive blind instrument, directed and governed by the immediate impression on it of the will of God; but they do not even give that luminary, all-glorious as it is, more than the second rank amongst his works, reserving the first for that stupendous production of divine power, the mind of man. (Grose.)

In Pottinger's Beloochistan mention is made of an extraordinary hill in this neighbourhood, called Kohé Gubr, or the Guebre's mountain. It rises in the form of a lofty cupola, and on the summit of it, they say, are the remains of an atush kudu or fire temple. It is superstitiously held to be the residence of deeves or sprites, and many marvellous stories are recounted of the injury and witchcraft suffered by those who essayed in former days to ascend or explore it.

At the city of Yezd in Persia, which is distinguished by the appellation of the Darub Abadut, or Seat of Religion, the Guebres are permitted to have an atush kudu or fire temple (which, they assert, has had the sacred fire in it since the days of Zoroaster) in their own compartment of the city; but for this indulgence they are indebted to the avarice, not the tolerance, of the Persian government, which taxes them at 25 rupees each man.

The religious reverence paid to fire by the ancient Persians is still retained by their descendants the Parsees, who now chiefly reside about Bombay in Hindostan, and at Yezd in Persia. These and apparently some other natives of India make long and weary pilgrimages to the "everlasting fire," near Bakau, in Shirwan, on the shores of the Caspian Sea, which is continually supplied by gas issuing from the earth. In very early periods the Persians adored the sun. Zoroaster, without disturbing the ancient reverence for the sun, seems to have first introduced the worship of fire, that the believers, when the sun was obscured, might not be without the symbol of the divine presence. For this purpose he furnished a fire which he pretended to have obtained from heaven, and from which the sacred fires in all the places of worship were kindled. This introduction led to the erection of temples in which the sacred fire might be preserved. In early times the Persians had no temples, but worshipped upon their mountains, because, by a building, the beams of the sun would be wholly or partially excluded. The modern Parsees may be seen at Bombay, every morning and evening, crowding to the esplanade to salute the sun at its appearance and departure.

Hanway observes that the Ghebers suppose the throne of the Almighty is seated in the sun, and hence they worship that luminary.

Early in the morning they (the Parsees or Ghebers of Oulam) go in crowds to pay their devotions to the sun, to whom upon all the altars there are spheres consecrated, made by magic, resembling the circles of the sun, and when the sun rises their orbs seem to be inflamed and to turn round with a great noise. They have every one a censer in their hands, and offer incense to the sun. (Rabbi Benjamin.)

Yezd is the chief residence of those ancient natives who worship the sun and the fire, which latter they have carefully kept lighted, without being once extinguished for a moment, above 3000 years, on a mountain near Yezd, called Ater Quedah, signifying the House or Mansion of the Fire. He is reckoned very unfortunate who dies off that mountain. (Stephen.)

The Peruvians ascribed all their improvements to Manco Capac, called the Inca, and his consort Mama Ocollo, who pretended to be the children of the Sun, and delivered their instructions in his name and by his authority. The Inca assumed not only the character of a legislator, but of a messenger from heaven; hence his precepts were received not only as the injunctions of a superior, but as the mandates of the Deity. His race was held to be sacred; and in order to preserve it distinct, without being polluted by any mixture of less noble blood, the sons of Manco Capac married their own sisters, and no person was ever admitted to the throne who could not claim such a pure descent. To those children of the Sun, for that was the name bestowed on the children of the first Inca, the people looked up with reverence due to beings of a superior order. They were deemed to be under the immediate protection of the Deity, from whom they issued, and by him every order of the reigning Inca was supposed to be dictated. This persuasion rendered the power of the Inca very absolute, and every crime committed against him was punished capitally. Manco Capac turned the veneration of his followers entirely towards natural objects. The sun, as the great source of light, of joy, and fertility, attracted their principal homage. The moon and stars, as cooperating with him, received secondary honours.

The Sun was worshipped under the various names of Ham or Cham, Chemosh, Zamos, Osiris, Vulcan, Sol, Phœbus, Apollo, &c., and was considered as the god of day, the dispenser of light, heat, and fertility, and the good principle with which darkness, or evil, would wage continued warfare till the final consummation, when light, or goodness, should eventually triumph. His symbol, fire, was maintained with the utmost care upon the altars, and even participated in the worship paid to him.

Layard says that a marked distinction may be traced between the religion of the earliest and latest Assyrians. Originally it may have been a pure Sabæanism, in which the heavenly bodies were worshipped as mere types of the power and attributes of the Supreme Deity. Of the great antiquity of this primitive worship there is abundant evidence. It obtained the epithet of perfect, and was believed to be the most ancient of religious systems, having preceded that of the Egyptians.

On the earliest monuments we have no traces of fire-worship, which was a corruption of the purer form of Sabæanism; but in the Khorsabad bas-reliefs, as well as on a multitude of cylinders of the same age, we have abundant proofs of its subsequent prevalence in Assyria.

Representations of the heavenly bodies as sacred symbols are of constant occurrence in the most ancient sculptures. In the bas-reliefs we find figures of the sun, moon, and stars suspended round the neck of the king when engaged in the performance of religious ceremonies. These emblems are accompanied by a small model of the horned cap worn by winged figures, and by a trident or bident.

The sun, moon, and trident of Siva, raised on columns, adorn the entrance to temples in India, such as that of Bangalore.

Balbec is supposed to be the same city which Macrobius, in his "Saturnalia," mentions under the name of Heliopolis of Cœlo-Syria, and to which, he tells us, the worship of the sun was brought, in very remote times, from the other city of the same name in Egypt. Heliopolis, in Greek, means "the City of the Sun;" and the signification of the Syriac term Balbec is "the Vale of Bal,"—the oriental name for the same luminary when worshipped as a god.

Gliddon says the name of Babylon, "Bab-El," is literally "Gate of the Sun;" as we now say, "Sublime Porte" of the Ottoman, or "Celestial Gates" of the Chinese autocracy.

Volney observes that the oriental name Babel for Babylon signifies "Port," that is to say, "the palace of Bel, or Belus."

The Sun was worshipped under the name of Mithras by the Persians, and by the Egyptians under the name of Osiris.

Each prong of the trident represents an obelisk,—the expounder of the laws which the planets obey in their revolutions round the sun. The point of the prong is the pyramidal apex of the obelisk. The obelisk is also typical of infinity, as the sides are continually approaching to parallelism, though they can never become parallel.

The horned cap represents the outline of the hyperbolic reciprocal curve of contrary flexure, which becomes more pointed, like the dome of contrary flexure, as the radius is subdivided into equal parts; or more truncated as the radius is divided into greater parts. Both forms, like the horn of Isis, are typical of eternity, as the last parallelogram along the axis may again be subdivided into an indefinite number of parallelograms, which may be extended, like a sliding telescope, to an indefinite distance.

The temple of Belus had eight terraces. The mystical

Mount Merù of the Hindoos had seven zones or regions. In Thibet a cone or pyramid is invariably placed before the devotees preparing to offer sacrifice, as a type of their sacred Mount Merù. In the eastern parts of Bengal a similar practice prevails. There is in every village a representation of the world-temple, made of earth, with steps. The whole is plastered with clay; and on stated festivals the statue of some favourite deity is placed upon the summit. Thus we see the object for which these structures were originally designed, and the idea which they symbolise. All primitive nations have attached particular sanctity to particular mountains, which they believed to be either the residence of their divinities, or to have been especially honoured by some manifestation of the divine presence. The Greeks had their Ida and Olympus; the Hindoos their Mount Merù. The mountain was typical of the pyramid, and the pyramid typical of the laws of gravitation, which extended from the earth to the heavens. In Mexico the mountains themselves have been formed by the hands of men into terraced pyramids. These temples, or gigantic altars, were dedicated to the Deity, whom they originally worshipped, and symbolical of the laws that govern the universe.

The Puranas, the mythological Hindoo poems, which form a supplement of their Vedas, have a tradition of the migration of Charma or Ham, with his family and followers, driven from his country by the curse of Noah; that having quitted their own land they arrived, after a toilsome journey, upon the banks of the Nile, where, by command of their goddess Padma Devi (the goddess residing upon the lotus), Charma and his associates erected a pyramid in her honour, which they called Padma-Mandiva, or Padma-Matha; the word Mandiva expressing a temple or palace, and Matha a college or habitation of students (for the goddess herself instructed Charma and his descendants in all useful arts). This pyramid, and the settlement belonging to it, was called Babel, and by the Greeks in a later age Byblos. We learn from the same source that this migration took place subsequently to the building of the Padma-Mandiva, or first Babel, on the banks of the Euphrates.

Another migration is also spoken of in the Puranas, the result of a general war between the worshippers of Vishnoo and Isuara, under which name water and fire were respectively typified; this is said to have commenced in India in the earliest ages, and thence to have spread over the whole world. In this struggle the Yoingees, or earth-born, were worsted; and by the interposition of the Deity, whose worship they opposed, were compelled to quit the country. These also took refuge in Egypt, carrying with them the groundwork of the Egyptian mythology.

Were the Yoingees, the pyramidal builders instructed in all useful arts, and spread over the world in the earliest ages, the same as the powerful hierarchy, the pyramidal builders, the constructors of canals for commerce and irrigation, and instructors in the useful arts, that has been traced by their monuments and standards erected in remote ages round the entire globe?

The Yoingees were the earth-born, who metaphorically dared to build a tower that should reach the heavens; they were compelled by the direct interposition of the Deity to quit the country, and were dispersed over the world like the wandering masons.

The worshippers of Isuara, the Hindoo Hercules, were worsted in their attempt to reach the heavens. So the giants, the Isuaraists, were defeated in their attempt to scale the heavens.

The Padma-Matha was a temple and college. The tower of Belus, the Egyptian pyramid, Mexican teocalli, and Burmese pagoda were temples.

The quadrangular sides of the courts that enclosed the temples formed the habitation of the priesthood — the colleges.

In attempting to ascertain the origin of an early race, when acknowledged historical records are wanting, we must not overlook the important testimony contained in the legendary traditions, anterior to all regular historical records, which are to be found occupying the place of history during the infancy of nations. We must, of course, receive such legends with scrupulous caution and the utmost latitude of interpretation. Still it is too useful an auxiliary, and possesses too many of the components of truth to be rejected. The records preserved by the priestly order,—though we may be disposed to question the extreme antiquity and indubitable authority claimed by them, —nay, even the oral traditions of a primitive people, handed down from father to son, and from generation to generation,—will often throw a ray of light upon the most obscure subjects, and present us, disguised indeed in allegory and loaded with fable, the doubtful outline of some great fact in the history of man, which might otherwise have defied conjecture and baffled research. Infinitely important to the inquirer into remote antiquity is the attentive observation of such religious ceremonies and observances as having, in a certain sense, survived the modes of faith from which they sprang, are denounced by the heedless spectator as idle and superstitious mummery, and may possibly be but imperfectly comprehended even by those who regard their performance as a sacred duty. A close scrutiny may often, however, have the effect of revealing the historical import of such, and enabling us by their assistance not merely to elucidate a doctrine, but to establish a fact.

Other symbols were common to India and Egypt. The most common of these was the lotus, adopted as a religious emblem by nations remote from each other. It is found in this capacity upon the banks of the Ganges, on the columns of Persepolis, and on the waters of the Nile. Thence it was transported into Greece, where it appears in the form of the mystical boat in which Hercules is fabled to have traversed the ocean, and which was called by the Greek mythologists "the Cup of the Sun." The Hindoo and Egyptian mythologies transplanted into Greece assume a more essentially material character than before. Here the powers of nature and the attributes of humanity are alone to be found impersonated by their divinities, with scarcely any perceptible recognition of a Supreme Being. Thus, Hercules was represented by the Greeks as the son of Jupiter, who is identical with the Isuara of Hindoo mythology and the Osiris of the Egyptian; while the Hindoos considered him as an avatara, or incarnation of the divinity; not a distinct person, but one with the being from whom he emanated,—a distinction totally unknown to the Greeks.

Salverte, who writes on the "Philosophy of Magic," is of opinion that to the great body of the priesthood no more was made known than the process by which the wonders of their art were to be wrought; while the rationale of these processes—all that could properly be called the science of the matter—was reserved for the higher order of sages,—a class few in number, and bound by the strongest ties of interest to maintain the mystery in which the knowledge entrusted to them was enveloped.

To these various precautions was added the solemnity of a terrible oath, the breach of which was infallibly punished with death. The initiated were not permitted to forget the long and awful torments of Prometheus, guilty of having given to mortals the possession of the sacred fire. Tradition also relates that, as a punishment for having taught men mysteries hitherto hidden, the gods cast thunderbolts on Orpheus,—a fable probably derived from the nature of the death of one of the priests of the Orphic mysteries that bore the name of the founder of the sect. Until the downfal of paganism the accusation of having revealed the secrets of initiation was the most frightful that could be laid to the charge of any individual; especially in the minds of the multitude, who, chained down to ignorance and submission by the spirit of mysticism, firmly believed that were the perjured revealers permitted to live, the whole nation would be sacrificed to the indignation of the gods.

Thus knowledge, straitened in action, was concentrated in a small number of individuals, deposited in books written in hieroglyphics, or in characters legible only to the adepts, and the obscurity of which was further increased by the figurative style of the sacred language. Sometimes even the facts were only committed to the memory of the priests, and transmitted by oral tradition from generation to generation. They were thus rendered inaccessible to the community; because philosophy and chemistry, being destined to serve a particular object, were scarcely heard of beyond the precincts of the temples, while the development of their secrets involved the unveiling of the religious mysteries. The doctrines of the Thaumaturgists were reduced by degrees to a collection of processes which were liable to be lost as soon as they ceased to be habitually practised. There existed no scientific bond by means of which one science preserves and advances another, and thus the ill-combined doctrines were destined to become obscure, and finally extinguished, leaving behind them only the incoherent vestiges of ill-understood and ill-executed processes.

"A condition of things such as then existed, we do not scruple to say," continues Salverte, "is the gravest injury that can happen to the mind of man, from the veil of mystery cast by religion over physical knowledge. The labours of centuries, and the scientific traditions derived from the remotest antiquity, are lost, in consequence of the inviolable secrecy observed concerning them. The guardians of science are reduced to formularies, the principle of which they no longer understand; so that, at length, in error and superstition, they rise little above the multitude, which they too long and too successfully have conspired to keep in ignorance.

When the books of Numa, nearly five centuries after his death, were discovered at Rome, the priests used their influence to have them burned, as dangerous to religion. "Why," asks Salverte, "but because chance instead of throwing them into the hands of the priest had first given them to the inspection of the profane, and the volumes exposed in too intelligible a manner some practices of the occult science cultivated by Numa with success?"

The following histories or traditions about the pyramids and their builders are quoted from Vyse's work.

"Abd-al-Latif, who wrote a work on the pyramids, says, 'I have read in some of the books of the ancient Sabæans that one of the two great pyramids is the tomb of Agathodæmon, and the other of Hermes, who are said to have been two great prophets, of whom Agathodæmon was the most famous and ancient. It is also said that people used to come from all parts of the world on a pilgrimage to these tombs.'

"Other Arabian authors, as Jamal Ed Din Mohammed Al Watwati Al Kanini Al Watwati (718 A. H.), mention that the Sabæans performed pilgrimages to the pyramids.

"Shehab Eddin Ahmed Ben Yahya (died between 741 and 749 A. H.) mentions that the Sabæans performed regular pilgrimages to the Great Pyramid, and also visited others which are less perfect.

"Soyuti (died 911 A. H.) mentions from Al Watwati Al Warrak, that the Sabæans, in performing pilgrimages to the pyramids, sacrificed hens and black calves, and burnt incense.

"He states, from Menardi, that many of the pyramids were destroyed by Karakousch; that those that remained were tombs, and contained dangerous passages, some of which communicated with Fioum; that they were sepulchres of ancient monarchs, and were inscribed with their names and with the secrets of astrology and of incantation; that it was not known by whom they were constructed. Soyuti then says, 'that Seth took possession of Egypt, and that one of his sons, Kinan, was Hermes; that he was endowed with great wisdom, and travelled through the world, being under the special protection of providence; that he was likewise a great warrior, and conquered all the East, and introduced Sabæanism, which inculcated a belief in one God, the observance of prayer seven times a day, sacrifices, fasts, and pilgrimages to the pyramids. He is supposed to have written the first treatise on astronomy; to have brought the people of Egypt from the mountains, where they had retired for fear of the waters, and to have taught them to cultivate the plains, and also to regulate the inundations of the Nile. He afterwards travelled into Upper Egypt, Nubia, and Abyssinia.

"Makrizi, who died 845 A. H., quotes from Ibrahim Alwatwati Al Warrak, that there was a great uncertainty about the history of Hermes of Babel; that according to some accounts he was one of the seven keepers in the temples, whose business it was to guard the seven houses; and that he belonged to the temple of the planet Mercury, and acquired his name from his office, for Mercury signifies in the Teradamian language Hermes. He is also said to have reigned in Egypt. It is added that he was renowned for his wisdom; and that he was buried in a building called Abou Hermes; and that his wife, or, according to some other accounts, his son and successor, was buried in another; and that these two monuments were the pyramids, and were called Haraman.

"Makrizi quotes from another author, that the construction of the two pyramids to the westward of Fostat (Cairo) was considered one of the wonders of the world; that they were squares of four hundred cubits, and faced the cardinal points. One was supposed to have been the tomb of Agathodæmon; the other that of Hermes, who reigned in Egypt for 1000 years; both of them were said to have been inspired persons, and to have been endowed with prophetic powers. That according to other accounts, these monuments were the tombs of Sheddad Ben Ad, and of other monarchs who conquered Egypt.

"Makrizi concludes by saying that every thing connected with the pyramids was mysterious, and the traditions respecting them various and contradictory; at the same time that they commanded such admiration and astonishment that they were actually worshipped.

"Al Akbari says the Sabæans perform pilgrimages to the pyramids, and say, 'Abou Chawl, we have finished our visit to thee.'"

Colonel Chesney discovered many pyramids in Syria to which pilgrimages were performed.

Sprenger informs us that Unscouski mentions, in Müller's "Sammlung Russicher Geschichte erstes Stück," p. 144, that he witnessed the celebration of the new year by the Lamas of the Calmucs in the following manner. "A tent of Chinese cloth was pitched in an open space marked out with red lines, to which the priest came in procession, from the westward, with his attendants, amongst whom six manyis (young priests) carried sacred standards, each of them being supported by persons in red garments, bearing a model of a pyramid and two large trumpets; and then fifty others, in yellow dresses, preceded with drums and cymbals the rest of the priests, who were guarded by armed Calmucs. The procession moved round the tent, and then assembled in the space before it, where the models of the pyramids were placed, which the priest worshipped by prostrating himself three times on the ground."

Sprenger also mentions, that in the Syrian Chronicle of Bar Hebræus (translated into Latin by Burns) Enoch is said to have invented letters and architecture; under the title of Trismegistus, or of Hermes, to have built many cities and established laws, to have taught the worship of God and astronomy, to give alms and tithes, to offer up first-fruits, libations, &c., to abstain from unlawful food and drunkenness, and to keep fasts at the rising of the sun, on new moons, and at the ascent of the planets. His pupil was Agathodæmon (Seth): according to other accounts, Asclepiades, a king renowned for wisdom, who, when Enoch was translated, set up an image in honour of him, and thereby introduced idolatry. The Egyptians are supposed to be descended from these persons. According to Hadgi Walfah, they derived their knowledge from the Chaldæans, who are said to have been the same as the Persian magi, and to have originally come from Babylon. The statues of the Grecian Hermes, which seem to agree in name with the pyramids (Haram), were not images, but symbols of the Deity, and of the generative principle of nature, in the form of obelisks. Statues of this kind, sacred to Hermes, were erected by the Greeks in honour of distinguished heroes; and the same allegorical allusion might have been kept in view when the pyramids were constructed as tombs. The Egyptian account, however, of Hermes is very obscure.

"We are all acquainted," says Gliddon, "with the wonder of the world—the eternal pyramids, whose existence astounds our credence, whose antiquity has been a dream, whose epoch is a mystery. What monuments on earth have given rise to more fables, speculations, errors, illusions, and misconceptions?"

The content of the pyramid of Cheops in cubic feet

$$= \tfrac{1}{3}\text{ area base} \times \text{height}$$
$$= \tfrac{1}{3}\,\overline{746}^2 \times 456 = 84590432.$$

The magnitude of the earth being to that of the sun as 1 : 1328460. Then 84590432 ÷ by 1328460 = 64 cubic feet for the magnitude of the earth compared to the content of the whole pyramid, which represents the magnitude of the sun; and 64 cubic feet = 4 feet cubed = a cube having the side = 4 feet.

Coutelle says the stones of the pyramid seldom exceed 9 feet by 6½. Supposing the breadth to equal the depth, then the content of such a stone will $= 9 \times \overline{6{\cdot}5}^2 = 380$ cubic feet, and $64 \times 6 = 384$; so the stone of 380 cubic feet would = 6 times the magnitude of the earth = 300 times the magnitude of the moon; for the magnitude of the moon is to the magnitude of the earth as ·02 : 1, or as $\frac{1}{50}$: 1.

Thus the magnitude of the earth would be represented by a cubic stone having the length of the side = 4 feet, or content = 64 cubic feet. The magnitude of the moon would be represented by a cubic stone having the length of the side = 1·08 feet, or content = 1·28 cubic feet. The magnitude of the sun would be represented by the content of the whole pyramid, which equals nearly 70,000,000 times 1·28 cubic feet, or 70,000,000 times the magnitude of the moon.

Such is the relative magnitudes of the two most conspicuous of the heavenly bodies as seen from the earth.

By this means of comparison some conception may be formed of the enormous magnitude of the great luminary placed in the centre of the solar system.

The diameter of the earth	-	=	7926 miles
The diameter of the sun	-	=	882000 „
The mean distance of the earth from the sun	- - - -	=	95000000 „

Hence the mean distance of the earth from the sun will = 215½ semi-diameters of the sun.

By the tables of Arago the diameter of the sun is to the diameter of the earth :: 109·93 : 1, or semi-diameters as 219·86 : 2.

By Herschel's they are about 222 : 2.

Thus by supposing 2 steps to represent the semi-diameter of the earth, then about 220 would represent the semi-diameter of the sun, if the steps were all equal in height.

By comparing the content of the whole pyramid from the base to the apex with the content of the part wanting from the platform to the apex, if it could be found, and then comparing the whole pyramid with the sun and the part wanting with a planet, some conception might be formed of the dimensions of that planet.

The content or magnitude of pyramid $\propto$ the cube of the axis or height.

The magnitude of the sun and planets $\propto$ the cube of their semi-diameters.

It will be seen, from the discrepancies of the various measurements of the platform, how unsatisfactory must be the estimate of the part wanting from the platform to the apex of the pyramid.

Nouet and his colleagues make the height from the present platform to the apex equal 19 feet French.

Coutelle makes the side of the platform equal 32 feet. Taking the height to the side of the base as 5 : 8 would make the height from the platform to the apex $= \frac{5}{8} 32 = 20$ feet. The least height of the steps $= 1·686$ feet. If so, the whole number of steps from the base to the apex would be between 215 and 220.

Thus the part wanting from the platform compared to the whole pyramid, will be about 100 times the magnitude of the earth compared to the magnitude of the sun. Or it would exceed the magnitude of Uranus, and be less than the magnitude of Neptune compared with the magnitude of the sun.

The diameter of Jupiter is to the diameter of the sun :: 11·56 : 109·93, which is nearly as 1 : 10. If the pyramid were supposed to be truncated at one tenth the height from the apex, the part so cut off would represent the magnitude of Jupiter, the magnitude of the sun being represented by the whole pyramid.

By this means the magnitude of the moon and of Jupiter, the greatest of the planets, have been compared with the magnitude of the sun.

The magnitude of Jupiter exceeds that of the earth about 1300 times. The magnitude or bulk of the sun equals 1384472 times that of the earth. The diameter of the sun $=111$ times the diameter of the earth. The diameter of the moon equals about $\frac{1}{4}$ the diameter of the earth.

The magnitude of the moon equals $\frac{1}{80}$th part that of the earth. The magnitude of the sun exceeds $1\frac{1}{3}$ million times that of the earth, and equals about 70 million times the bulk of the moon, or about 1000 times the bulk of Jupiter.

The objections to this calculation will be, that although the number of steps from the base to the apex of the pyramid may be admitted to equal the number of semi-diameters of the sun, yet the steps, being unequal in height, cannot represent the semi-diameters.

So it will be requisite to show how the steps may all be represented of equal height, and how they have been made to diminish gradually from the base to the summit; so that the content of the pyramid being made to represent the $\frac{1}{2}$ circumference of the earth, the number of steps might represent the semi-diameter of the earth's orbit. *Fig.* 57. A. All the steps of the external pyramid will be equal in height, and the length of each step = the distance from the apex, such as has been drawn, in *Fig.* 40, to represent the variation of the time when a body falls to the centre of force.

Fig. 57. A. Make the produced perpendicular height of the pyramid of Cheops equal the side of the base. Join this perpendicular with the side of the base. Thus a triangle will be formed exterior to the pyramid. Divide the side of this triangle into as many equal parts as the required number of steps of the pyramid. In the figure the number of steps only equals 20.

From the equal divisions of the side of the triangle draw lines to the centre of the base, which will cut the triangle formed by the sides of the pyramid into as many unequal

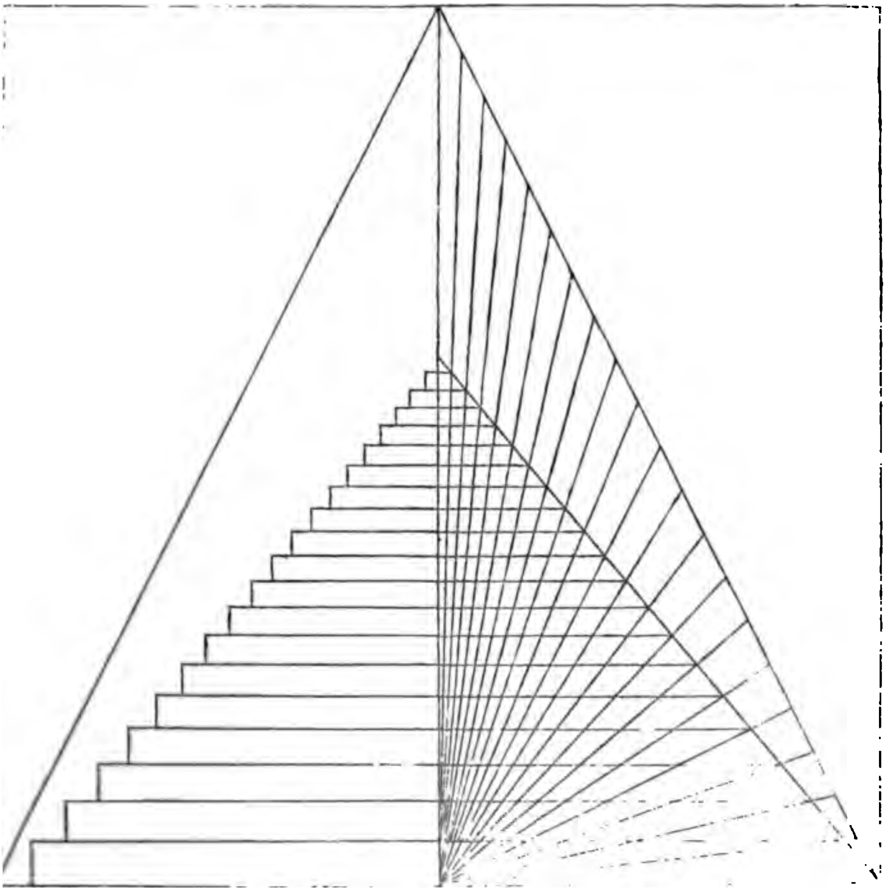


Fig. 57. A.

parts. From these points of intersection draw lines parallel to the base. The distance between these lines will be the height of the series of steps or terraces of the pyramid, which gradually diminish from the base to the apex, and apparently in the same ratio as the steps of the pyramid of Cheops decrease; for, according to Coutelle, the greater height is 4·628 feet, and the least 1·686 feet.

The height of the exterior triangle = the side of the base of the pyramid = the side of the circumscribing square of the triangle. The square of the height or side of the square = the area of the base. The cube of the side = 3 times the content of a pyramid having the height = side of base = 16 plethrons.

All the terraces of which will be of equal height if lines be drawn parallel to the base from the equal divisions of the side or perpendicular of triangle.

Thus we shall have the outline of a pyramid having the side of the base = the height, and each terrace of equal height. The number of terraces of equal height would represent the number of semi-diameters of the sun that equal the distance of the sun from the earth.

When the exterior triangle is equilateral, the height of the greatest and least step will be in a less ratio than when the height of the triangle = the side of the base.

When the exterior triangle is made equal the inclined side of the pyramid, the ratio between the greatest and least step will be still less.

Herodotus makes the height of the pyramid to equal the side of the base.

In order that a pyramid might represent time, we supposed a body, in descending from the earth to the sun, to describe a $\frac{1}{2}$ diameter of the sun in the time corresponding to a stratum or terrace of the pyramid, the steps or terraces being all equal in height, and velocity $\propto \frac{1}{D^2}$.

But the steps or terraces of the Great Pyramid are unequal in height, decreasing from the base to the apex; therefore the time of describing a $\frac{1}{2}$ diameter of the sun will decrease in a greater ratio than when the velocity was supposed to $\propto \frac{1}{D^2}$.

So the terraces of the pyramid might denote that a body falling to the centre was acted upon by a force which produced a velocity that varied in a greater inverse ratio than D^2.

The pyramid of Cheops might be called the pyramid of the Sun, as it denotes the time of descent from the earth to the sun. The number of steps accord with the number of $\frac{1}{2}$ diameters of the sun, which = the $\frac{1}{2}$ diameter of the earth's orbit, and the pyramid itself = the $\frac{1}{2}$ circumference of the earth.

Less pyramid = pyramid of Cheops.

Height of less pyramid = $\frac{5}{8}$ side of base.

Height of greater pyramid = side of base.

Since their bases are equal, their contents will be as their heights.

Therefore time of descent to centre, when velocity $\propto$ in a greater inverse ratio than D^2, will be to time of descent to centre, when velocity $\propto \frac{1}{D^2}$ :: less pyramid : greater pyramid $:: \frac{5}{8} : 1 :: 5 : 8$.

Or time of descent to centre by the force of gravity will $= \frac{5}{8}$ time of descent when velocity $\propto \frac{1}{D^2}$.

If the number of steps that represent the number of $\frac{1}{2}$ diameters of the sun, that equal the $\frac{1}{2}$ diameter of the orbit of the earth, were equal the number of $\frac{1}{2}$ diameters of the earth that equal the diameter of the sun; then the steps might indicate, not only the distance of the earth from the sun in terms of the sun's diameter, but also the distance of the earth from the sun, and the diameter of the sun in terms of the diameter of the earth.

Diameter of the earth : diameter of the sun

$:: 1 : 109{\cdot}93$ (Arago)

say as $1 : 109{\cdot}5$

$2 \times 109{\cdot}5 = 219$, or 219 semi-diameters of the earth = diameter of the sun.

$219 \times \frac{1}{2}$ diameter of the sun,

$= 219 \times \frac{1}{2}\, 314949 = 34486915$ leagues French

and 34515000 leagues

= the mean distance of the earth from the sun.

Again 219 semi-diameters of the earth = diameter of the sun
$219 \times \frac{1}{4}$ diameter of the earth $= \frac{1}{2}$ diam. of the sun,
therefore $219 \times 219 \times \frac{1}{4}$ diameter of the earth
$= \overline{219}^2 \times \frac{1}{4}$ diameter of the earth $= \frac{1}{2}$ diameter of the earth's orbit.

If 219 semi-diameters of the earth = diameter of the sun,
and 219 „ „ sun $= \frac{1}{2}$ diameter of the earth's orbit,
219 quarter diameters of the earth $= \frac{1}{2}$ diameter of the sun
then 219^2 „ „ earth
= 219 semi-diameters of the sun
= semi-diameter of the orbit of the earth.

Thus the square of the number of $\frac{1}{2}$ diameters of the earth that equal the diameter of the sun will equal as many quarter diameters of the earth as equal the semi-diameter of the earth's orbit.

Also the number of semi-diameters of the earth that equal the diameter of the sun will equal as many semi-diameters of the sun as equal the semi-diameter of the earth's orbit. Hence the following proportions:—

219 : $\frac{1}{4}$ diameter of the earth :: 219^2 : $\frac{1}{2}$ diameter of the sun,
or 1 : $\frac{1}{2}$ „ „ :: 219 : diameter „

Also $\frac{1}{2}$ diameter of the earth : diameter of the sun :: $\frac{1}{2}$ diameter of the sun : $\frac{1}{2}$ diameter of orbit of the earth,
or $\frac{1}{2}$ diameter of the earth : $\frac{1}{2}$ diameter of the sun :: diameter of the sun : $\frac{1}{2}$ diameter of the orbit of the earth.

If $109{\cdot}5 \times \frac{1}{2}$ diameter of the earth $= \frac{1}{2}$ diameter of the sun
$109{\cdot}5 \times$ diameter of the sun $= \frac{1}{2}$ diameter of the orbit
$\overline{109{\cdot}5}^2 \times$ diameter of the earth $= \frac{1}{2}$ „ „
or diameter of the earth : diameter of the sun :: diameter of the sun :: $\frac{1}{2}$ diameter of the orbit.

The distance of the moon from the earth is about 109·5 diameters of the moon.

Hence diameter of the moon : $\frac{1}{2}$ diameter of the orbit of the moon :: diameter of the earth : diameter of the sun :: diameter of the sun : $\frac{1}{2}$ diameter of the orbit of the earth.

Compare 219, the assumed number of $\frac{1}{2}$ diameters, with the number of $\frac{1}{2}$ diameters of the earth that equal the diameter of the sun, and with the number of $\frac{1}{2}$ diameters of the sun that equal the $\frac{1}{2}$ diameter of the orbit of the earth.

According to Herschel,

Diameter of earth : diameter of sun :: 7926 : 882000,
or as 1 : 111·26.

Diameter of the sun : mean distance of the earth from the sun :: 882000 : 95000000 miles,
or as 1 : 107·71 „

Then 2 × 111·26 = 222·52
and 2 × 107·71 = 215·42
2)437·94
218·97

Or 222·52 semi-diameters of the earth = diameter of sun, and 215·42 semi-diameters of the sun = $\frac{1}{2}$ diameter of the orbit of the earth.

Here the mean number = 218·97,
the assumed number = 219·5.

By Arago's tables,

Diameter of the earth : diameter of sun :: 1 : 109·93
Diameter of the earth = 2865 leagues French.

The diameter of the sun is not given in leagues; but 109·93 × 2865 = 314949·45 leagues for the diameter of the sun.

Mean distance of the earth from sun = 34515000 leagues;
∴ 34515000 ÷ 314949·45 = 109·58 = the number of diameters of the sun that equal the mean distance of the earth from the sun.

Then 109·93 × 2 = 219·86
and 109·58 × 2 = 219·16
2)439·02
219·51

Or 219·86 semi-diameters of the earth = diameter of sun, and 219·16 semi-diameters of the sun = $\frac{1}{2}$ diameter of the orbit of the earth.

The mean number here = 219·51
the assumed number = 219.

By Herschel's tables the number of $\frac{1}{2}$ diameters of the earth that equal the diameter of the sun exceeds by 7·1 the number of $\frac{1}{2}$ diameters of the sun that equal the $\frac{1}{2}$ diameters of the orbit of the earth; but the mean = 218·97.

By Arago's tables the number of $\frac{1}{2}$ diameters of the earth that equal the diameter of the sun may be said to equal the number of $\frac{1}{2}$ diameters of the sun that equal the $\frac{1}{2}$ diameters of the orbit of the earth; the mean being 219·48.

The assumed number, 219, equals the number of $\frac{1}{2}$ diameters of the earth that equal the diameter of the sun, and the number of $\frac{1}{2}$ diameters of the sun that equal the $\frac{1}{2}$ diameters of the orbit of the earth.

$\overline{219}^2 \times \frac{1}{4}$ diameter of the earth

$= \overline{219}^2 \times \frac{1}{4}\,7926 = 95034721$

and 95000000 miles

= the mean distance of the earth from the sun (*Herschel*); but 219 × $\frac{1}{2}$ diameter of the earth will not equal the diameter of the sun; for it would require 222·52 semi-diameters of the earth to equal the diameter of the sun; or 222·52 × $\frac{1}{2}$ 7926 to equal 882000 miles.

There appears to be a difference in the calculations of these two astronomers regarding the diameters of the earth, sun, and orbit of the earth. Possibly the race that constructed this pyramid might have found a difficulty in agreeing as to the comparative diameters of the earth, sun, and orbit of the earth, and so left the pyramid truncated, or incomplete.

This pyramidal monument, probably a temple dedicated to the Sun by the Sabæans, who worshipped that glorious orb, the attendant planets, and sidereal system, proves that a highly developed civilisation existed anterior to Roman or Grecian antiquity. Manetho says that Venephes built pyramids in the second dynasty. Lepsius makes the fourth Manethonic dynasty to begin 3400 years B. C. The last dynasty of the old kings, which ended with the invasion of the Hyksos, 1200 years before Homer, was the twelfth Manethonic dynasty.

It is in the East that we must look for a development of science and civilisation such as the pyramids prove to have existed at very remote epochs in Egypt and Babylon.

Still further east, China, that long secluded empire, claims an antiquity for science which has long been doubted by Europeans. Her claim to antiquity of civilisation, which none can dispute, dates anterior to the dawn of European history; which civilisation has continued, with little interruption, for thousands of years, to the present age.

Before leaving this subject, let us try if the $\frac{1}{2}$ diameter of the moon's orbit can be expressed in terms of the diameters of the earth and moon proximately.

Diameter of earth : diameter of moon :: 1 : 0·27 (*Arago*),
say as 1 : 0·26,

then $\frac{1}{\cdot 26} = 3{\cdot}85$, and $2 \times 3{\cdot}85 = 7{\cdot}7$;

or 7·7 semi-diameters of the moon = diameter of the earth.

$$\overline{7{\cdot}7}^2 \times \tfrac{1}{2} \text{ diameter of the earth}$$
$$= 59{\cdot}29 \text{ semi-diameters of the earth,}$$

and 59·9643 semi-diameters of the earth = mean distance of the moon from the earth = 237000 miles (*Herschel*).

Since $\overline{7{\cdot}7}^2 \times \frac{1}{2}$ diameter of the earth = $\frac{1}{2}$ diameter of the moon's orbit,
and $7{\cdot}7 \times \frac{1}{4}$ diameter of the moon = $\frac{1}{2}$ diameter of the earth,
$\therefore \overline{7{\cdot}7}^2 \times 7{\cdot}7 \times \frac{1}{4}$ diameter of moon = $\frac{1}{2}$ diameter of orbit;
or $\overline{7{\cdot}7}^3 \times \frac{1}{4}$ diameter of the moon = $\frac{1}{2}$ diameter of the moon's orbit.

Thus the square of the number of $\frac{1}{2}$ diameters of the moon that equal the diameter of the earth will equal as many $\frac{1}{2}$ diameters of the earth as equal the $\frac{1}{2}$ diameter of the moon's orbit.

Or the cube of the number of $\frac{1}{2}$ diameters of the moon that equal the diameter of the earth will equal as many $\frac{1}{4}$ diameters of the moon as equal the $\frac{1}{2}$ diameter of the orbit of the moon.

$7{\cdot}7 \times \frac{1}{2}$ diameter of the moon = diameter of the earth,

$\overline{7{\cdot}7}^2 \times \frac{1}{2}$ diameter of the earth = $\frac{1}{2}$ diameter of the orbit of the moon,

$\therefore$ ½ diameter of the moon : ½ diameter of the earth :: 7·7 × diameter of the earth : ½ diameter of the orbit of the moon.

If 3·85 diameters of the moon = diameter of the earth,

$\overline{3{\cdot}85}^2$ × 2 diameters of the earth = ½ diameter of orbit of the moon,

$\overline{3{\cdot}85}^3$ × 2 diameters of the moon = ½ diameter of the orbit of the moon.

The square and cube of the ½ diameters of the moon are used in the calculation of the ½ diameters of the moon's orbit, as the 1st and 2nd powers of the ½ diameters of the earth were used for calculating the ½ diameters of the earth's orbit.

If 7·7 × ½ diameter of the moon = diameter of the earth, and $7{\cdot}7^2$ × ½ diameter of the earth = ½ diameter of the moon's orbit,

then $7{\cdot}7^3$ × ¼ diameter of the moon = ½ diameter of the moon's orbit.

The area of the moon's orbit would include about 60^2 or 3600 times the area included by the circumference of the earth.

The ½ diameter of the moon is about ⅛ the diameter of the earth. The ½ diameter of the moon's orbit is about ¼ the diameter of the sun. So the diameter of the sun will nearly equal twice the diameter of the moon's orbit, and the circumference of the sun will nearly include four times the area of the moon's orbit.

$\overline{219}^2$ or 47961 × ¼ diam. of earth = ½ diam. of earth's orbit,
or 23980 × ½ diam. of earth = ,, ,, ,,
and 60 × ½ diameter of the earth = ½ diameter of the moon's orbit,

$\therefore$ 23980 ÷ 60 = 399, say 400.

Then the distance of the earth from the sun will equal 400 times the distance of the moon from the earth.

The other planetary distances can be proximately expressed in terms of the diameters of the sun and planets.

"An oracle appointing the cubical altar of Apollo to be doubled was," as Maclaurin supposes, "of greater advantage to geometry than to the Athenians then afflicted with the plague; as it gave occasion to Plato to consider the famous problem of the duplication of the cube, and produced the solid geometry. It afterwards received great improvements from the incomparable Archimedes, who squared the area of the parabola, and made some progress in the mensuration of the circle, and enriched this science with many discoveries worthy of so excellent a genius."

Fig. 58. The side of a square : diagonal :: 1 : $2^{\frac{1}{2}}$
square of side : square of diagonal as 1 : 2,
∴ square of hypothenuse × side of cube = double the cube.

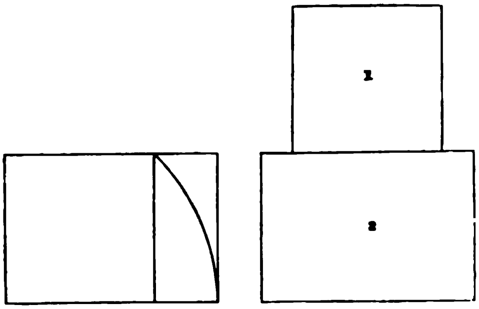


Fig. 58.

Hence, when a right-angled triangle has the two sides equal, the square of hypothenuse × one side = the cubes of the two sides.

Also, hypothenuse^2 × side = cube of side + side × rectangle by sum and difference of hypothenuse and side;

$$(\text{as when side} = 6,\ \text{hypothenuse} = 72^{\frac{1}{2}})$$
$$= \text{cube of side} + 6 \times (72^{\frac{1}{2}} + 6) \times (72\ - 6)$$
$$= \text{cube of side} + \text{cube of side}.$$

Thus the content of 2 will be double the content of 1, and the base of 2 double the base of 1. The area of their bases will be as 1 : 2, and their heights equal. Thus the cubic altar of Apollo may be said to be doubled.

Content of 2 will be double the cube 1 ; but 2 will not be a cube.

A cube may be made nearly double another cube, but not accurately so ;

for if $a^3 : b^3 :: 1 : 2$

$$b^3 = \frac{a^3}{2}$$

$$b = \frac{a}{2^{\frac{1}{3}}} \text{ an impossible quantity.}$$

But in cubing the measurements of ancient monuments one cube will be estimated at double another cube.

As $6{\cdot}3^3$ may be said to be double 5^3;
since $6{\cdot}3^3 = 250{\cdot}047$,
and $5^3 = 125$.

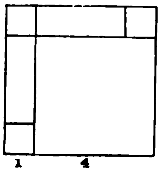


Fig. 59.

So that when numbers are as $6{\cdot}3 : 5$, the cube of the first may be called double the cube of the second.

Fig. 59. $\frac{1}{2}$ cube of any side is less than the cube of $\frac{4}{5}$ side by $\frac{3}{2}$ cube of $\frac{1}{5}$ side.

As cube of $10 = 10^3 = 1000$

$$\left.\begin{array}{r} \frac{1}{2} \text{ cube of } 10 = 500 \\ \text{Cube of } \frac{4}{5} \text{ side} = (\frac{4}{5}10)^3 = 8^3 = 512 \end{array}\right\} \text{difference} = 12$$

$\frac{3}{2}$ cube of $\frac{1}{5}$ side $= \frac{3}{2}(2)^3 = 12 =$ difference.

Or cube of $40 = 40^3 = 64000$

$$\left.\begin{array}{l} \frac{1}{2} \quad \text{,,} \qquad\quad = 32000 \\ \text{Cube of } \frac{4}{5}40 = 32^3 \ = 32768 \end{array}\right\} \text{difference} = 768$$

$\frac{3}{2}$ cube of $\frac{1}{5}40 = \frac{3}{2}(8)^3 = \frac{3}{2}512 = 768 =$ difference.

$\therefore$ $\frac{1}{2}$ cube of side $=$ cube of $\frac{4}{5}$ side $-$ $\frac{3}{2}$ cube of $\frac{1}{5}$ side.

Cube of side $=$ twice cube of $\frac{4}{5}$ side $-$ thrice cube of $\frac{1}{5}$ side; when side $= 5$, $\frac{4}{5} = 4$, $\frac{1}{5} = 1$,

$$5^3 = 2 \times 4^3 - 3$$
$$125 = 128 - 3.$$

Difference between the cubes of 5 and 4

$$= 4^3 - 3 \times 1^3$$
$$= 64 - 3 = 61,$$

or difference $= 5 \times 5 + 5 \times 4 + 4 \times 4$
$= 25 + 20 + 16 = 61.$

Difference between any two cubes
$=$ difference between the 2 sides
$\times$ (rectangle by 2 sides $+$ squares of 2 sides).

As $9^3 - 6^3 = 729 - 216 = 513$.

$$\begin{aligned}\text{Difference} &= 3 \times (9 \times 6 + 9 \times 9 + 6 \times 6)\\ &= 3 \times (54 + 81 + 36)\\ &= 3 \times 171 = 513.\end{aligned}$$

These 3 strata applied to 3 sides of the cube of 6 will $=$ cube of 9.

$$9^3 = 729$$

$$\left.\begin{aligned}8^3 &= 512\\ 6^3 &= 216\\ 1^3 &= 1\end{aligned}\right\} 729$$

$$10^2 = 100$$

$$\left.\begin{aligned}8^2 &= 64\\ 6^2 &= 36\end{aligned}\right\} 100$$

$$10^2 = 8^2 + 6^2$$

$$9^3 = 8^3 + 6^3 + 1^3.$$

If diagonal of a square $= 10$, the square of a side will $= \frac{1}{2}(10)^2 = 50$, and square of diagonal $= 10^2 = 100$; the squares will be as $1 : 2$

$$9^3 = 729$$

$$\tfrac{1}{2}\, 9^3 = 364{\cdot}5$$

$$364{\cdot}5^{\frac{1}{3}} = 7{\cdot}14, \text{ \&c.}$$

$$7{\cdot}14^3, \text{\&c.} : 9^3 :: 364{\cdot}5 : 729 :: 1 : 2.$$

Cubes of sides are as $1 : 2$.

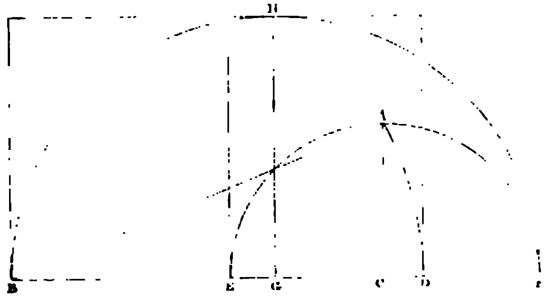


Fig. 60.

***Fig.* 60.** ABC is a right-angled triangle.

$BD = AB =$ hypothenuse
$BF = BC + CF =$ sum of 2 sides
$BE = BC - CF =$ the difference
FB is bisected in G
$GH = GB = GF =$ mean of 2 sides.

Then mean $\times$ (difference^2 + hypothenuse^2) = sum of cubes of 2 sides.

Or $GH \times (BE^2 + BD^2) = AC^3 + BC^3$.

When the 2 sides are equal their difference vanishes.

Then side $\times$ hypothenuse^2 = cubes of 2 sides.

When the 3 sides of a right-angled triangle are as 3, 4, 5.

Squares of 2 sides $= 3^2 + 4^2 = 5^2 =$ hypothenuse^2.

Hypothenuse$^3 = 5^3 = 125$.

Sum of cubes of 3 sides $= 3^3 + 4^3 + 5^3 = 216 = 6^3$.

Hypothenuse^3 : sum of cubes of 3 sides $:: 5^3 : 6^3$.

Sum of cubes of 3 sides : cube of sum of 2 sides

$$:: 3^3 + 4^3 + 5^3 : (3+4)^3$$
$$:: 216 : 343$$
$$:: 6^3 : 7^3.$$

Sum of squares of 3 sides : square of sum of 2 sides as

$$3^2 + 4^2 + 5^2 : (3+4)^2$$
$$:: 50 : 49.$$

Mean of 2 sides $\times$ (difference^2 + hypothenuse^2) = cubes of 2 sides.

Or $3{\cdot}5 \times (1^2 + 5^2) = 91$.

Cubes of 2 sides $= 3^3 + 4^3 = 91$.

Let the sides of the triangle be 9 and 15.

Mean $= \frac{1}{2}(9 + 15) = 12$.

Difference $= 15 - 9 = 6$.

Hypothenuse$^2 = 9^2 + 15^2 = 81 + 225 = 306$.

Then mean $\times$ (difference^2 + hypothenuse^2) = cubes of 2 sides.

Or $12 \times (6^2 + 306) = 4104$.

Cubes of 2 sides $= 9^3 + 15^3 = 729 + 3375 = 4104$.

Hence in any right-angled triangle the mean of the 2 sides × (square of their difference + square of hypothenuse) equals sum of cubes of the 2 sides.

Or $m \times (d^2 + h^2) = $ sum of cubes of 2 sides $= s$

$$d^2 + h^2 = \frac{s}{m}$$

$$h^2 = \frac{s}{m} - d^2$$

$$h^3 = \left(\frac{s}{m} - d^2\right)^{\frac{3}{2}}.$$

When the two sides of the triangle are equal, $m =$ side of cube, and d vanishes.

As the angle at B decreases, or the difference between the 2 sides increases, the sum of the cubes of the 2 sides will approach to equality with the cube of the hypothenuse.

Or mean × (difference^2 + hypothenuse^2) will approach to $\frac{1}{2}$ hypothenuse × (hypothenuse^2 + hypothenuse^2) $= \frac{1}{2}$ hypothenuse^3 + $\frac{1}{2}$ hypothenuse^3 = hypothenuse^3.

The sum of the squares of the 2 sides of a right-angled triangle always equals the square of the hypothenuse.

As the angle at B decreases, or the difference between the 2 sides increases, the sum of the two sides will approach to equality with the hypothenuse.

As the angle at B decreases, the cube of the sum of 2 sides, and the sum of the cubes of the 2 sides, both approach to equality with the cube of the hypothenuse.

When the 2 sides are equal, the cube of the sum of the 2 sides will = 8 times the cube of 1 side; or 4 times the cubes of the 2 sides, or 4 times one side × hypothenuse^2, or one side × (2 hypothenuse)^2.

Then side × hypothenuse^2 will = 2 cubes of side,
and hypothenuse^3 will = 2·83 nearly.

The less square (*fig.* 61.) will represent the cube of 1 side = 1.

The greater square will represent the cube of the hypothenuse = 2·83 nearly.

The diagonal of the less square = side of the greater square = hypothenuse.

The areas of the squares are as 1 : 2, for the square of the greater side = the square of the hypothenuse.

Greater cube = hypothenuse$^3 = (2^{\frac{1}{2}})^3 = 2^{\frac{3}{2}} = 2{\cdot}83$ nearly.

less cube = cube of side = 1.

difference of cubes = (hypothenuse $-\frac{1}{2}$ side) × hypothenuse^2

$$=(\sqrt{2}-\tfrac{1}{2})\times(2^{\frac{1}{2}})^2=(\sqrt{2}-\tfrac{1}{2})\times 2=2^{\frac{3}{2}}-1$$

$$=2{\cdot}83-1=1{\cdot}83.$$

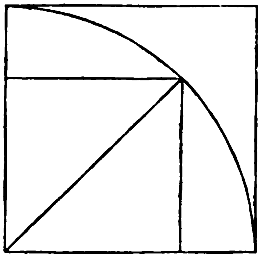


Fig. 61.

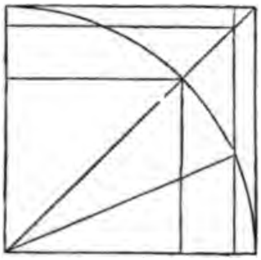


Fig. 62.

Generally, difference of 2 cubes = less side × rectangle by sum and difference of sides + square of greater side × difference of sides.

Let the cubes be 3^3 and 5^3; rectangle by sum and difference of sides

$$=(3+5)\times(5-3)=8\times 2=16$$

less side × rectangle $=3\times 16=48$

square of greater side × difference of sides

$$=5^2\times(5-3)=25\times 2=50$$

and $48+50=98$

difference of cubes $=5^3-3^3=125-27=98$.

Fig. 62. shows that a series of right-angled triangles, having radius for hypothenuse, may be inscribed in a quadrantal area.

The circumscribing square = hypothenuse^2.

Inscribed square = square of 1 side, and the rectangle by the sum and difference of the sides of the squares will = the square of the other side.

In this series of triangles the squares of the 2 sides = square of hypothenuse = radius^2, a constant quantity.

Or the circumscribing square will represent the cube of the hypothenuse, and the inscribed square the cube of one side of a triangle.

Fig. 63. Every square described on the base of a triangle will be within the square of the hypothenuse, and all their diagonals will be in the same straight line, which is the diagonal of the square of the hypothenuse.

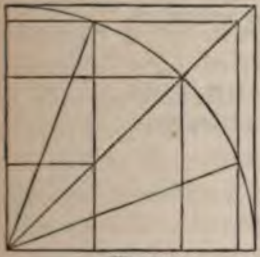


Fig. 63.

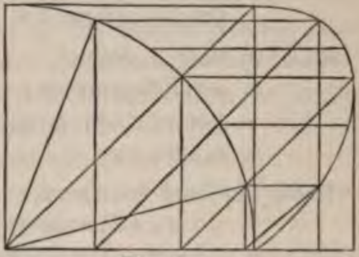


Fig. 64.

Fig. 64. Each corresponding square described on the perpendicular will have the diagonal parallel to the diagonal of the square of the hypothenuse; and the extremities of these diagonals beyond the quadrant will trace a pear-like curve having axis = radius, and ordinate $\propto$ sine $-$ versed sine.

The square described on the base of a triangle, and the square described on the corresponding perpendicular, will together = hypothenuse^2 = radius^2.

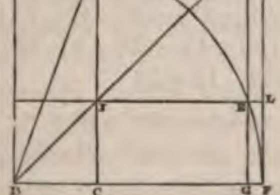


Fig. 65.

Fig. 65. Square of one side of triangle = rectangle by sum and difference of hypothenuse and the other side.

Arc F E = arc D A.

Radius of quadrant = AB = hypothenuse of triangle ABC.

$$\begin{aligned}
\text{Sine}^2 &= \text{radius}^2 - \text{cosine}^2.\\
&= (\text{radius} + \text{cosine}) \times (\text{radius} - \text{cosine})\\
&= (\mathrm{BF} + \mathrm{BC}) \times \mathrm{CF}\\
&= \text{parallelograms } \mathrm{DL} + \mathrm{LC} = \text{square } \mathrm{KG}\\
\text{cosine}^2 &= \text{radius}^2 - \text{sine}^2\\
&= (\text{radius} + \text{sine}) \times (\text{radius} - \text{sine})\\
&= (\mathrm{BF} + \mathrm{BG}) \times \mathrm{GF}\\
&= \text{parallelograms } \mathrm{DM} + \mathrm{MG} = \text{square } \mathrm{IB}\\
\text{radius}^2 &= \text{sine}^2 + \text{cosine}^2\\
&= \text{parallelograms } \mathrm{DL} + \mathrm{LC} + \mathrm{DM} + \mathrm{MG}\\
&= \text{square } \mathrm{KG} + \text{parallelograms } \mathrm{DM}, \mathrm{MG}\\
&= \text{square } \mathrm{DF}.
\end{aligned}$$

If a = sine, and b = cosine,

$$\begin{aligned}
&(a+b) \times (a-b) = a^2 - b^2\\
&= \mathrm{KG} - \mathrm{IB} = \mathrm{KE} + \mathrm{EC}\\
&(a-b) \times (a-b) = a^2 + b^2 - 2ab\\
&= \mathrm{KG} + \mathrm{IB} - (\mathrm{KC} + \mathrm{BE}) = \mathrm{AE}.
\end{aligned}$$

$$\begin{aligned}
\text{Difference} &= (a^2 - b^2) - (a^2 + b^2 - 2ab)\\
&= 2ab - 2b^2\\
&= \mathrm{KC} + \mathrm{BE} - 2\mathrm{IB}\\
&= \mathrm{KI} + \mathrm{IG}.
\end{aligned}$$

Thus $(a+b) \times (a-b)$ exceeds
$(a-b) \times (a-b)$ by $2ab - 2b^2 = \mathrm{KI} + \mathrm{IG}$.

If $a = 6$, and $b = 2$,

$$\begin{aligned}
\text{difference} &= (a^2 - b^2) - (a^2 + b^2 - 2ab)\\
&= (6^2 - 2^2) - (6^2 + 2^2 - 24)\\
&= (36 - 4) - (36 + 4 - 24)\\
&= 32 - 16 = 16
\end{aligned}$$

$$\begin{aligned}
(a+b) \times (a-b) &= a^2 - b^2\\
&= 6^2 - 2^2 = 32\\
(a-b) \times (a-b) &= a^2 + b^2 - 2ab\\
&= 6^2 + 2^2 - 24 = 16.
\end{aligned}$$

The distance of the moon from the earth is about 109·5 diameters of the moon.

Hence diameter of the moon : $\frac{1}{2}$ diameter of orbit of the moon

:: diameter of the earth : diameter of the sun.

:: diameter of the sun : $\frac{1}{2}$ diameter of orbit of the earth.

Since $\frac{1}{2}$ diameter of the moon : $\frac{1}{2}$ diameter of orbit of the moon

:: $\frac{1}{2}$ diameter of the sun : $\frac{1}{2}$ diameter of orbit of the earth.

:: 1 : 219.

Therefore the pyramid of Cheops will represent the time of descent from the earth to the moon through 219 semi-diameters of the moon, as well as the time of descent from the earth to the sun through 219 semi-diameters of the sun.

The bases of the pyramids will in both cases be in the centre or orbit of the earth; but in the descent to the sun the apex of the external pyramid will be in the centre of the sun, and in the descent to the moon the apex of the external pyramid will be in the centre of the moon. (*Fig.* 57. A.)

The axis of the external pyramid is supposed to be divided into 219 equal parts, or 219 semi-diameters: this pyramid represents the time of descent through 219 semi-diameters when velocity is supposed to $\propto \frac{1}{D^2}$. But the time of descent is only $\frac{5}{8}$ that time, which is represented by the internal pyramid, that of Cheops, where the 219 distances along the axis are unequal, diminishing from the base to the apex, so that the time through successive unequal distances of the internal pyramid, which correspond to the semi-diameters of the external pyramid, decrease in a greater ratio than the time through the successive semi-diameters of the external pyramid: so the velocity will increase in a greater inverse ratio than D^2.

The external pyramid has the height equal side of base, like the pyramid of Belus. Herodotus says the height of the pyramid of Cheops is equal to the side of the base.

We supposed the pyramid of Cheops might have been dedicated to the sun, because it represented the semi-diameter of the sun and the semi-diameter of the earth's orbit, as well as the time of descent from the earth to the sun; but now it appears that this pyramid will also represent the semi-diameter of the moon, and the semi-diameter of the moon's orbit, as well as the time of descent from the earth to the moon. So the pyramid of Cheops might have been dedicated to both the sun and moon.

As the sides of the pyramid front the cardinal points, the terraces on the eastern side would face the rising sun, which would be visible from the terraces before it could be seen from the base of the pyramid. The Sabæan priests, if placed on the terraces, could announce the precise time when the people stationed at the base should perform their adoration to the rising sun. On the western side the same reverence might have been observed to the setting sun.

It may be remarked that Herodotus calls the terraces little altars. Perhaps on these altars offerings of the first-fruits of the earth, ripened by the genial influence of the sun, were made to that splendid luminary.

Adoration somewhat similar might have been made to the moon, especially to the new and full moon.

If the axis of the external pyramid were equal to twice the axis of the internal pyramid, then the pyramids would be as 2 : 1. The axis of the external pyramid would represent the distance, and the internal pyramid the time.

In order to determine the axis of the external pyramid, a similar pyramid to that of Cheops, having 219 terraces of unequal height, should first be made; next the external pyramid having the axis divided into 219 equal parts, which would be determined by construction; thus the axis of this external pyramid would represent the distance, and the internal pyramid the time of descent to that distance.

In the valley of the Nile, which has played so important a part in the history of mankind, Lepsius informs us that ascertained shields of kings go back to the fourth Manethonic dynasty. This dynasty begins 3400 years before the Christian era, and 2300 years before the emigration of the Heraclidæ to Peloponnesus. The last dynasty of the old kings, which ended with the invasion of the Hyksos, 1200 years before Homer, was the twelfth Manethonic dynasty, to which belong Ameranah III., the builder of the original labyrinth, and maker of the lake of Mœris. After the expulsion of the Hyksos the new kingdom began with the eighteenth dynasty, 1600 years before Christ. The great Ramses, called by Herodotus Sesostris, was the second ruler of the nineteenth dynasty. The canal of Suez was begun by Sesostris to facilitate the access to the Arabian copper mines, which were worked under Cheops, one of the fourth dynasty of Egyptian kings.

All the Egyptian monuments would have been of little or no avail as sources of history, unless they bore some records for the information of the reader of a future age. Of this the Egyptians were fully aware in the earliest times. According to their annals, Tosorthoros, the second king of the second dynasty, more than 3000 years B. C., who was the first to build with hewn stone, devoted much attention to the development of the art of writing; and from the time of Cheops — also more than 3000 years B. C. — we find in the monuments a completely formed system of writing, the use of which was evidently by no means confined to the priests.

The manner in which the Egyptians availed themselves of this art is worthy of notice. Not satisfied, like the Greeks and Romans, with a single inscription on some prominent part of their temples or tombs, they engraved them with astonishing precision and elegance — considering the hardness and roughness of the stone, together with the pictorial character of the writing — upon all the walls, pillars, roofs, architraves, friezes, and posts, both inside and outside.

Writing was in very early times applied also to literary purposes. From the very first use of the papyrus and the time of the pyramids at Memphis, we find writers occupied themselves in describing on leaves the wealth and power of their rulers. That they even then had public annals appears from the historical accounts that have come down to us. We now possess two original fragments of such annals, belonging to the commencement of the New Empire, and therefore extending upwards of 500 years farther back than the earliest literary remains of any other ancient nation. The great number of these fragments gives credibility to the statement of Diodorus, that a library was built at Thebes in the time of Ramses Miamun, who flourished in the fourteenth century B. C. This is confirmed by Champollion's observations among the ruins on the spot. Lepsius tells us that he himself has seen the tombs of two librarians, father and son, who lived under that king, and were called superintendents of the books. Clemens Alexandrinus says, the Egyptians in his time had forty-two sacred books; the latest of which, according to Bunsen, was earlier than the time of the Psammetichi, certainly not later. It can, therefore, be no matter of surprise that 400,000 volumes or scrolls should in a short time have been collected in the library founded at Alexandria by Ptolemy Philadelphus.—(*Humboldt.*)

Lepsius found divisions of time from the 21,600th part of a day up to their greatest period of 36,525 years. Between these extremes there were cycles of every length, determined with greater precision than those of any other ancient nation. Their seasons consisted of four months. They recognised and registered in their calendar, not only the old lunar year, but also the common year of 365 days, and the exact year of about 365¼ days, which commenced with the heliacal rising of Sirius.

The emblem of the scribe's palette, reed-pen, and ink bottle, are found in the legends of the fourth dynasty, about 3400 B. C., which proves that, in that remote day, the art of writing was already familiar with the builders of the pyramids.

But the builders themselves have given the best proof that writing was familiar to them, since their works are the most ancient and stupendous monuments of the surprising degree of cultivation the arts and sciences had attained at a very remote epoch.

The art of writing must have long preceded the attainment of the astronomical knowledge recorded by the pyramids, and claims an antiquity never suspected by the Greeks or Romans.

According to Lepsius, Mœris built the last of the 69 pyramids, and reigned 2154 B. C. This is supposed to be the termination of the pyramidal period, which ceased when Lower Egypt was overrun by the shepherd hordes.

The sources of the Nile are as much involved in mystery as every thing else connected with the strange country of Egypt. The statue under which it was represented was carved out of black marble, to denote its Ethiopian origin, but crowned with thorns, to symbolise the difficulty of approaching its fountain-head. It reposed appropriately on a sphynx, the type of enigmas, and dolphins and crocodiles disported at its feet. The solution has baffled the scrutiny and self-devotion of modern enterprise as effectually as it did the inquisitiveness of ancient despots and the theories of ancient philosophers. Alexander and Ptolemy sent expeditions in search of it; Herodotus gave it up; Pomponius Mela brought it from the antipodes, Pliny from Mauritania, and Homer from heaven. Bruce thought he had detected its infancy in the fountains of the Blue River. This was only a foundling, however, — a mere tributary stream.

PART V.

PYRAMID OF CEPHRENES. — CONTENT EQUAL TO $\frac{5}{12}$ CIRCUMFERENCE, CUBE EQUAL TO $\frac{1}{5}$ DISTANCE OF MOON. — THE QUADRANGLE IN WHICH THE PYRAMID STANDS. — SPHERE EQUAL TO CIRCUMFERENCE. — CUBE OF ENTRANCE PASSAGE IS THE RECIPROCAL OF THE PYRAMID. — THE PYRAMIDS OF EGYPT, TEOCALLIS OF MEXICO, AND BURMESE PAGODAS WERE TEMPLES SYMBOLICAL OF THE LAWS OF GRAVITATION AND DEDICATED TO THE CREATOR. — EXTERNAL PYRAMID OF MYCERINUS EQUAL TO $\frac{1}{19}$ CIRCUMFERENCE EQUAL TO 19 DEGREES, AND IS THE RECIPROCAL OF ITSELF. — CUBE EQUAL TO $\frac{1}{4}$ CIRCUMFERENCE. — INTERNAL PYRAMID EQUAL TO $\frac{1}{24}$ CIRCUMFERENCE. — CUBE EQUAL TO $\frac{1}{5}$ CIRCUMFERENCE. — THE SIX SMALL PYRAMIDS. — THE PYRAMID OF THE DAUGHTER OF CHEOPS EQUAL TO $\frac{1}{180}$ CIRCUMFERENCE EQUAL TO 2 DEGREES, AND IS THE RECIPROCAL OF THE PYRAMID OF CHEOPS. — THE PYRAMID OF MYCERINUS IS A MEAN PROPORTIONAL BETWEEN THE PYRAMID OF CHEOPS AND THE PYRAMID OF THE DAUGHTER. — DIFFERENT PYRAMIDS COMPARED.—PYRAMIDS WERE BOTH TEMPLES AND TOMBS. — ONE OF THE DASHOUR PYRAMIDS EQUAL TO $\frac{1}{3}$ CIRCUMFERENCE, CUBE EQUAL TO TWICE CIRCUMFERENCE.—ONE OF THE SACCARAH PYRAMIDS EQUAL TO $\frac{5}{18}$ CIRCUMFERENCE. — CUBE EQUAL TO $\frac{1}{6}$ DISTANCE OF MOON. — GREAT DASHOUR PYRAMID EQUAL TO $\frac{5}{9}$ CIRCUMFERENCE. — CUBE EQUAL TO $\frac{1}{3}$ DISTANCE OF MOON. — HOW THE PYRAMIDS WERE BUILT. — NUBIAN PYRAMIDS. — NUMBER OF EGYPTIAN AND NUBIAN PYRAMIDS. — GENERAL APPLICATION OF THE BABYLONIAN STANDARD.

The Pyramid of Cephrenes.

All that Herodotus says of the dimensions of the pyramid of Cephrenes is, that they are far inferior to those of Cheops' pyramid, for we measured them.

The following are the measurements recently made:—

JOMARD.

Present height = 138 metres =452·64 feet English
Former height =455·64

Northern side of base = 207·9 = 682
Western side of base = 210 = 688.

Belzoni.

Height = 456 feet
Side of base = 684 feet.

Vyse.

Present height = 447·6 feet
Former height = 454·3 „
Present base = 690·9 „
Former base = 707·9 „
Square of platform on the top about 9.

Wilkinson.

Present height = 439 feet
Former height = 466 „
Northern side of base = 684 „
Western side of base = 695 „

Jomard supposes this pyramid to have lost about 3 feet from the top, which, if added to the height he has given, 452·64 feet, will make the height to the apex = 455·64 feet, and $456\frac{5}{8}$ feet = $1\frac{5}{8}$ stade.

This will make the height to the apex of the pyramid of Cephrenes equal to the height to the platform of the pyramid of Cheops.

Let the height to apex = $1\frac{5}{8}$ stade
= $456\frac{5}{8}$ feet = 394 &c. units
and base = 684 by 703 feet
= 592 by 608 units,

then height × base

$$= 394 \text{ \&c.} \times 592 \times 608 = \tfrac{5}{4} \text{ circumference}$$

pyramid = $\frac{1}{3}$ of $\frac{5}{4}$ = $\frac{5}{12}$ circumference = 150 degrees.

Thus the height to the apex, $1\frac{5}{8}$ stade, will nearly accord with the height assigned by Jomard, Belzoni, and Vyse; and the base, 684 by 703 feet, will somewhat exceed the base of Wilkinson.

Wilkinson's dimensions, height to apex 466 feet, and base

684 × 695 feet, would also very nearly = $\frac{5}{12}$ circumference, but then Wilkinson's height to the apex would be much greater than the other measurements, and would make the height to the apex of Cephrenes pyramid = the height to the apex of the pyramid of Cheops.

$468\frac{1}{3}$ feet = 10 plethrons = the height to the apex of the pyramid of Cheops; and 466 feet, according to Wilkinson, would = the height to the apex of the pyramid of Cephrenes.

Thus the dimensions we have assigned to the pyramid of Cephrenes will be that one side of the base = $2\frac{1}{2}$ stades = 15 plethrons = 607·5 units, and the other side = 15 plethrons less 15 units = 592·5 units.

Perimeter of base = (608 + 592) × 2
= 2400 units
= 60 plethrons less 30 units
height to apex = $1\frac{5}{8}$ stade
= 10 plethrons less 10 units.

The pyramid of Cephrenes is said by Jomard to rise not from the level of the natural rock, but out of an excavation or deep cut made in the solid rock all round the pyramid.

This pyramid, says Greaves, is bounded on the north and west sides by two very stately and elaborate pieces that have not been described by former writers. About 30 feet in depth, and more than 1400 in length, out of a hard rock, these buildings have been cut in perpendicular, and squared by the chisel, as I suppose, for the lodgings of the priests. They run along at a convenient distance, parallel to the two sides of this pyramid, meeting in a right angle.

The side of the quadrangle, or one side of half the quadrangle described, exceeds 1400 feet.

Five stades = 1405 feet.

The side of the quadrangle that enclosed the tower of Belus was = twice the side of the base of that pyramid.

Supposing the side of the base of the pyramid of Cephrenes to equal half the side of the quadrangle of Greaves, or $2\frac{1}{2}$ stades, or 702·5 feet, such a base would nearly agree with 707·9 feet, the former base of Vyse.

If each side of rectangle on the north and west sides = 1410 feet = 1220 units,

then $1220^3 = 16$ times circumference
and $90 \times 16 = 1440$ circumference
= distance of Mercury from the Sun.

Thus the distance of Mercury will = 90 cubes, and distance of Belus = 150 times the distance of Mercury = 150×90 cubes.

The distance of Saturn = 25 times the distance of Mercury = 25×90 cubes.

The assigned base of pyramid $= 592 \times 608$ units; $\frac{1}{2}(529+608)=600$.

If the side of square base of pyramid = 601 units, and height × base = $\frac{5}{4}$ circumference, then 5 times the cube of the side of the base $= 5 \times 601^3 = 1085409005$ units.

Distance of moon = 9·55 circumference = 1085730026 units.

Hence 5 times the cube of the side of the base of the pyramid of Cephrenes will = 9·55 circumference = distance of the moon from the earth.

The cube of Cheops will be to the cube of Cephrenes as 5 : 4.

Distance of Mercury from the sun will = 150 times the distance of the moon from the earth, $= 150 \times 5 = 750$ cubes of Cephrenes, $= 150 \times 4 = 600$ cubes of Cheops.

Should one side of the base of the pyramid = 610 units, and the other side = 592 units, the cube of the greater side will $= 610^3 = 2$ circumference; the mean of the 2 sides $= \frac{1}{2}(610+592)=601$.

The cube of the mean will $= 601^3 = \frac{1}{5}$ distance of the moon.

A sphere having a diameter = 601 units will = circumference.

If the base of pyramid be a square having a side = 601 units, and height = 392, &c., then height × base = 392 &c. $\times\, 601^2 = \frac{5}{4}$ circumference. Pyramid $= \frac{5}{12}$ circumference.

Cube of height : cube of side of base :: 392^3, &c. : 601^3 :: $\frac{1}{18}$: $\frac{1}{5}$ distance of moon :: 5 : 18.

Cube of height = $\frac{5}{18}$ cube of side of base.
Cube of side of base = $\frac{1}{5}$ distance of moon
= $\frac{60}{5}$ = 12 radii of the earth.

If 12 radii divided the circumference of the earth into 12 equal parts, then pyramid would = 5 of these parts, and the cube of the side of the base would = the 12 radii.

The inclined side of the pyramid will = 494 &c. units,
and 494^3 &c. = $\frac{1}{9}$ distance of the moon.

So cube of height : cube of inclined side :: 392^3, &c. : 494^3, &c. :: $\frac{1}{18}$: $\frac{1}{9}$
:: 1 : 2.

Cube of inclined side : cube of side of base :: $\frac{1}{9}$: $\frac{1}{5}$ distance of the moon :: 5 : 9.

Cube of inclined side = $\frac{5}{9}$ cube of side of base.

Pyramid = 5 times 30 degrees.

Cube of side of base = $\frac{1}{5}$ of 60 radii
= $\frac{1}{5}$ distance of the moon.

Cube of perimeter of base = $\frac{64}{5}$ distance of the moon
2 " " = $\frac{512}{5}$ " "
4 " " = $\frac{4096}{5}$ " "
5 cubes of 4 times perimeter = 4096 " "
$2\frac{1}{2}$ cubes " " = 2048 " "
and distance of Jupiter = 2045 " "

Thus $2\frac{1}{2}$ cubes of 4 times perimeter of pyramid of Cephrenes = distance of Jupiter = 2 cubes of 4 times perimeter of pyramid of Cheops.

Sphere having diameter = 601 units = side of base of Cephrenes = $601^3 \times \cdot 5236$ = circumference, or sphere of Cephrenes = circumference = twice the pyramid of Cheops.

Cube of Cephrenes = 601^3 = $\frac{1}{5}$ distance of the moon
Cylinder = $\frac{3}{2}$ circumference
Sphere = $\frac{2}{2}$ "
Cone = $\frac{1}{2}$ "

Cone of Cephrenes = pyramid of Cheops

Cylinder = $\frac{3}{2}$ circumference = height × area of the base of Cheops.

Cube of side of base = $\frac{1}{5}$ distance of the moon
5 cubes „ = distance „
Cube of 4 times perimeter = $\frac{4096}{5}$
5 cubes = diameter of the orbit of Jupiter.
$(5 \times 601)^3 = \frac{1}{5} \times 5^2 = 25$ distance of the moon
6 cubes of 5 times side of base
= 150 times distance of the moon
= distance of Mercury
16 cubes = 400 times distance of the moon
= distance of the earth.
$(10 \times 601)^3 = \frac{1000}{5} = 200$ distance of the moon
2 cubes of 10 times side of base
= 400 times distance of the moon
= distance of the earth
3 cubes = 600 distance of the moon
distance of Mars = 604 „ „
Sphere of Cephrenes = circumference = pyramid of Cholula
Cone „ = $\frac{1}{2}$ = pyramid of Cheops.
Sphere, diameter $2 \times 601 = 8$ circumference
Sphere, diameter = perimeter of base $= 4 \times 601 = 64$ circumference.

Cube of Cephrenes = $\frac{1}{5}$ distance of the moon
Cube of Cheops = $\frac{1}{4}$ „ „
Pyramid of Cephrenes = $\frac{5}{12}$ circumference
Pyramid of Cheops = $\frac{6}{12}$ „
Cylinder having height = diameter of base = 601 will
$= 601^3 \times {\cdot}7854 = \frac{3}{2}$ circumference
Sphere = $\frac{2}{2}$ „
Cone = $\frac{1}{2}$ „
Cylinder having height = diameter of base = 2×601
will = 12 circumference
Sphere = 8 „
Cone = 4 „
Cylinder having height = diameter of base = 4×601
= perimeter of base
will = 96 circumference
Sphere = 64 „
Cone = 32 „

15 cylinders = 1440 circumference = distance of Mercury
40 „ = 3840 „ = distance of the earth
$\frac{1}{10}$ cylinder = 9·6 „ = distance of the moon.

Distance of moon : distance of Mercury :: distance of Mercury : distance of Belus

$$1 : 150 :: 150 : 150^2$$
$$\cdot 1 : 15 :: 15 : 15^2 \times 10.$$

distance of Mercury = 15 cylinders
distance of Belus = $15^2 \times 10$ „

5 cylinders having height = diameter of base = twice perimeter of base will = 3840 circumference = distance of the earth.

Height × area of base of pyramid
= 393, &c. × 601^2 = $\frac{5}{4}$ circumference,
Pyramid = $\frac{1}{3}$ of $\frac{5}{4}$ = $\frac{5}{12}$ „

Vyse makes the former height = 454·3 feet = 392·8 units,
former base = 707·9 = 612 „

If height × area base = 400, &c. × 615^2 = $\frac{4}{3}$ circumference,
Pyramid = $\frac{1}{3}$ of $\frac{4}{3}$ = $\frac{4}{9}$ „

The heights and sides of bases of the two pyramids will be proportionate to each other.

So that if the first pyramid were completely cased, the cased pyramid might be = to the latter, supposing the bases were square, which seems doubtful.

The first pyramid would be to the latter pyramid as

$$\frac{5}{12} : \frac{4}{9} \text{ circumference},$$
$$:: 45 : 48$$
$$:: 15 : 16$$

When reference is made to the pyramid of Cephrenes, the content is supposed = $\frac{5}{12}$ circumference, and cube of one side, or of the mean of two sides of base = $\frac{1}{5}$ distance of the moon.

Wilkinson makes the sides

684 by 695 feet
=591·4 by 610·9 units
say 592 by 610 „
610^3 = 2 circumference
mean = 601
601^3 = $\frac{2}{5}$ distance of the moon.

Height × area of base
$= 393, \&c. \times 592 \times 610 = \frac{5}{4}$ circumference
Pyramid $= \frac{1}{3}$ of $\frac{5}{4} = \frac{5}{12}$ „

It will probably be found that the sides of the base of some of the pyramids are unequal.

From the two great pyramids we learn that the quadrant was divided into 3 equal parts; or the circumference into 12, the zodiacal division.

The pyramid of Cheops $= \frac{6}{12}$ circumference,
„ Cephrenes $= \frac{5}{12}$ „
Or the parallelopipedon of Cheops $= \frac{6}{4} = 6$ quadrants.
„ „ Cephrenes $= \frac{5}{4} = 5$ „

The pyramid of Cheops $= \frac{1}{2}$ circumference $= 180$ degrees
„ „ Belus $= \frac{1}{2}$ a sign $= 15$ „
which are as 12 : 1.

The pyramid of Belus : pyramid of Cephrenes :: $\frac{1}{24} : \frac{5}{12}$ :: 1 : 10.

The pyramid of Cheops : pyramid of Cephrenes :: $\frac{6}{12} : \frac{5}{12}$ :: 6 : 5 signs, if the equator be supposed to be divided into 12 equal parts or signs.

The distance of the moon from the earth = 5 cubes of Cephrenes.

The distance of the earth from the sun = 400 times the distance of the moon from the earth $= 400 \times 5 = 2000$ cubes of Cephrenes.

In Vyse's measurements of the interior of the pyramid of Cephrenes, the length of the entrance passage from the first covering stone to the horizontal passage = 104 feet 10 inches.

Total length of the entrance passage to the bottom of the incline = 104 feet.

105·2 feet = 91·2 units
$10 \times 91{\cdot}2 = 912$ „
and $912^3 = \frac{20}{3}$ circumference =
$91{\cdot}2^3 = \frac{20}{3000}$ circumference $= 2\frac{2}{5}$ degrees
$= \frac{1}{150}$ circumference $= \frac{12}{5}$ degree.

Pyramid of Cephrenes $= \frac{5}{12}$ circumference = 150 degrees.

$\therefore$ the cube of the length of the entrance passage is the reciprocal of the content of the pyramid of Cephrenes.

The Birman solid hyperbolic temples are symbolical of the law of the velocity described by a body gravitating to the centre of force. The Egyptian solid pyramidal temples are typical of the law of the time corresponding to that velocity.

On each side of the hyperbolic temple, as the Shoemadoo at Pegu, are dwellings for the priests, who still officiate at the altar; but the former science of the priesthood has departed.

Along the sides of the quadrangular area in which stands the pyramid of Cephrenes are dwellings for the priests, excavated out of the solid rock; but the hierarchy exists no longer, and the knowledge accumulated for ages, and held sacred by the priesthood, has perished.

The Birman pagodas are solid structures, without any opening.

Vyse computes the space occupied by the chambers in the pyramid of Cheops at $\frac{1}{1590}$ of the whole.

The teocallis of Mexico are solid pyramidal temples. Montezuma was emperor and high-priest. The temple of Mexitli had five terraces. It was on the platform of this teocalli that the Spaniards, the day preceding the "noche triste," or "melancholy night," attacked the Mexicans, and, after a dreadful carnage, became masters of the temple. It stood within a great square, surrounded by a wall of hewn stone. "Close to the side of the wall," says De Solis, "were habitations for the priests, and of those who, under them, attended the service of the temple; with some offices, which altogether took up the whole circumference, without retrenching so much from that vast square but that eight or ten thousand persons had sufficient room to dance in it upon their solemn festivals.

In the centre of the square stood a pile of stone, which in the open air exalted its lofty head, overlooking the towers of the city, and gradually diminishing till it formed half a pyramid. Three of its sides were smooth, the fourth had stairs wrought in the stone, — a sumptuous building, and extremely well proportioned. It was so high that the staircase contained 120 steps; and of so large a compass, that on the top it terminated in a flat 40 feet square.

Pyramid of Mycerinus.

Herodotus states that Mycerinus left a pyramid less than that of his father, wanting on all sides, for it is quadrangular, 20 feet; it is 3 plethrons on every side, and one half is made of Ethiopian stone.

Instead of the side of the base being 3 plethrons, suppose the perimeter of the base to equal 30 plethrons, or 5 stades. Then each side will equal 7½ plethrons, or $\frac{5}{4}$ stade, or 351·21 feet, or 304 units, which is 20 units less than half the side of Cheops' pyramid. For the side of the base of Cheops' pyramid = 648 units, and ½ 648 = 324 units, from which take 20 units, and we have 304 units left for the side of the base of Mycerinus' pyramid. These 20 units may have been called feet by Herodotus. If the priesthood in his time knew the value of the Babylonian unit, it appears they never made him acquainted with it, for in his tables neither this measure nor its equivalent is ever mentioned, though this unit formed the basis of his table of measures. Its value may probably have been unknown to all, except the elect of the sacred colleges of a philosophical priesthood.

At whatever period, remarks Maurice, the Egyptian hieroglyphics were first invented, their original meaning was scarcely known, even to the priests themselves, at the æra of the invasion of Cambyses. And at the time when the Macedonian invader erected Alexandria, probably out of the ruins of Memphis, the knowledge of them was totally obliterated from their minds.

The difference between the sides of these two pyramids may be expressed by saying

The perimeter of the base of the pyramid of Mycerinus equals half the perimeter of the base of the pyramid of Cheops, less 80 units, or less 20 units on every side.

The perimeter of Cheops = 64 plethrons
=2592 units
½=1296 „

∴ The perimeter of Mycerinus
=1296−80=1216 units
and side of base = 304 „

5 stades=30 plethrons=1215 units
=perimeter of the base.

Jomard's dimensions of this pyramid are,

Base, measured on the north side, 100·7 metres=330 feet English.

Height 53 metres=173·84 feet; but height not determined with great accuracy.

Angle made by the plane of the face with the plane of the base, about 45°.

Vyse makes the former base =354·6 feet
„ present height=203
„ former height =218

Wilkinson's present base =333
„ present height =203·7

by calculation with the angle of 51° given by Vyse.

Pliny makes the distance between the angles, or side of the base=363 feet.

By Arbuthnot's table a Roman foot = 11·604 inches English.

363 × 11·604 inches=351·02 feet English
and $\frac{5}{4}$ stade =351·25

If the angle of inclination of the side =45°, according to Jomard, the height will=half the side of the base.

Assuming the height=$\frac{5}{8}$ stade
=175·625 feet=152 units,
side of base will =$\frac{5}{4}$ stade
=351·25 feet=304 units,
and 152, &c. × $\overline{305}^2$ =$\frac{1}{8}$ circumference of earth
pyramid=$\frac{1}{3}$ of $\frac{1}{8}$ =$\frac{1}{24}$ circumference.

Such a pyramid would combine the height of the teocalli, $\frac{5}{8}$ stade, with the content of the tower of Belus, $\frac{1}{24}$ circumference, and the height $\frac{5}{8}$ stade would be $\frac{5}{8}$ that of the tower.

These supposed dimensions of this pyramid,

height $=175 \cdot 6$ feet,
base $=351 \cdot 25$,

accord nearly with Jomard's height, 173·84 feet, and with Vyse's former base, 354·6 feet, as well as with Pliny's base, 351·02 feet, and also with the 304 units, or 351·25 feet, obtained by comparing the side of this pyramid with that of Cheops.

But the difference between the heights of Vyse and Jomard $=55$ feet, and the difference between their bases 24 feet.

Cube of side of base $=305^3=\frac{1}{4}$ circumference
Cube of twice side $=610^3=2$
Cube of height $=152^3$, &c. $=\frac{1}{32}$.

If this pyramid had formerly been a teocalli having the height to the side of base as $\frac{5}{8} : \frac{5}{4}$ stade, or 152, &c. : 305 units, or $175\frac{5}{8} : 351\frac{1}{4}$ feet.

Supposing such a teocalli to have had 4 terraces of equal heights, and the height of the 4 to equal $\frac{5}{8}$ stade, and the height to the apex to equal the height of 5 terraces, or $175\frac{5}{8}+\frac{1}{4}\,175\frac{5}{8}$ feet $=220$ feet.

Then the content of this pyramid, having the same base, would exceed that of the pyramid having the height to side of base as $\frac{5}{8} : \frac{5}{4}$ stade by $\frac{1}{4}$.

Or content $=\frac{1}{24}+\frac{1}{4}$ of $\frac{1}{24}$ circumference
$=\frac{5}{96}$ or $\frac{1}{19}$ circumference nearly,
and $\frac{1}{19}$ circumference $=19$ degrees nearly,
for $19 \times 19=361$.

Such a pyramid would accord with the base and former height by Vyse's measurement.

By this supposition the mode we have adopted for measuring the content of a teocalli is practically illustrated.

Here the circumscribing triangle of the pyramid and teocalli are equal, and so are their contents, for the content of the pyramid or teocalli $=\frac{1}{3}$ (the square of the base of the circumscribing triangle × the height).

The bulk of the pyramid has been more carefully and compactly built than the two larger ones, and the stones have been better finished, and are of a greater size. It has been carried up in steps or stages, diminishing towards the top like those in the fourth and fifth pyramids; and the angular spaces have been filled up so as to complete the pyramidal form. (*Vyse.*)

The dimensions of such a pyramid will be

Height to apex $= \frac{5}{8} + \frac{1}{4}$ of $\frac{5}{8}$ stade
$= 175\cdot6 + 43\cdot9 = 219\cdot5$ feet $= 190$ units.
Side of base $= \frac{5}{4}$ stade $= 351\cdot25$ feet $= 304$ units;
then height × base

$$= 190, \&c. \times 305^2 = \frac{3}{360^{\frac{1}{2}}} \text{ circumference}$$

$$\text{pyramid} = \frac{1}{360^{\frac{1}{2}}} \text{ circumference} = 360^{\frac{1}{2}} \text{ degrees.}$$

Vyse's height to apex $= 218$ feet $= 188\cdot5$ units
side of base $= 354\cdot6$ feet $= 306\cdot5$
square of platform at the top about 9.

According to Vyse's dimensions the content of the pyramid of Mycerinus will $= \dfrac{1}{360^{\frac{1}{2}}}$ circumference $= 360^{\frac{1}{2}}$ degrees.

The perimeter of the base will $= \frac{5}{4} \times 4 = 5$ stades $= 30$ plethrons.

$$\text{Height} = \frac{5}{8} + \frac{1}{4} \text{ of } \frac{5}{8} = \frac{25}{32} = \frac{5^2}{2^5} \text{ stade.}$$

Also a pyramid having the height to apex $= \frac{5}{8}$ stade.
Side of base = twice the height $= \frac{5}{4}$ stade;
Or perimeter of base $= 5$ stades $= 30$ plethrons
will $= \frac{1}{24}$ circumference $= 15$ degrees
= the content of the tower of Belus.

Both these formulas will require a small correction, — the addition of a unit to a stade, as will be seen afterwards.

Side of base of pyramid $= 305$ units, and $305^3 = \frac{1}{4}$ circumference.

So 4 cubes = circumference.

Or if a cube be described on each of the 4 sides the sum of the cubes will = circumference

$$(2 \times 305)^3 = \frac{8}{4} = 2 \text{ circumference,}$$

or cube of sum of 2 sides = 2 circumference.

$$189^3 \text{ \&c.} = \tfrac{6}{100} \text{ circumference}$$
$$(10 \times 189 \text{ \&c.})^3 = \tfrac{6000}{100} = 60$$

cube of 10 times height = 60 circumference.

Inclined side will = 242 &c. units

$$242^3 \text{ \&c.} = \tfrac{1}{8} \text{ circumference}$$
$$(2 \times 242 \text{ \&c.})^3 = 1 \quad ,,$$

Cube of twice inclined side = circumference.

Cube of side of base : cube of inclined side :: $\frac{1}{4}$: $\frac{1}{8}$ circumference :: 2 : 1.

$$\text{Pyramid} = \frac{1}{(360)^{\frac{1}{2}}} \text{ circumference} = (360)^{\frac{1}{2}} \text{ degrees.}$$

So the pyramid of Mycerinus will be the reciprocal of itself.

Cube of perimeter of base $=(4 \times 305)^3 = 16$ circumference.

Cube of perimeter of base of Cheops = 16 distance of the moon.

Cubes of perimeters are as circumference : distance of the moon. Pyramid of Mycerinus : pyramid of Cheops

$$:: \frac{1}{(360)^{\frac{1}{2}}} \quad : \tfrac{1}{2} \text{ circumference}$$
$$:: \quad 2 \quad : 19 \quad ,,$$
$$:: \quad 1 \quad : \ 9{\cdot}5 \quad ,,$$

:: circumference : distance of the moon.

The pyramid of Mycerinus will be similar to the pyramid of Cheops, so the height will = $\frac{5}{8}$ side of base.

4 cubes of Mycerinus = circumference
4 ,, Cheops = distance of the moon.

Taking Jomard's base as that of the internal pyramid,

side of base = 330 feet = 285 units
and 283^3 &c. = $\frac{1}{5}$ circumference.

Content of external : content of internal pyramid :: $\frac{1}{4}$: $\frac{1}{5}$ :: $\frac{1}{19}$: $\frac{1}{24}$ circumference.

Thus we shall have the content of the external pyramid = $\frac{1}{19}$ circumference

Cube of side of base $= \frac{1}{4}$ circumference
Content of internal pyramid $= \frac{1}{24}$ „
Cube of side of base $= \frac{1}{5}$ „

The external and internal pyramids will be similar, having height $= \frac{5}{8}$ side of base.

The content of the internal pyramid will = that of the tower of Belus $= \frac{1}{24}$ circumference.

Content of external pyramid $= \frac{1}{19}$ circumference = 19 degrees, or $=$ circumference$^{\frac{1}{2}} = 360^{\frac{1}{2}} = 19$ degrees.

Herodotus says the pyramid of Mycerinus was built up to the middle with Ethiopian stone. The casing has been taken away at different times: some of it was removed a few years ago to assist in the construction of the arsenal at Alexandria. The lower part of the casing consisted of polished granite, as the ancient historians have described; but the eleven or twelve courses towards the bottom are not worked smooth, but form a sort of rusticated base, inclining like the rest of the pyramid.

The style of building of the pyramid of Cephrenes is said to be inferior to that of Cheops, the stones used in its construction being less carefully selected, though united with nearly the same kind of cement. Nor, says Wilkinson, was all the stone of either pyramid brought from the quarries of the Arabian mountains, but the outer tier or casing was composed of blocks hewn from their compact strata. This casing, part of which still remains on the pyramid of Cephrenes, is, in fact, merely formed by levelling or planing down the upper angle of the projecting steps, and was consequently commenced from the summit.

The pyramid of Mycerinus is described as being built in almost perpendicular degrees, to which a sloping face has afterwards been added. The outer layers, many of which still remain, were of red granite, of which material the lowest row of the pyramid of Cephrenes was also composed, as is evident by the block and fragments which lie scattered about its base.

In measuring the content of the teocalli, this sloping face, which included the outer layers of the pyramid, has been included; since the inclining side of the teocalli, according to estimation, is that straight line which touches all the exterior angles of the terraces, or degrees, and terminates at the apex and ground base of the teocalli.

The following measurements of the small pyramids at Gizeh are those made by Col. Vyse, who, in his description of the pyramids, has given the measurements of the interior chambers and passages of all the pyramids.

The fourth central and sixth western pyramids south of the third pyramid, that of Mycerinus, are both built of large square blocks put together in the manner of Cyclopian walling, and are at present in steps or degrees. These two pyramids are of equal dimensions and similar in construction, each having four terraces, like a teocalli. Both are in a dilapidated state.

Height to the top platform, 69·6 feet,
$\frac{1}{4}$ stade $= 70\frac{1}{4}$ feet.
Side of the base of the lowest terrace $= 102{\cdot}5$ feet.

Suppose the height to the apex = the height of 5 terraces $= 70 + \frac{1}{4}\,70 = 87{\cdot}5$ feet $= 75$ &c. units $= \frac{5}{16}$ stade.

Let the base of the circumscribing triangle $= 128$ feet $= 111$ units $= 2\frac{1}{4}$ plethrons, then height × base $= 75$ &c $\times 111^2 = 3$ degrees.

$$\text{Pyramid} = \tfrac{1}{3}\,3 = 1 \text{ degree, or } \tfrac{1}{360} \text{ circumference.}$$

Thus the fourth and sixth pyramids or teocallis, &c., each = 1 degree.

The fourth pyramid is much dilapidated on the northern front; but the masonry on the other sides is very fine, and the stones exceeding large and apparently of great antiquity. Like the sixth pyramid it has been built in regular stages.

Side of base = 111 units
Height = 75 ,,

110^3 &c. $= \frac{5}{4000}$ distance of the moon
$(10 \times 110 \text{ \&c.})^3 = \frac{5000}{4000} = \frac{5}{4}$
$(2 \times 10 \times 110 \text{ \&c.})^3 = \frac{5}{4} \times 2^3 = \frac{40}{4} = 10.$

Cube of 20 times side, or of 5 times perimeter
= 10 times distance of the moon

15 cubes $=$ 150 times distance of the moon
$=$ distance of Mercury

$(2 \times 2 \times 10 \times 110\ \&c.)^3 = 10 \times 2^3 = 80$ distance of moon
5 cubes of 40 times side
$=$ 400 times distance of the moon
$=$ distance of the earth.

$75^3\ \&c. = \frac{3}{800}$ circumference
$(10 \times 75\ \&c.)^3 = \frac{3000}{800} = \frac{30}{8}$
$(2 \times 10 \times 75\ \&c.)^3 = \frac{30}{8} \times 2^3 = 30.$

Cube of 20 times height
$=$ 30 times circumference.

$(4 \times 2 \times 10 \times 75\ \&c.)^3 = 30 \times 4^3 = 1920.$

2 cubes of 80 times height $=$ 3840 circumference,
$=$ distance of the earth.

The fifth pyramid is to the south-east of the third.

Height to apex $=$ 93·3 feet $=$ 80 units.
Side of base $=$ 145·9 feet $=$ 125 units.

Height × base $= 80\ \&c. \times 125^2 = \frac{1}{90}$ circumference
$=$ 4 degrees.

Pyramid $= \frac{4}{3}$ of a degree $= \frac{1}{270}$ circumference,
and height to side of base :: 80 : 125 :: 5 : 8 nearly.

Or height $= \frac{5}{8}$ side of base nearly.

Perimeter of base $=$ 500 units,
and height $=$ 80 „

$501^3\ \&c. = \frac{10}{9}$ circumference
$(3 \times 501)^3 = \frac{10}{9} \times 3^3 = 30$ circumference;

or cube of 3 times perimeter $=$ 30 times circumference.

The fifth pyramid had at the time of Richardson a flat top, which was covered with a single stone. The two pyramids to the west of this, but in the same line, consist each of four receding platforms, like the Mexican teocallis. The several divisions of these pyramids are ascended by high narrow steps to the summit, which is a platform.

The third pyramid, that of Mycerinus, appears also to have been originally a teocalli, and that at a later period the terraces of the ancient teocalli had been built up so as to form a plain-sided pyramid.

We know of no pyramid of which the fifth pyramid will be the reciprocal. Such a pyramid should $= \frac{3}{4}$ circumference, = 270 degrees.

The seventh, eighth, and ninth pyramids are situated to the eastward of the great pyramid.

The seventh (northern) and eighth (central) pyramids are both in very ruined condition. The dimensions of both are supposed by Vyse to be equal.

Height to apex = 111 feet, and side of base = 172·5 feet.

If the height be supposed = 105 feet = 92 units, and side of base = 164 feet = 142 units,

Then height × base = 92 &c. $\times\ 142^2 = \frac{1}{60}$ circumference,
= 6 degrees.

So each pyramid will $= \frac{1}{3}$ of $\frac{1}{60} = \frac{1}{180}$ circumference,
= 2 degrees.

Thus the side of base of each pyramid will $= 3\frac{1}{2}$ plethrons,
= 141·75 units.

Height will $= 2\frac{1}{2}$ plethrons = 91·25 units.
141^3 &c. $= \frac{1}{40}$ circumference.
Cube of side $= \frac{1}{40}$ circumference.
91^3 &c. $= \frac{1}{180}$ circumference.
Cube of height $= \frac{1}{180}$ circumference.

Several of the casing stones of the central pyramid had been roughly chiselled into the proper angle, and then worked down to a polished surface after they had been built; and in many places the operation had not been entirely performed. They were as firmly laid as the blocks in the Great Pyramid, and the masonry of the buildings had a great resemblance. It is to be remembered that tradition assigns the building of this pyramid to the daughter of Cheops.

The pyramid of Cheops $= \frac{1}{2}$ circumf. = 180 degrees.
The pyramid of his daughter $= \frac{1}{180}$ circumf. = 2 degrees.
The height of the pyramid of Cheops $= 1\frac{2}{3}$ stade.
The side of base $= 2\frac{2}{3}$ stades.
The height of the pyramid of his daughter $= 2\frac{1}{2}$ plethrons.
The side of base $= 3\frac{1}{2}$ plethrons.

The height : side of base of Cheops' pyramid :: 5 : 8;
of his daughter :: 5 : 7.

The side of the base of the great pyramid = 16 plethrons.

The perimeter of the base of the small pyramid $= 3\frac{1}{2} \times 4$
$= 14$ plethrons.

Having since found that the pyramid of Mycerinus is a mean proportional between the pyramid of Cheops and the pyramid of his daughter. So that if all the three pyramids be similar, we can determine the height and side of base of the pyramid of Cheops' daughter.

The three pyramids are

Cheops : Mycerinus :: Mycerinus : Daughter
180 : $360^{\frac{1}{2}}$ :: $360^{\frac{1}{2}}$: 2 degrees.

The three pyramids being similar, the cubes of the sides of bases will be as their contents.

Cube of Cheops : cube of Mycerinus :: cube of Mycerinus : cube of Daughter

$$648^3 : 305^3 :: 305^3 : 144^3$$

$\frac{1}{4}$ distance of moon : $\frac{1}{4}$ circumference :: $\frac{1}{4}$ circumference : $\frac{1}{4} \times \frac{1}{\text{distance of moon}}$

$\frac{1}{4}$ 9·5 circumference : $\frac{1}{4}$ circumference :: $\frac{1}{4}$ circumference : $\frac{1}{4} \times \frac{1}{9{\cdot}5}$ circumference.

Hence the cube of the side of base of pyramid of Cheops' daughter will $= 144^3 = \frac{1}{4} \times \frac{1}{9{\cdot}5}$ circumference $= \frac{1}{38}$ circumference.

Since the three pyramids are similar, and height of each $= \frac{5}{8}$ side of base

$\therefore$ height of pyramid of Cheops' daughter $= \frac{5}{8}\,144 = 90$ units.

Height × area base

$$= 90 \times 144^3, \text{\&c.} = \tfrac{1}{60} \text{ circumference.}$$

Pyramid $= \frac{1}{3}$ of $\frac{1}{60} = \frac{1}{180}$ circumference $= 2$ degrees.

$\therefore$ Height will be 90 and side base 144
instead of 92 ,, 142

The pyramid of the Daughter is the reciprocal of the pyramid of Cheops.

The pyramid of Mycerinus is the reciprocal of itself.

The pyramid of Mycerinus is a mean proportional between the pyramid of Cheops and the pyramid of his daughter.

The three pyramids are all similar.

The height of each $= \frac{5}{8}$ side of base.

Hence knowing the side of the base of pyramid of Cheops, the dimensions of all the three pyramids can be determined.

Wilkinson mentions "that on the east side of the great pyramid stand three smaller ones, built in degrees or stages, somewhat larger than the three on the south side of the pyramid of Mycerinus. The centre one is stated by Herodotus to have been erected by the daughter of Cheops. It is 122 feet square, which is less than the measurement given by the historian of $1\frac{1}{2}$ plethron, or about 150 feet; but the difference may be accounted for by its ruined condition."

Wilkinson makes the side of the base of the pyramid of Cheops' daughter to equal 122 feet. Vyse makes the side of the base to equal 172·5 feet.

If side of base = 122 feet = 105·5 units

$$104^3, \text{\&c.} = \tfrac{1}{100} \text{ circumference}$$

$$(10 \times 104, \text{\&c.})^3 = \tfrac{1000}{100} = 10 \text{ circumference,}$$

or cube of 10 times side of this base

$$= 10 \text{ times circumference.}$$

Side of base of pyramid = 143 units;

$$143^3 = \tfrac{1}{40} \text{ circumference.}$$

The dimensions of the seventh pyramid are,

Height to the apex (supposed) 111 feet.
Side of the base (supposed) 172·5 feet.
Height 111 feet = 96 units.
Side of base 172·5 feet = 149 units.

Let the height = 98 units,
and side of base = 152 units $= \frac{5}{8}$ stade.

Then height × base

$=98$, &c. $\times 152^2 = \frac{1}{50}$ circumference $= 7{\cdot}2$ degrees.

Pyramid $= \frac{1}{150}$ circumference $= 2{\cdot}4$ degrees

$= \frac{1}{150}$ circumference $= \frac{12}{5}$ degrees;

$152{\cdot}5^3 = \frac{1}{32}$ circumference and 150^3, &c. $= \frac{1}{320}$ distance of moon.

The second pyramid, that of Cephrenes,

$= \frac{5}{12}$ circumference $= 150$ degrees.

Thus the seventh pyramid will be the reciprocal of the second, that of Cephrenes, as the eighth is the reciprocal of the first, that of Cheops.

Perimeter of the eighth pyramid $= \frac{5}{8} \times 4 = 2\frac{1}{2}$ stade = one of the sides of the base of the pyramid of Cephrenes.

The fourth and sixth pyramids are both teocallis, each having four terraces, and the content of each $= \frac{1}{360}$ circumference $= 1$ degree.

The teocalli of Cholula has four terraces, and the content $= 1$ circumference $= 360$ degrees.

Hence the fourth and sixth pyramids are the reciprocals of the teocalli of Cholula.

The third pyramid, that of Mycerinus $= \frac{1}{360^{\frac{1}{2}}}$ circumference $= 360^{\frac{1}{2}}$ degrees, and is $\therefore$ the reciprocal of itself.

Height : side of base of Cephrenes

$:: 392 : 601 :: 5 : 7{\cdot}64.$

Height to side of base of seventh pyramid

$:: 98 : 152 :: 5 : 7{\cdot}75.$

Height to side of base of Mycerinus as $5 : 8$.

If the three pyramids were similar, and height of each $= \frac{5}{8}$ side of base, then cubes of sides of base will be as

$601^3 : 305^3 :: 305^3 : 155^3$, &c.

$\frac{1}{5}$ distance of moon : $\frac{1}{4}$ circumference :: $\frac{1}{4}$ circumference :: $\frac{1}{30{\cdot}4}$ circumference;

$\frac{9\cdot5}{5}$ circumference : $\frac{1}{4}$ circumference :: $\frac{1}{4}$ circumference : $\frac{1}{30\cdot4}$ circumference.

Thus side of base of seventh $=155$, &c.

height $=\frac{5}{8}$ 155, &c. $=97$.

Height × area base

$=97\times155^2$, &c. $=\frac{1}{50}$ circumference

Pyramid $=\frac{1}{3}$ of $\frac{1}{50}=\frac{1}{150}$ circumference $=2\cdot4$ degrees.

Vyse makes the former height of Cephrenes somewhat more than $\frac{5}{8}$ side of base.

So it would seem that the pyramid of Cephrenes is dissimilar to Mycerinus, though it may have been similar to the seventh.

If so the cubes of the sides of the bases of the three pyramids will not be as their contents.

Thus the pyramid of Cephrenes and the seventh will be reciprocals, and may be similar to each other, though dissimilar from the pyramid of Mycerinus; still the pyramid of Mycerinus will be a mean proportional between Cephrenes and the seventh.

Cephrenes : Mycerinus :: Mycerinus : seventh.

$150 : 360^{\frac{1}{2}} :: 360^{\frac{1}{2}} : 2\cdot4$ degrees.

Cube of side of base of Cheops $=648^3=\frac{1}{4}$ distance of the moon.

$\frac{5}{8}$ cube $=\frac{5}{8}\times\frac{1}{4}=\frac{5}{32}$;

pyramid $=\frac{1}{3}$ of $\frac{5}{32}=\frac{5}{96}$

$$=\tfrac{5}{96}\times9\cdot55=\frac{47\cdot75}{96}=\tfrac{1}{2}\text{ circumference.}$$

Cube of side of base of Cephrenes $=601^3=\frac{1}{5}$ distance of the moon.

$\frac{5}{8}$ cube $=\frac{5}{8}\times\frac{1}{5}=\frac{5}{40}=\frac{1}{8}$;

pyramid $=\frac{1}{3}$ of $\frac{1}{8}=\frac{1}{24}$

$$=\tfrac{1}{24}\times9\cdot55=\frac{9\cdot55}{24}\text{ circumference ;}$$

but pyramid $=\frac{5}{12}=\frac{10}{24}$ circumference,
which is greater than $\frac{9\cdot55}{24}$.

Thus the pyramid of Cephrenes exceeds $\frac{1}{3}$ of $\frac{5}{8}$ cube of side of base, and, therefore, is dissimilar to the pyramid of Cheops or Mycerinus.

The angle of inclination of the side of Cheops is less than the angle of inclination of the side of Cephrenes.

Should the side of base of a pyramid = 601 units, and height = $\frac{5}{8}$ side = 375 &c. units; then height × area of base = $\frac{1}{8}$ distance of the moon;

pyramid = $\frac{1}{3}$ of $\frac{1}{8}$ = $\frac{1}{24}$ distance of the moon;
tower of Belus = $\frac{1}{24}$ circumference.

Thus a pyramid having side of base = that of Cephrenes, and height $\frac{5}{8}$ side of base, will = $\frac{1}{24}$ distance of the moon.
Pyramid of Cheops has height = $\frac{5}{8}$ side of base,
and content = $\frac{1}{2}$ circumference.

These two pyramids will be similar,

and as $\frac{1}{2}$ circumference : $\frac{1}{24}$ distance of the moon
circumference : $\frac{1}{12}$ „ „
„ : $\frac{60}{12}$ radii of the earth
„ : 5 „ „
3·1416 : 2·5
5 : 4 nearly.

Cubes of sides of bases are as $\frac{1}{4}$: $\frac{1}{5}$ distance of the moon
5 : 4 „ „

Such a pyramid would be to the tower of Belus as $\frac{1}{24}$ distance of the moon : $\frac{1}{24}$ circumference
as „ „ : circumference.

Pyramid of Cephrenes : tower of Belus as $\frac{5}{12}$: $\frac{1}{24}$ circumference :: 10 : 1.

Having found that a pyramid has two dimensions, one internal, the other external, let us try how nearly two such pyramids of Cephrenes may be made to accord with the measurements of Vyse.

Former height 454·3 feet = 392 units
Present height 447·6 „ = 376 „

Former base 707·9 feet = 612 units
Present base 690·9 „ = 579 „

Internal Pyramid.

Let height × base $= 376 \times 601^2 = \frac{1}{8}$ distance of the moon
pyramid $= \frac{1}{3}$ of $\frac{1}{8} = \frac{1}{24}$
cube of side of base $= 601^3 = \frac{1}{5}$ distance of the moon.

External Pyramid.

Let height × base = 381 &c. $\times\ 610^2 = \frac{10}{8}$ circumference
pyramid $= \frac{1}{3}$ of $\frac{10}{8} = \frac{10}{24} = \frac{5}{12}$
cube of side of base $= 610^3 =$ 2 circumference.

The internal and external pyramids will be similar, having height $= \frac{5}{8}$ side of base, which is the proportion of the internal and external pyramids of Cheops; therefore the two pyramids of Cephrenes are similar to the two pyramids of Cheops, or the four pyramids are all similar.

The two pyramids of Cephrenes will be external : internal :: $\frac{10}{24}$ circumference : $\frac{1}{24}$ distance of the moon
:: 10 „ : distance „

External pyramid of Cephrenes : tower of Belus :: $\frac{10}{24}$: $\frac{1}{24}$ circumference :: 10 : 1.

Internal pyramid of Cheops : tower of Belus :: $\frac{1}{2}$: $\frac{1}{24}$ circumference :: 12 : 1.

External pyramid of Cephrenes : internal pyramid of Cheops :: $\frac{5}{12}$: $\frac{6}{12}$ circumference
:: 5 : 6.

Internal pyramid of Cephrenes : external pyramid of Cheops :: $\frac{1}{24}$: $\frac{1}{18}$ distance of the moon.
:: 3 : 4.

In all the four pyramids cube of height : cube of side of base :: $5^3 : 8^3$ :: 125 : 512
:: 1 : 4 nearly.

External cube of Cephrenes : internal pyramid of Cheops :: 2 : $\frac{1}{2}$ circumference :: 4 : 1.

Internal cube of side of base of Cephrenes : external cube of side of base of Cheops :: $\frac{1}{5}$: $\frac{8}{30}$ distance of the moon
:: 3 : 4.

The ninth southern pyramid is in much better preservation than the seventh and eighth.

The height to apex = 101·8 feet, and side of base = 160 ft.

101·8 feet = 88 &c. units;
160 feet = 138 units.

Height × base $= 88 \times 139^2$ &c. $= \frac{3}{200}$ circumference.
Pyramid $= \frac{1}{200}$ circumference $= \frac{9}{5}$ degree.
The great pyramidal teocalli at Dashour $= \frac{5}{9}$ circumference
= 200 degrees,
the reciprocal of which will be the ninth pyramid.

139^3 &c. $= \frac{1}{400}$ distance of the moon, or $= \frac{12}{500}$ circumf.
and distance of the moon $= \frac{1}{400}$ distance of the earth,

$$\therefore 139^3 \text{ \&c.} = \frac{1}{160000} = \frac{1}{400^2} \text{ distance of the earth.}$$

$(10 \times 139 \text{ \&c.})^3 = \frac{1000}{400} = \frac{5}{2}$ distance of the moon.
$(10 \times 10 \times 139 \text{ \&c.})^3 = \frac{5000}{2} = 2500$.

3 cubes of 100 times side = 7500 distance of the moon,
= distance of Uranus.
9 cubes „ „ = distance of Belus.

Height = 88 units.

$88^3 = \frac{3}{500}$ circumference
$(10 \times 88)^3 = \frac{3000}{500} = 6$
$(10 \times 10 \times 88)^3 = 6000$.

6 cubes of 100 times height = 36000 circumference
= distance of Saturn
12 cubes - - - = „ Uranus.
36 cubes - - - = „ Belus.

Thus 9 cubes of 100 times side of base
= 36 cubes of 100 times height = distance of Belus,
∴ cube of height : cube of side :: 1 : 4.

Height × area of base $= 88 \times 139^2$ &c. $= \frac{3}{200}$ circumfer.
Pyramid $= \frac{1}{3}$ of $\frac{3}{200} = \frac{1}{200}$ „

$\frac{1}{2}$ side of base = 69 &c.
Inclined side = 112

Cube of $\frac{1}{2}$ side of base $= 69^3$ &c. $= \frac{3}{1000}$ circumference
Cube of height $= 88^3$ &c. $= \frac{6}{1000}$,,
Cube of inclined side, say $111^3 = \frac{12}{1000}$,,
Cube of side of base $= 139^3$ &c. $= \frac{24}{1000}$,,
The cubes will be as 1, 2, 4, 8
or as $1, 2, 2^2, 2^3$.

Cube of side of base
= twice cube of inclined side
= 4 times cube of height
= 8 times cube of $\frac{1}{2}$ side of base.

Pyramid : pyramid of Cheops :: $\frac{1}{200}$: $\frac{1}{2}$ circumference
:: 1 : 100.

$$\text{Sum of cubes} = \frac{3+6+12+24}{1000} = \frac{45}{1000}$$

$\frac{1}{3}$ sum $= \frac{15}{1000}$ circumference = height × area of base of pyramid.

$\frac{1}{9}$ sum $= \frac{5}{1000} = \frac{1}{200}$ circumference = pyramid.

Thus cube of side of base
= double the cube of inclined side.
Cube of height
= double the cube of $\frac{1}{2}$ side of base.
Cube of side of base
= 4 times cube of height.
Cube of inclined side
= 4 times cube of $\frac{1}{2}$ side of base.

If the cube of the side of base = 4 times cube of height, the cube of the hypothenuse, or inclined side, would not exactly = 2 cubes of height.

If height = 50
and side of base = 79·4,
then cube of side of base will = 4 times cube of height.
Inclined side will = 63·8;
but 63^3 will = twice cube of height.

So that if the height of a pyramid nearly = $\frac{5}{8}$ side of base, the cubes will be nearly as 1, 2, 4, 8.

Height of Cheops' pyramid = 406 &c. units,
and $\frac{5}{8}$ side of base = $\frac{5}{8}$ 648 = 405.
Inclined side = 518 &c.
when height = 405.

Side of base = 648 and $648^3 = \frac{1}{4}$ distance of moon
Inclined side = 518 &c. „ $514^3 = \frac{1}{8}$ „
Height = 405 „ 408^3 &c. $= \frac{1}{16}$ „
$\frac{1}{2}$ side of base = 324 „ $324^3 = \frac{1}{32}$ „

By deducting 4 units from inclined side, and adding 4 units to height,
the cubes will be as 1, 2, 4, 8.

Four is an important number in the pyramid of Cheops.
$(2 \times 514)^3$ = distance of the moon.
$(2 \times 648)^3$ = diameter of the orbit of the moon.

Sum of cubes $= \dfrac{8+4+2+1}{32} = \dfrac{15}{32}$ distance of the moon.

Calling distance of the moon = 9·6 circumference,

sum of cubes will = 4·5 circumference
$\frac{1}{3}$ sum of cubes = $\frac{3}{2}$ circumf. = height × area of base
$\frac{1}{9}$ sum of cubes = $\frac{1}{2}$ circumf. = pyramid.

Height × area of base = $\frac{5}{8}$ cube of side of base = $\frac{5}{8} \times \frac{1}{4}$ = $\frac{5}{32}$ distance of the moon = $\frac{5}{32} \times 9{\cdot}6 = \frac{48}{32} = \frac{3}{2}$ circumference

(calling circumference $= \dfrac{1}{9{\cdot}6}$ distance of the moon).

Pyramid = $\frac{1}{3}$ of $\frac{3}{2} = \frac{1}{2}$ circumference,
or, pyramid = $\frac{1}{3}$ of $\frac{5}{8} = \frac{5}{24}$ cube of side of base
$= \frac{5}{24} \times \frac{1}{4} = \frac{5}{96}$ distance of the moon.
Pyramid = $\frac{1}{2}$ circumference = $\frac{5}{96}$

circumference $= \dfrac{10}{96} = \dfrac{1}{9{\cdot}6}$ distance of moon.

Distance of moon = 9·6 times circumference.

We have called the distance of the moon = 9·55 times circumference, and distance of Mercury = 1440 times circumference, or 150 distances of the moon, to avoid fractions.

But 150 × 9·6 = 1440 circumferences, without a fraction.

We have made the distance of the moon in pyramid of Cheops = 9·57 circumference,

which is less than 9·6
and greater than 9·55.

Since 2 distance of moon = cube of twice side of base

$$= (2 \times 648)^3 = 6^{12}$$

$$2 \text{ circumference} = \frac{1}{9{\cdot}57 \text{ \&c.}} \times 6^{12}.$$

$$\text{Circumference} = \frac{1}{2 \times 9{\cdot}57 \text{ \&c.}} \times 6^{12} = 113689008 \text{ units.}$$

Thus, if the distance of the moon = 9·6 circumference,

$$\begin{aligned} \text{pyramid would} &= \tfrac{1}{3} \text{ of } \tfrac{5}{8} \text{ cube of side} \\ &= \tfrac{1}{3}\, 405 \times 648^2; \\ \text{but pyramid} &= \tfrac{1}{3}\, 406 \text{ \&c.} \times 648^2 \\ &= \tfrac{1}{2} \text{ circumference.} \end{aligned}$$

$$\text{Distance of the moon} = \frac{6^{12}}{2}.$$

$$\begin{aligned} \text{If pyramid} &= \tfrac{5}{96} \text{ distance of the moon} \\ &= \frac{5}{96} \times \frac{6^{12}}{2} \\ &= \frac{5}{16} \times \frac{6^{11}}{2} \end{aligned}$$

circumference = $\tfrac{5}{16} \times 6^{12} = 113374080$ units, which is too little.

Since 406 &c. × $648^2 = \tfrac{3}{2}$ circumference,
and $405 = \tfrac{5}{8}\, 648$,

$\therefore 405 \text{ \&c.} \times 648^2 \text{ \&c.} = \tfrac{3}{2}$
$\tfrac{8}{5}\, 405 \text{ \&c.} \times 648^2 \text{ \&c.} = 648^3 \text{ \&c.} = \tfrac{3}{2} \times \tfrac{8}{5} = \tfrac{24}{10}$
$(2 \times 648 \text{ \&c.})^3 = \tfrac{24}{10} \times 2^3 = \tfrac{192}{10} = 19{\cdot}2.$

Cube of twice side of base = 19·2 circumference
= twice distance of the moon.

$\tfrac{1}{2}$ cube = 9·6 circumference = distance of the moon.

Hence, if side of base = 648 &c. units, and distance of moon = 9·6 circumference, then cube of side of base would = $\tfrac{1}{4}$ distance of the moon = $\tfrac{1}{4}$ 9·6 = 2·4 circumference.

Pyramid having height = $\frac{5}{8}$ side of base would = $\frac{1}{2}$ circumference of the earth.

Circumference would = $243 \times 684^2 = 113689008$ units.

Thus pyramid will = $\frac{1}{3}$ of $\frac{5}{8}$ cube of side of base = $\frac{1}{3}$ of $\frac{5}{8}$ of $\frac{1}{4}$ distance of the moon = $\frac{5}{96}$ distance of the moon = $\frac{1}{2}$ circumference of the earth.

So 10 distance of the moon = 96 circumference.

Distance of Mercury = 150 distance of the moon
= 96 × 15 = 1440 circumference.

Height : side of base :: 5 : 8
Pyramid : cube of side of base :: $\frac{1}{2}$: 2·4 circumference
:: 10 : 48
:: 5 : 24

684^2 stades = circumference
648^3 &c. units = $\frac{1}{4}$ distance of the moon.

Cheops' and ninth pyramid will be similar; and cubes of their sides will be as their contents, as 100 : 1.

Cube of side of base of Cheops : cube of side of base of ninth
:: $\frac{1}{4}$ distance of the moon : $\frac{24}{1000}$ circumference
:: $\frac{9{\cdot}6}{4}$ circumference : $\frac{24}{1000}$ circumference
:: 9600 : 96 :: 100 : 1.

A pyramid = $\frac{1}{20}$ circumference will be a mean proportional between Cheops' and ninth,

as $\frac{1}{2} : \frac{1}{20} :: \frac{1}{20} : \frac{1}{200}$ circumference.

Cube of 10 times side of base of ninth pyramid = $\frac{24}{1000} \times 10^3 = 24$ circumference.

Cube of side of base of Cheops'= $\frac{1}{4}$ distance of the moon = 2·4 circumference.

Thus cube of 10 times side of base of ninth pyramid = 10 times cube of side of base of Cheops'= 24 circumference $= 10 \times \frac{1}{4} = \frac{10}{4} = \frac{5}{2}$ distance of the moon.

Cube of 100 times side of base of ninth pyramid = 10 times cube of 10 times side of base of Cheops'= 24000 circumference = $\frac{5000}{2}$ = 2500 distance of the moon.

3 or 30 cubes = 72000 circumference
= 7500 distance of the moon
= distance of Uranus.

9 or 90 cubes = 216000 circumference
= 22500 distance of the moon
= distance of Belus.

Cube of 100 times side of base of ninth pyramid = 24000 circumference = 2500 distance of the moon.

3 cubes = distance of Uranus
9 cubes = „ Belus.

$$100^3 : 208^3 :: 1 : 9.$$

Therefore cube of 208 times side of base = 216000 circumference = 22500 distance of the moon = distance of Belus.

Sphere = distance of Neptune
Pyramid = „ Uranus

or cube of 52 times perimeter = distance of Belus
= cube of Babylon.

Pyramid height = side of base
= $\frac{1}{3}$ cube = distance of Uranus.

Pyramid height = $\frac{5}{8}$ side of base will be similar to the ninth,

and = $\frac{5}{8}$ distance of Uranus,
= 45000 circumference ;

such a pyramid would be to the pyramid of Cheops
: 45000 circumference : $\frac{1}{2}$ circumference
: 90000 „ : 1 „

both pyramids being similar.

The base of such a pyramid would = the square of Babylon, and height = $\frac{5}{8}$ side of base
content = $\frac{5}{8}$ distance of Uranus
= $\frac{5}{24}$ „ Belus
= $\frac{5}{24}$ cube of Babylon.

Pyramid of Cheops = $\frac{5}{24}$ cube of Cheops.

Cube of Cheops : cube of Babylon :: $\frac{1}{4}$: 22500 distance of the moon :: 1 : 90000.

Height of tower of Belus = side of base = 1 stade, and content = $\frac{1}{24}$ circumference.

Pyramid having height = side of base = side of Babylon = 120 stades will = 72000 circumference = distance of Uranus.

$$\begin{aligned}\text{Tower} : \text{pyramid} &:: \tfrac{1}{24} : 72000 \text{ circumference}\\ &:: 1 : 1728000\\ &:: 1^3 : 120^3.\end{aligned}$$

The pyramids being similar, their contents will be as the cubes of their sides or heights, as $1^3 : 120^3$ stades.

The height and side of base of the pyramid that represents the distance of Uranus will = 120 times the height and side of base of the tower of Belus.

Content will = $\frac{1}{3}$ cube of Babylon.

Hence, by taking the distance of the moon = 9·6 circumference, the planetary distances can be expressed in terms of both the circumference of the earth and the distance of the moon without any fractions.

The cube of side of base of Cheops' will = $\frac{1}{4}$ distance of the moon = 2·4 circumference.

Height = $\frac{5}{8}$ side of base,
and content = $\frac{1}{2}$ circumference = $\frac{10}{192}$ distance of the moon
= $\frac{5}{10}$ „ = $\frac{5}{96}$;
pyramid = $\frac{5}{24}$ cube of side of base.

10 times cube of side of base = $\frac{5}{2}$ distance of the moon
20 times cube of side = 5 „ „
48 circumference = 5 „ „
3×4^2 „ = 5 „ „

Side of base of Cheops' pyramid
=648 units=$2\frac{2}{3}$=$\frac{8}{3}$ stade.

Height=$\frac{5}{8}$ side of base=405 units
=$\frac{5}{8}$ of $\frac{8}{3}$=$\frac{5}{3}$ stade.

Height × area base
$=\frac{5}{3}\times\left(\frac{8}{3}\right)^2=\frac{5}{3}\times\frac{64}{9}=\frac{320}{27}$ cubic stades
$=\frac{320}{27}\times 243^3$ cubic units.

Pyramid=$\frac{1}{3}$ content=$\frac{1}{2}$ circumference, very nearly.

The addition of part of a unit to height and side of base will be required to make pyramid $= \frac{1}{2}$ circumference.

Thus side of base $= \frac{8}{3}$ stade,

height $= \frac{5}{8}$ side of base $= \frac{5}{3}$ stade,

content of pyramid $= \frac{320}{81} \times 243^3$

$$= 320 \times \frac{243^3}{3^4},$$

but $\frac{1}{2}$ circumference will lie between

$$320 \text{ and } 321 \times \frac{243^3}{3^4},$$

$$\text{or } 320 \text{ and } 321 \times 3^{11}$$

$$\text{for } \frac{243^3}{3^4} = \frac{(3^5)^3}{3^4} = \frac{3^{15}}{3^4} = 3^{11}$$

$$320 \times 3^{11} = 56687040 \text{ units}$$

$$\begin{aligned} \text{and } \tfrac{5}{32} \times 6^{11} &= 56687040 \\ \tfrac{1}{2} \text{ circumference} &= 56844504 \\ \therefore \quad \tfrac{5}{32} \times 6^{11} &= 320 \times 3^{11} \\ 6^{11} &= 3^{11} \times 320 \times \tfrac{32}{5} \\ &= 3^{11} \times 32^2 \times \tfrac{10}{5} \\ &= 3^{11} \times 32^2 \times 2 \\ &= 3^{11} \times (2^5)^2 \times 2 \\ &= 3^{11} \times 2^{11} \\ &= 6^{11} \end{aligned}$$

6^{12} = diameter of orbit of moon in units

$= (2 \times 648)^3$ = cube of twice side of Cheops' base;

3^5 transposed, doubled and squared,

$= 684^2$ = circumference of earth in stades;

$3^5 \times 684^2$ = circumference of earth in units.

Pyramid of Cheops $= \frac{1}{3}$ of $\frac{5}{3} \times (\frac{8}{3})^2$

$$\begin{aligned} &= \tfrac{5}{9} \times \tfrac{64}{9} \\ &= 5 \times \frac{8^2}{9^2} \text{ cubic stades} \\ &= 5 \times \frac{8^2}{9^2} \times 243^3 \text{ cubic units} \end{aligned}$$

$$=5\times\frac{8^2}{9^2}\times(3^5)^3$$

$$=5\times\frac{8^2}{3^4}\times 3^{15}$$

$$=5\times 8^2\times 3^{11}$$

$$=5\times 2^6\times 3^{11}$$

or $=5\times 4^3\times 3^{11}=56687040,$

when corrected $=\frac{1}{2}$ circumference $=56844504$

Pyramid $=\frac{1}{3}$ of $\frac{5}{8}$ cube of side of base

$$\frac{5}{8}\text{ cube}=\frac{5}{8}\times\left(\frac{8}{3}\right)^3=\frac{5}{8}\times\frac{8^3}{3^3}$$

$$=5\times\frac{4^3}{3^3}\text{ cubic stades}$$

$$=5\times\frac{4^3}{3^3}\times 243^3\text{ cubic units}$$

$$=5\times\frac{4^3}{3^3}\times 3^{15}$$

$$=5\times 4^3\times 3^{12}$$

Pyramid $=\frac{1}{3}=5\times 4^3\times 3^{11}$

Cube of side $=(\frac{8}{3})^3$ stade $=\frac{1}{4}$ distance of moon

Cube of 2 side $=\frac{16^3}{3^3}$ stade $=2$ distance of moon

$$=\frac{16^3}{3^3}\times 243^3\text{ units}$$

$$=\frac{16^3}{3^3}\times 3^{15}$$

$$=16^3\times 3^{12}=2^{12}\times 3^{12}=6^{12}$$

Cube of 2 side of base $=$ diameter of orbit of moon

$=6^{12}=19{\cdot}2$ circumference

Cylinder - $=15$

Sphere - $=10$

Cone - $=5$

Pyramid of Cheops : Cone :: 1 : 10

" : Sphere :: 1 : 20

" : Cylinder :: 1 : 30

Cube of side of base $=\frac{1}{4}$ distance of moon $=2\ 4$ circumference
Cylinder - - - $=\frac{15}{8}$
Sphere - - - $=\frac{10}{8}$
Cone - - - $=\frac{5}{8}$

Pyramid of Cheops : Cone :: 8 : 10
" : Sphere :: 8 : 20
" : Cylinder :: 8 : 30

Cone having height = diameter of base
= side of base of Cheops' pyramid will $=\frac{5}{4}$ pyramid
$=\frac{5}{4}\times\frac{1}{2}=\frac{5}{8}$ circumference.

Cone having $\frac{5}{8}$ height will

$$=\frac{5}{4}\times\frac{5}{8}=\frac{25}{32}=\frac{5^2}{2^5} \text{ pyramid}$$

$$=\frac{5^2}{2^5}\times\frac{1}{2}=\frac{5^2}{2^6}=\frac{5^2}{8^2} \text{ circumference.}$$

Height of cone $=\frac{5}{8}$ diameter of base
Content $=(\frac{5}{8})^2$ circumference.

Pyramid of Cheops : cone having diameter = side of base of pyramid of Cheops, and height = height of Cheops :: 32 : 25 :: 2^5 : 5^2.

Cone having same height as the last and diameter of base = diagonal of base of Cheops' pyramid will

$$=2\times\frac{5^2}{2^5}=\frac{5^2}{2^4} \text{ pyramid}$$

$$=\frac{5^2}{2^4}\times\frac{1}{2}=\frac{5^2}{2^5} \text{ circumference.}$$

Diameters being as 1 : $2^{\frac{1}{2}}$,
Cones are as 1 : 2,
Heights being equal.

Cone having height and diameter of base = diagonal of base of pyramid will

$$=\frac{5}{4}\times(2^{\frac{1}{2}})^3=5\times\frac{8^{\frac{1}{2}}}{16^{\frac{1}{2}}}=\frac{5}{2^{\frac{1}{2}}} \text{ pyramid}$$

$$=\frac{5}{2^{\frac{1}{2}}}\times\frac{1}{2}=\frac{5}{8^{\frac{1}{2}}} \text{ circumference.}$$

Cone having height and diameter of base = twice side of pyramid will

$$=\frac{5}{2^{\frac{1}{2}}}\times(2^{\frac{1}{2}})^3=5\times\frac{8^{\frac{1}{2}}}{2^{\frac{1}{2}}}=10 \text{ pyramid}$$

$$=10\times\tfrac{1}{2}=5 \text{ circumference.}$$

Pyramid having height and side of base = twice side of base of Cheops' pyramid will

$$=\tfrac{1}{3} \text{ twice distance of moon}$$
$$=\tfrac{1}{3}\ 19{\cdot}2=6{\cdot}4 \text{ circumference.}$$

$$\text{Cone} : \text{pyramid} :: 5 : 6{\cdot}4 :: 25 : 32$$
$$:: 5^2 : 2^5$$

:: 5 to second power : 2 to the fifth.

$$\text{Sphere} : \text{pyramid} :: 2\times5^2 : 2^5$$
$$\text{Cylinder} : \text{pyramid} :: 3\times5^2 : 2^5$$
$$\text{Cube} : \text{pyramid} :: 3 : 1.$$

The proportions are only proximate and will require correction.

Since cone : pyramid : : 25 : 31·82, &c.

The pyramids of Saccarah are numerous and of irregular formation, some towering aloft, others greatly decayed, some constructed of brick, and some of stone. Champollion considers the brick pyramids of more ancient date than those of stone.

There are several large pyramids at Saccarah and Dashour. The largest one at Saccarah is about 350 feet high, and has only four retreating steps or terraces.

The teocalli of Cholula has four terraces.

The pyramids of Djizeh, like those of Abousir, Saccarah, and Dashour, are placed at various distances from each other.

The multitude of pyramids scattered over the district of Saccarah, observes Denon, prove that this territory was the necropolis (city of the dead) to the south of Memphis, and that the village opposite to this, in which the pyramids of Djizeh are situated, was another necropolis, which formed the northern extremity of Memphis. The extent of the ancient city may thus be measured.

The remains of some of the kings of Egypt, who were sovereigns and pontiffs, may have been deposited within a pyramidal temple; as the remains of the popes, who were sovereigns and pontiffs, are still interred within the temple of St. Peter's at Rome.

Doubtless both in the old and new world, tumuli, which are but rude imitations of pyramids, have been raised as sacred memorials over the ashes of kings and chiefs.

The custom of depositing the remains of man in or near some sacred place is not confined to any country. Some Mahomedans carry a corpse a journey of many months to be deposited near a sacred shrine. The Hindoos carry their dead and dying great distances to the sacred Ganges.

The Moslem emperors have erected many splendid mausolea as monuments to their posthumous fame; as the Burra-Gombooz at Bejapore, which exceeds the dome of St. Paul's at London in diameter, and is only inferior to that of St. Peter's at Rome. It was constructed in the lifetime of the monarch, Mahomed Shah, and under his own auspices.

So the pyramids, like the modern cathedrals, may have been erected as temples, and used as mausolea. They were used as temples of worship and places of sacrifice when the Spaniards arrived in America; and remains of the dead have been found in some Mexican teocallis.

The hyperbolic temple still continues to be used in the Burmese empire as a place of worship, but not of sacrifice.

Bohlen mentions that the Burmese priests are embalmed exactly in the Egyptian fashion. The intestines are taken out of the body, the cavity of which is filled with spices, and the whole is protected from the external air by a covering of wax. The arms are then placed on the breast, the body is swathed in bandages varnished with gum, covered with gold leaf, and at the expiration of one year it is burned; the remains are then placed in a pyramidal-formed building.

The sepulchres of the Egyptian kings were not always in a remote and sequestered place, like the valley of Bidân-el-

Molouk, but even within the precincts of the temple. Thus all the Saite kings were buried near the temple of 'Athenæa, and within its enclosing wall. Here also was a tomb of Osiris.

When a Scythian king dies, says Herodotus, they smear his body all over with wax, after having opened it and taken out the intestines. The cavity is filled with chopped cypress, pounded aromatics, parsley, and aniseed, and then the incision is sewn up.

One of the pyramids at Dashour, according to Davidson, has a base, each side of which is 700 feet, a perpendicular height of 343 feet, and 154 steps. There is an entrance into the north side, which leads down by a long sloping passage, and then by a horizontal base to a large room, the upper part of which is constructed of stones of polished granite, each projecting six inches beyond that below, and thus forming in appearance pretty nearly a pointed arch.

Height = 343 feet, side of base = 700 feet.

Let the height to apex $= \frac{1}{2}$ the side of the base, and height to apex : side of base

$:: \frac{5}{4} : \frac{5}{2}$ stade :: 351·25 feet : 702·5 feet
:: 303·25 units : 607·5 units
:: 305 : 610 when corrected;
then height × base
$= 305$ &c. $\times 610^2 =$ circumference.

Pyramid $= \frac{1}{3}$ circumference = 120 degrees.

The reciprocal pyramid should $= \frac{1}{120}$ circumference = 3 degrees.

A pyramid representing $\frac{1}{3}$ circumference of the earth will have for the side of the base 705·39 feet, and height 352·69 feet.

The perimeter of the base will $= \frac{5}{2} \times 4 = 10$ stades = 60 plethrons, or = 60 plethrons + 10 units, when corrected.

The height will $= \frac{1}{8}$ perimeter.

$\frac{5}{4} \times (\frac{5}{4})^2$ stade $= \frac{1}{8}$ circumference
pyramid $= \frac{1}{24}$ „
$\frac{5}{4} \times (\frac{5}{2})^2$ stade = „
pyramid $= \frac{1}{3}$ „

These formulas will require to be corrected by the addition of unity to a stade.

Here we find the solution of the frequent recurrence of 5 stades and $\frac{5}{8}$ stade in the measurements of the sacred monuments in both hemispheres.

Five stades being a whole number was not so mysterious as the fraction $\frac{5}{8}$ stade, which has so often and unexpectedly crossed our path of inquiry.

Taking the measurements with the small correction, we have

$5 \times 2 = 10$ stades $= 60$, or 3 score plethrons
5 stades $= 30$, or $\frac{1}{2}$ of 3 score plethrons.

Thus a square base having a perimeter of 3 score plethrons or 10 stades, and height $= \frac{1}{2}$ the side will $=$ circumference of the earth in units, and pyramid $= \frac{1}{3}$ circumference.

A square base having a perimeter of $\frac{1}{2}$ of 3 score plethrons or 5 stades, and height $= \frac{1}{2}$ the side, will $= \frac{1}{8}$ circumference, and pyramid $= \frac{1}{24}$ circumference.

The cube of the side of base of pyramid $= 610^3 =$ twice circumference.

If perimeter of a square

$=$ 30 pleths., side $= 7\frac{1}{2} = 303{\cdot}75$ units, and $305^3 = \frac{1}{4}$ circum.
$=$ 60 ,, $= 15 = 607{\cdot}5$,, $610^3 = 2$,,
$=$ 120 ,, $= 30 = 1215$,, $1220^3 = 16$,,

Hence the cubes of 305, 610, 1220 units, which respectively $= \frac{1}{4}$, 2, 16 circumference, will have the perimeters of their bases somewhat greater than 30, 60, 120 plethrons.

The tower of Belus has the height $=$ the side of the base $= 1$ stade.

1×1^2 stade $= \frac{1}{8}$ circumference
pyramid $= \frac{1}{24}$,,

Here unity must be subtracted from a stade,

for 243×243^2 exceeds $\frac{1}{8}$ circumference
but 242 &c. $\times 242^2 = \frac{1}{8}$,,
and pyramid $= \frac{1}{24}$,,

So the formula for the tower will be the side of the base $= 1$ stade less unity, and height $=$ the side of the base.

Thus a square base having a perimeter of $\frac{1}{2}$ of 3 score plethrons less one stade, and height = the side, will = $\frac{1}{8}$ circumference, and pyramid = $\frac{1}{24}$ circumference.

Perimeter = 4 stades $=4 \times 6=24=30-6$ plethrons = $\frac{1}{2}$ of 3 score plethrons less 1 stade.

The Dashour pyramid has 154 steps.

The distance of the earth from the sun = 220 semi-diameters of the sun.

Distance of the earth = 400 distance of the moon
,, Venus = 281 ,, ,,
400 : 281 :: 220 : 154,

or distance of Venus = 154 semi-diameters of the sun.

There is a pyramid at Saccarah, the sides of which, on an average, are said to be about 656 feet, and the height 339 feet.

This is the pyramid which contains hieroglyphics in relief round the doorway of a small chamber.

Height = 339 feet, and side of base = 656 feet.

If the height = 338 feet = 292 units
and side of base = 654·5 feet = 567 units = $2\frac{1}{3}$ stade = 14 plethrons; then height × base
$= 292 \times 567^2 = \frac{5}{6}$ circumference = 300 degrees,
pyramid = $\frac{1}{3}$ of $\frac{5}{6}$ $=\frac{5}{18}$,, = 100 ,,

Perimeter of base $=4 \times 14=56$ plethrons
height = 7 ,, + $8\frac{1}{2}$ units.

The reciprocal of this pyramid will equal $\frac{1}{100}$ circumference = $\frac{18}{5}$ degrees.

This pyramid : the pyramid of Cephrenes :: $\frac{5}{18}$: $\frac{5}{12}$ circumference :: 100 : 150 degrees :: 2 : 3.

The Dashour pyramid : Cheops' pyramid :: $\frac{1}{3}$: $\frac{1}{2}$ circumference :: 120 : 180 degrees :: 2 : 3.

Six times the cube of the side of the base of the pyramid at Saccarah $=6 \times 566^3$ &c. = 9·55 circumference = distance of the moon from the earth.

Thus 6 cubes will = distance of the moon
150×6 ,, = ,, Mercury
$150^2 \times 6$,, = ,, Belus from the sun.

6 cubes of 566 = distance of the moon
3×6 or 18 cylinders, diameter 4×566 = dist. of Mercury.

$\frac{6}{8}$ or $\frac{3}{4}$ cube of 2×566 = distance of the moon
$\frac{3}{2}$ " " = diameter of orbit

$\frac{6}{8}$ or $\frac{3}{4}$ cylinder, diameter 16×566 = distance of the earth
$\frac{3}{2}$ " " " = diameter of orbit.

cube of side of base = $\frac{1}{6}$ distance of the moon
cube of perimeter = $\frac{64}{6}$
cube of 4 perimeters = $\frac{4096}{6}$
6 cubes = 4096
3 cubes = 2048
and distance of Jupiter = 2045.

Thus 3 cubes of 4 times perimeter of base = distance of Jupiter.

$$566^3 = \tfrac{1}{6} \text{ distance of moon}$$
$$(6 \times 566)^3 = \tfrac{1}{6} \times 6^3 = 36.$$

25 cubes of 6 times side of base
= 800 times distance of the moon
= diameter of the orbit of the earth.

Height × area of the base of the cased pyramid of Cheops = $\frac{1}{6}$ distance of the moon = cube of side of base of Saccarah pyramid.

There is another pyramid at Dashour that has a base line of 600 feet: at the height of 184 feet the plane of the side is changed, and a new plane of inclination completes the pyramid with a height of 250 feet more. The platform is 30 feet square. The entrance passage, which is on the north face, cuts the side of the pyramid at right angles; and as the inclination of the passage is 20 degrees, according to Jomard, it follows that the side of the pyramid makes an angle of 70 degrees with base.

In its present state the pyramid consists of 198 steps, 68 large steps from the ground to the angle, and 130 smaller ones from the angle to the top. *Fig.* 66. A.

The platform at the top is 30 feet square, so the height from the platform to the apex will be 15 feet, or very nearly.

Stated height to platform = 184 + 250 = 434 feet.

Therefore height from base to apex will = 434 + 15 = 449 feet.

DE, the bsse of the teocalli, = 600 feet.

The circumscribing triangle ABC having the height FA = 449 feet, and the sides AB, AC, drawn from the apex A touching the sides of the teocalli DG, EH, inclined 70 degrees at the height, FI = 189 feet, will have a base BC = 820 feet.

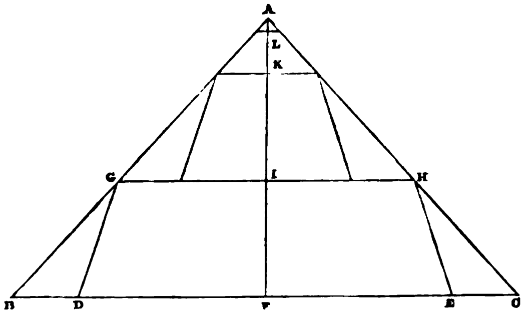


Fig. 66. A.

The stated height FL to platform = 184 + 250 = 434 feet.

Let us take 5 from 250, and add 5 to 184,

then 189 + 245 = 434 feet
or FI + IL = FL.

Thus the whole height FL to the platform will remain = 434 feet.

The height to apex will = FI + IL + LA = FA
= 189 + 245 + 15 = 449
= FI + AI = FA
= 189 + 260 = 449.

AF : AI :: BC : GH
449 : 260 :: 820 : 474 feet.
AF = 449 feet = 380 units = $(\frac{5}{4})^2$ stade
AI = 260 „ = 225 „
BC = 820 „ = 708·75 „ = $17\frac{1}{2}$ plethrons
GH = 474 „ = 410.

Height A F × base B C
$= 380 \times 706^2 = \frac{5}{3}$ circumference $= 600$ degrees
pyramid A B C $= \frac{1}{3}$ of $\frac{5}{3} = \frac{5}{9}$ „ $= 200$ „
$379^3 = \frac{1}{20}$ distance of the moon.

Height A I × base G H
$= 225$ &c. $\times 410^2 = \frac{1}{3}$ circumference $= 120$ degrees
pyramid A G H $= \frac{1}{3}$ of $\frac{1}{3} = \frac{1}{9}$ „ $= 40$ „
pyramid A B C − pyramid A G H = frustum G H B C
$\frac{5}{9} - \frac{1}{9} = \frac{4}{9}$ circumference
or $200 - 40 = 160$ degrees.

Pyramid A B C + frustum G H B C
$= 200 + 160 = 360$ degrees
= twice the pyramid of Cheops.

Pyramid A B C + pyramid A G H
$= 200 + 40 = 240$ degrees
= twice the other pyramid at Dashour.

Pyramid A B C = 200 degrees
= twice the pyramid at Saccarah.

The frustum G H B C, if completed, would be 10 degrees greater than the pyramid of Cephrenes, and 20 degrees less than the pyramid of Cheops.

The height to apex $2\frac{1}{2} \times \frac{5}{8} = \frac{25}{16} = (\frac{5}{4})^2$ stade $= 380$ units.

Perimeter of base $= 4 \times 17\frac{1}{2} = 70$ plethrons.

This pyramid will be to that of Cheops as 200 : 180 degrees :: 10 : 9; and to that of Cephrenes as 200 : 150 degrees :: $\frac{5}{9} : \frac{5}{12}$ circumference :: 4 : 3.

It may be remarked that the number of steps at present are 198, and the pyramid = 200 degrees.

The pyramid is built of a hard white stone, which contains fossils. Its sides face the cardinal points.

This structure, which is partly a pyramid and partly a teocalli, will explain how the pyramids were built.

The height to the apex $= 2\frac{1}{2} \times \frac{5}{8}$ or $5 \times \frac{3}{16}$ stade.

Suppose F K, the height from the base to the platform of a second terrace, $= 2 \times \frac{5}{8}$ or $\frac{5}{4}$ stade, then the height from a second platform to the apex will $= \frac{1}{2}$ of $\frac{5}{8} = \frac{1}{16}$ stade.

Or the height I A might be divided into any convenient number of terraces, and the stones raised from terrace to terrace till the teocalli was completed by building upwards. Then to give the structure the pyramidal form, the builders would begin at the highest platform and build up to the apex; then from the next platform in the descent they would build up the angular spaces so that the pyramidal part would be completed from the apex down to the second terrace, and so in succession till they had finished the pyramid from the apex to the base G H, by building downwards — as they had completed the teocalli by building upwards. At G H, when $\frac{1}{5}$ the pyramid was finished, the building ceased, and the remaining $\frac{4}{5}$ was left incomplete, which might probably have been the original intention, for the structure combines the teocalli with the pyramid and different proportions of the earth's circumference.

This mode of building is in accordance with the method described by Herodotus, who says, "the pyramid of Cheops was first built in form of steps or little altars. When they had finished the first range they carried stones up thither by a machine; from thence the stones were moved by another machine to the second range, where there was another to receive them, for there were as many machines as ranges or steps." Others say they transferred the same machine to each range. Both accounts have been related to us.

The upper part of the pyramid was first finished, then the next part, and last of all the part nearest the ground.

These are all the Egyptian pyramids of which we have found any stated measurements.

If the side of base BC = 713 units, then $3 \times 713^3 = 9{\cdot}55$ circumference = distance of the moon.

Thus 3 times the cube of the base BC will = distance of the moon from the earth.

150 times 3 cubes will = distance of Mercury.

150^2 times 3 cubes will = distance of Belus from the sun.

Should the side of the base BC = 713 units, it will exceed 706 by 7 units; so that, if the side of the base be increased, the height must be diminished in order that the pyramid ABC may equal $\frac{5}{9}$ circumference, or 200 degrees.

Four times the cube of the side of the base of the pyramid of Cheops = the distance of the moon.

Hence the great Dashour cube will be to the Cheops' cube as 4 : 3.

The cube of the side of this great Dashour pyramid = $\frac{1}{3}$ distance of the moon.

The other Dashour pyramid, which has 164 steps, = $\frac{1}{3}$ circumference of the earth.

So 3 cubes = distance of the moon,
and 3 pyramids = circumference of the earth.

Side of base DE = 600 feet = 519 units

$521^3 = \frac{5}{4}$ circumference

$514^3 = \frac{1}{8}$ distance of the moon.

Cube of 3 times side of base BC = $(3 \times 713)^3 = 3^3 \times \frac{1}{3}$ = 9 times distance of the moon.

Let BC = 713 units. (*Fig.* 66. B.)
GH = 414
AF = 374
DE = 521

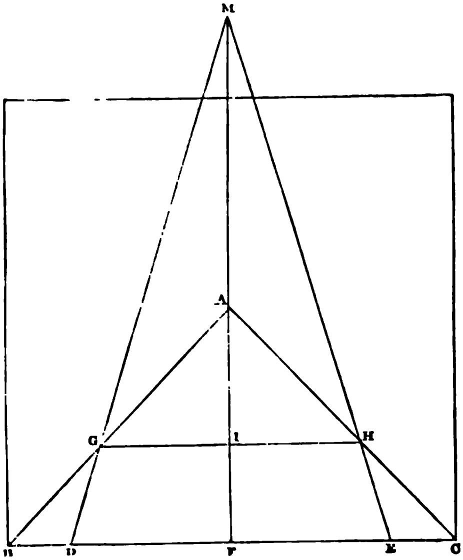


Fig. 66. B.

Height AF × base BC
$= 374 \times 713^2 = \frac{5}{3}$ circumference.

Pyramid ABC $= \frac{1}{3}$ of $\frac{5}{3} = \frac{5}{9}$ circumference $= 200$ degrees.

BC$^3 = 713^3 = \frac{1}{3}$ distance of the moon.

So 3 times the cube of BC = distance of the moon.

The cube of the side GH $= 414^3$ &c. $= \frac{5}{8}$ circumference.

A pyramid $= \frac{1}{3}$ of $\frac{5}{8} = \frac{5}{24}$ circumference $= 5$ times the pyramid of Belus.

The cube of the side DE $= 521^3$ &c. $= \frac{5}{4}$ circumference.

A pyramid $= \frac{1}{3}$ of $\frac{5}{4} = \frac{5}{12}$ = pyramid of Cephrenes.

Thus the cube of the side DE is double the cube of the side GH.

The cube of twice the side GH $= (2 \times 414)^3 = 5$ circumf.
The cube of perim. of base GH $= 40$ „
The cube of twice the side DE $= (2 \times 521)^3 = 10$ „
The cube of perim. of base DE = $= 80$ „

In order that GH = 414 may be within the circumscribing triangle ABC, the height FI will be somewhat less than 158 units, since the height to the apex AF is less than 380 units by 6.

Let AI = 221 units;

Height AI × base GH $= 221 \times 414^2 = \frac{1}{3}$ circumference.

Pyramid AGH $= \frac{1}{3}$ of $\frac{1}{3} = \frac{1}{9}$ circumference $= 40$ degrees.

FI = AF − AI $= 374 - 221 = 153$ units.

This great Dashour pyramid $= \frac{5}{9}$ circumference.

A pyramid having the same base and height = side of base $= \frac{1}{3}$ the cube $= \frac{1}{9}$ of 3 cubes $= \frac{1}{9}$ distance of the moon.

The pyramid AGH $= \frac{1}{9}$ circumference.

3 times the cube of the side of base BC
= distance of the moon from the earth.

150 times the distance of the moon
= distance of Mercury.

150 times the distance of Mercury
= distance of Belus.

This great pyramid of Dashour : 3 times the cube of the side of the base BC :: $\frac{5}{9}$ circumference : distance of moon
:: $\frac{5}{9}$ circumference : 9·55 circumference.

Pyramid AGH : pyramid $= \frac{1}{3}$ cube of BC :: $\frac{1}{9}$ circumference : $\frac{1}{9}$ distance of the moon.

If the sides DG, EH be produced till they meet at M, it will be found that MF = 838 units.

Height MF × base DE
$= 838 \times 521^2 = 2$ circumference.
Pyramid MDE $= \frac{1}{3}$ of $2 = \frac{2}{3}$ circumference.

Thus 3 times the pyramid MDE = 2 circumference, and 3 times the cube of BC = distance of the moon.

Pyramid ABC + pyramid AGH
$= \frac{5}{9} + \frac{1}{9} = \frac{6}{9} = \frac{2}{3}$ circumference;
∴ pyramid ABC + pyramid AGH = pyramid MDE.

The $2^{\frac{1}{3}}$ is a quantity impossible to express in numbers; but all the ordinates, as GH, DE, ∝ MF, the distance from M, and continually increase from 0 at M to 414 &c., or 1 at GH to 521 &c., or $2^{\frac{1}{3}}$, which is represented by the line DE.

$$1{\cdot}26^3 = 2{\cdot}000376.$$

So the cube root of 2 will be less than 1·26, or DE will be to GH in a less proportion than 1·26 : 1.

$\text{GH}^3 : \text{DE}^3 :: 1 : 2$
$\text{DE}^3 : (2\,\text{GH})^3 :: 1 : 4$
$\text{GH}^3 : (2\,\text{GH})^3 :: 1 : 8$
$\text{GH}^3 : (2\,\text{DE})^3 :: 1 : 16$
$\text{DE}^3 : (4\,\text{GH})^3 :: 1 : 32$
$\text{DE}^3 : (4\,\text{DE})^3 :: 1 : 64$
$\text{DE}^3 = \frac{5}{4}$ circumference.

The cube of perimeter $= (4\,\text{DE})^3 = \frac{5}{4} \times 64 = 80$ circumf.

$18 \times 80 = 1440$ circumference = distance of Mercury.

The 2 sides of the bases of the pyramids ABC, AGH, are BC, GH.

Their sum = BC + GH = 713 + 414 = 1127 units.

Cylinder having height = diameter of base
$1131 = 1131^3 \times {\cdot}7854 = 10$ circumference
1130^3 &c. $= \frac{4}{3}$ distance of the moon.

The difference of the sides = BC − GH = 713 − 414 = 299 units; and $300{\cdot}5^3 = \frac{1}{40}$ distance of the moon.

For the cube of Cephrenes $= 601^3 = \frac{1}{5}$ distance of moon.

So the cube of $300{\cdot}5 = \frac{1}{8}\,601^3 = \frac{1}{8}$ of $\frac{1}{5} = \frac{1}{40}$ distance of the moon.

The cube of the side DE + the cube of the side GH
$= \frac{5}{4} + \frac{5}{8}$ circumference $= 521^3$ &c. + 414^3 &c.

Sum of the two sides = DE + GH = 521 + 414 &c. = 935; $934^3 = \frac{3}{4}$ distance of the moon.

Cylinder having height = diameter of base 2 BC, 2 × 713 units, will = 20 circumference
= double the cylinder diameter BC + GH, 1131 units.

The cube, sphere, and cone, diameter 2 BC will be double the cube, sphere, and cone, diameter BC + GH.

72 cylinders diam. 2 BC = 1440 circumf. = dist. of Mercury
72 × 150 - - - - = Belus
9 cylinders diam. 4 BC - = Mercury
9 × 150 - - - - = Belus

Hence 3 cubes of BC - = distance of the moon
and 9 cylinders, diameter 4 BC = „ Mercury
5 cubes of side of base of Cephrenes = „ moon
and 3 × 5 = 15 cylinders, diam. = perimeter of base
= distance of Mercury
4 cubes of side of Cheops' - = „ moon
and 3 × 4 = 12 cylinders, diam. = perimeter
will = distance of Mercury
3 cubes of the side of the great Dashour pyramid
= distance of moon
and 3 × 3 = 9 cylinders, diam. = perimeter
will = distance of Mercury.

Since 3 cubes of BC = distance of moon,
so 3 × 3 or 9 cylinders, diameter 4 BC = distance of Mercury.
Also $\frac{9}{8}$ cylinder, diameter 8 BC = „ „
$\frac{24}{8} = 3$ - - - = „ earth.
Thus 3 cubes of BC - = „ moon
3 cylinders, diameter 8 BC = „ earth
3 × 3 or 9 - 4 BC = „ Mercury
So $\frac{3}{8}$ cube of 2 BC - = „ moon
$\frac{3}{4}$ cube - - - = 2 „ „
= diameter of orbit of the moon.

$\frac{3}{8}$ cylinder, diam. 16 BC - = distance of the earth
$\frac{3}{4}$ cylinder - - = 2 „ „
= diameter of orbit of the earth.

Cube of side of base - = $\frac{1}{3}$ distance of moon
Cube of perimeter - = $\frac{64}{3}$ „ „
Cube of 4 perimeters - = $\frac{4096}{3}$ „ „
3 cubes of 4 times perimeter = 4096 „ „
= 2 × 2048
and 2 × 2045
= 2 distance of Jupiter.

Thus 3 cubes = diameter of the orbit of Jupiter.

If the cubes of the sides of the base of the Saccarah and Dashour pyramids be $566^3 : 713^3$
as $\frac{1}{6} : \frac{1}{3}$ distance of moon
$1 : 2$.

Then if a = side of base, and b = height of Dashour
and c = „ d = „ Saccarah

$$a^3 \text{ will} = 2c^3$$
$$a = 2^{\frac{1}{3}}c$$
$$\text{if } b^3 = 2d^3$$
$$b = 2^{\frac{1}{3}}d$$
$$\text{then } a^2 \times b = (2^{\frac{1}{3}}c)^2 \times 2^{\frac{1}{3}}d = 2^{\frac{2}{3}}c^2 \times 2^{\frac{1}{3}}d = 2c^2d$$
$$\therefore \text{ pyramid } a^2 \times b = 2 \text{ pyramid } c^2d;$$
$$\text{but } b = 2^{\frac{1}{3}}d$$
$$b^3 = 2d^3.$$

Hence when the cubes of the sides of base are as $1 : 2$, and contents as $1 : 2$, the cubes of their heights will be as $1 : 2$, and cubes of hypothenuses as $1 : 2$.

$$\text{For hypothenuse}^2 = (a^2 + b^2)$$
$$\text{hypothenuse} = (a^2 + b^2)^{\frac{1}{2}}$$
$$\text{hypothenuse}^3 = (a^2 + b^2)^{\frac{3}{2}}$$
$$\text{or hypothenuse}^3 = (\text{sum of squares of 2 sides})^{\frac{3}{2}}$$
$$= (a^2 + b^2)^{\frac{3}{2}}$$
$$= \left((2^{\frac{1}{3}}c)^2 + (2^{\frac{1}{3}}d)^2\right)^{\frac{3}{2}}$$
$$= \left(2^{\frac{2}{3}} \times (c^2 + d^2)\right)^{\frac{3}{2}}$$
$$= 2 \times (c^2 + d^2)^{\frac{3}{2}}$$

Thus cube of hypothenuse of the greater triangle = twice cube of hypothenuse of the less triangle.

Hence it appears, by similar triangles, that when the sides of a right-angled triangle are double the sides of another, each to each, then the hypothenuse of the greater triangle will be double the hypothenuse of the less triangle.

When the squares of the sides are double, each to each, then the square of the hypothenuse of the greater triangle will be double the square of the hypothenuse of the less triangle.

When the cubes of the sides are double, each to each, then the cube of the hypothenuse of the greater triangle will be double the cube of the hypothenuse of the less triangle.

Hence height × base of Saccarah pyramid

$$= 293 \times 566^2, \text{\&c.} = \tfrac{5}{6} \text{ circumference}$$

Pyramid $= \tfrac{5}{18}$.

Height × base of Dashour pyramid

$$= 370 = 713^2 = \tfrac{5}{3} \text{ circumference}$$

Pyramid $= \tfrac{5}{9}$.

Cubes of heights are as

$$293^3 : 270^3 :: \tfrac{2}{9} : \tfrac{4}{9} \text{ circumference}$$
$$:: 1 : 2.$$

Cubes of sides of bases are as

$$566^3 : 713^3 :: \tfrac{1}{6} : \tfrac{1}{3} \text{ distance of moon}$$
$$:: 1 : 2.$$

Contents as

$$\tfrac{5}{18} : \tfrac{5}{9} \text{ circumference}$$
$$:: 1 : 2.$$

Cubes of hypothenuses are as

$$637^3 : 802^3 :: 1 : 2.$$

The Nubian pyramids are said to be about 80 in number, but generally of small dimensions. Some have propyla in front of one side. One portico is sculptured, and has an arched roof constructed with a keystone; the whole curve consists of five stones. There is an arched portico, similarly constructed, at Jebel Barkal, near the Nile, where there are also pyramids with propyla in front of them. There are also pyramids at Nouvri, a few miles north of Jebel Barkal. Waddington describes the largest as containing within it another pyramid of a different date, stone, and architecture. The inner is seen from a part of the outer one having fallen off. The base line of this pyramid is 159 feet (48·5 metres), according to Cailliaud, or 152 feet, according to Waddington, who states the height at 103 feet 7 inches.

Taking Caillaud's base and Waddington's height, we have

$$\begin{aligned} \text{height} &= 103{\cdot}6 \text{ feet,} \\ \text{say} &= 105 \text{ feet} = 91 \text{ units} = \tfrac{3}{8} \text{ stade,} \\ \text{side of base} &= 159 \text{ feet} = 137{\cdot}5 \text{ units.} \end{aligned}$$

Then $\frac{1}{3}$ height × area base

$$\begin{aligned} &= \text{content pyramid} = \tfrac{1}{3}\, 91 \times \overline{137}^2, \text{ \&c.} \\ &= \tfrac{1}{200} \text{ circumference of earth.} \end{aligned}$$

Or the content of such a pyramid will

$$= \tfrac{1}{100} \text{ of that of Cheops.}$$

$\frac{5}{8}$ stade is associated negatively with the height of this pyramid which

$$= 1 - \tfrac{5}{8} = \tfrac{3}{8} \text{ stade.}$$

We have not met with the dimensions of any other Nubian pyramid.

Pyramid $= \frac{1}{200}$ circumference $= \frac{9}{5}$ degrees.

Great Dashour pyramid $= \frac{5}{9}$ circumference $= 200$ degrees.

Consequently the pyramid at Nouvri is the reciprocal of the Great Dashour pyramid.

Most of the Nubian pyramids have not their sides placed opposite to the four cardinal points. None of them appears to have been entered.

The following description of the Nubian pyramids is extracted from "Egypt and Mehemet Ali."

"Two groups of pyramids stand near Djebel-Birkel, in Nubia; one contains only a few pyramids, but the other has twice as many in good condition. Among the former is one that has almost entirely fallen in, which is larger and different in form from the others; and it appears to be of a more remote age. The others, 17 in number, vary considerably in style from the Egyptian pyramids, but they are certainly not older, nor, indeed, are they very old. In fact, they look as smooth and uninjured as if they had been but just completed. I ascended one of them,—which may be done without difficulty, because each layer of stones forms a convenient step, and only the four corners, from top to bottom, are covered with a polished, rounded stone moulding,—and found on the summit a square wooden beam fixed in the wall, which had come to light by the falling of a stone, and, though thereby exposed to the wind and weather, was still as sound as if new.

"None of these pyramids are above 80 feet high, and they are comparatively smaller at the base than the Egyptian pyramids, and more tapering.

"Only a few of these pyramids had sculptures, which were softer and more voluptuous than the Egyptian style admits; one of these high reliefs represented a queen seated on a throne, the pedestal of which consisted of lions, with a rich covering thrown over them.

"I consider the majority of the pyramids of Nour to be the most ancient of all the Ethiopian monuments now extant. They are not so taper as the pyramids of Birkel, and consequently more nearly resembling the Egyptian; neither has any of them the peculiar projecting entrance of those at Birkel, nor do the layers of stone form steps by which to ascend them. On the whole the remains of rather more than forty may be distinguished, but only sixteen of them are in tolerable preservation, and even these are much injured by the weather, and in a dilapidated state. They are built entirely of rough-hewn sandstone and a kind of ferruginous pudding-stone, cemented with earth, and many of them appear to have been tumuli of mould, afterwards covered with stones. The nature of the circumjacent ground affords reason to conjecture that not only all these pyramids were encompassed by a canal communicating with the Nile, but even that several others traversed the place on which they stand. One of these monuments exceeds all its companions in extent, and its outer sides are so broken and shattered that we had no difficulty in ascending its summit. The form of this singular structure differs entirely from those that surround it; and it appears to have consisted of several stories, of various degrees of steepness. The entire height of this truncated pyramid, as it now stands, is nearly 100 feet, and its circumference about four times that extent."

A pyramid having height = side of base
= 113 feet = 98 &c. units
would = 1 degree, or $\frac{1}{360}$ circumference;
$2\frac{1}{2}$ plethrons less $2\frac{1}{2}$ units
$= 101{\cdot}5 - 2{\cdot}5 = 98{\cdot}75$ units.

Hence a pyramid having the height = side of base = $\frac{5}{2}$ plethron less $\frac{5}{2}$ unit, will = a degree nearly, or = $\frac{1}{3}\,\overline{98}^3$ &c. units.

If the height to the platform of this teocalli were 100 feet, then 13 feet would be the height of the apex of this hypothetical pyramid above the platform of the teocalli.

As we know of no complete dimensions of any of the small pyramids, we have given this, as an example among other similar cases, to show the general application of the method of calculation for ascertaining the contents of such pyramids in terms of the cubic unit, or circumference of the earth.

We only suppose this method of calculation to be applicable to some of the small pyramids—for pyramids and obelisks continued to be erected ages after their geometrical principles of construction were lost.

Again, if the height 113 feet were divided into 9 equal parts, like the tower of Belus, then the height of each terrace would = 113 ÷ 9 = 12·55 feet. So that the height of the apex above the platform would = 12·55 feet
= 11 units.

The contents of the two pyramids will be as $113^3 : 281^3$
$:: 1 : 15$ degrees
$:: \frac{1}{360} : \frac{1}{24}$ circumference.

The height of the tower of Belus = side base = 1 stade
= 281 feet.

Lepsius reckons 69 Egyptian pyramids in the vicinity of Memphis, all within a line of 56 miles, and 139 at and near Meröe, in Upper Nubia.

			SIDE OF BASE.
80 pyramids at	Meröe	sandstone,	60 to 20 feet
42 ,,	Noori	,,	100 to 20 ,,
17 ,,	Gabel Birkel	,,	80 to 23 ,,

The arch, both round and pointed, is coeval with the era of these last pyramids.

Gliddon remarks that the style of Egyptian architecture was grand and chaste, while the column now termed Doric, and attributed to the Greeks, was in common use in the reign of Osortasen, which precedes the Dorians by 1000 years.

The arch, both round and pointed, with its perfect keystone, in brick and in stone, was well known to the Egyptians long before this period; so that the untenable assertion, that the most ancient arch is that of the Cloaca Maxima at Rome falls to the ground.

In architecture, as in everything else, the Greeks and Romans obtained their knowledge from their original sources in Egypt, where still existing ruins attest priority of invention 1000 years before Greece, and 1500 years before Rome.

These topics are now beyond dispute, and may be found in the pages of the Champollion school. Until the last few years they were utterly unknown to history.

It is by these chronicles, or "foolish things," as Josephus calls the enduring pyramids, that the scientific claims of the ancients have been transmitted to posterity, ages after every other record had perished.

These monumental records of science and skill have been found in all parts of the world, constructed by colonies, combining a priesthood with the learning and science of an early age. These colonies may have been founded by some of the great Cyclopian family, known by the various designations of Shepherd Kings of Egypt, the Anakim of Syria, the Oscans of Etruria, and the Pelasgians of Greece. Such were the wandering masons, who appear to have been both architects and civil engineers, — to have travelled round the world, building cities, erecting temples for worship, and constructing canals for irrigation and commerce; thus making the barren land fruitful, and, at the same time, facilitating the transport of the productions of the soil, and so promoting the temporal welfare of man; while the priests or magi administered to his spiritual wants, and controlled him by laws which they made and enforced.

We have applied the Babylonian standard to the measurements of the numerous passages, chambers, and sarcophagi within the pyramids of Gizeh, made by Colonel Vyse, where the cubes represent planetary distances. But the numerous instances already given may be sufficient to show the mode of application, and the importance of accurate measurements of ancient monuments, designed by the builders as permanent records of the astronomical knowledge of a race unknown when history began.

PART VI.

AMERICAN TEOCALLIS. — MYTHOLOGY OF MEXICO BEFORE THE ARRIVAL OF THE SPANIARDS. — TEOCALLIS OF CHOLULA, SUN, MOON, MEXITLI. — THEIR MAGNITUDES COMPARED WITH THE TEOCALLIS OF PACHACAMAC, BELUS, CHEOPS, THE PYRAMIDS OF MYCERINUS AND CHEOPS' DAUGHTER, AND SILBURY HILL, THE CONICAL HILL AT AVEBURY. — THE INTERNAL AND EXTERNAL PYRAMIDS OF THE TOWER OF BELUS.—HILL OF XOCHICALCO.—TEOCALLI OF PACHACAMAC IN PERU. — RUINS OF AN AZTEC CITY. — THE BABYLONIAN BROAD ARROW. — THE MEXICAN FORMED LIKE THE EGYPTIAN ARCH. — DRUIDICAL REMAINS IN ENGLAND. — THOSE IN CUMBERLAND, AT CARROCK FELL, SALKELD, BLACK-COMB. — THOSE IN WILTSHIRE, AT WEST KENNET, AVEBURY, STONEHENGE. — EXTERNAL AND INTERNAL CONE OF SILBURY HILL. — MOUNT BARKAL IN UPPER NUBIA. — ASSYRIAN MOUND OF KOYUNJIK AT NINEVEH. — RECTANGULAR ENCLOSURE AT MEDINET-ABOU, THEBES. — THE CIRCLES AT AVEBURY. — CONICAL HILL AT QUITO, IN PERU. — TOMB OF ALYATTES, IN LYDIA. — CONICAL HILL AT SARDIS. — STONEHENGE CIRCLES AND AVENUE, CONICAL BARROWS. — OLD SARUM IN WILTSHIRE, CONICAL HILL. — THE CIRCLE OF STONES CALLED ARBE LOWES IN DERBYSHIRE. — CIRCLE AT HATHERSAGE, AT GRANED TOR, AT CASTLE RING, AT STANTON MOOR, AT BANBURY, IN BERKSHIRE. — HILL OF TARA. — KIST-VAEN. — STONES HELD SACRED.

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AMERICAN TEOCALLIS.

(*Described by Humboldt.*)

"AMONG the tribes of people who, from the seventh to the twelfth century of our era, appeared successively in the country of Mexico, five are enumerated,—the Toltèques, Cicimèque, Acolhues, Tlascalteques, and Aztèques, who, though politically divided, spoke the same language, observed the same worship, and constructed pyramidal edifices, which they regarded as the teocallis, or the houses of their gods. These edifices, though of dimensions very different, had all the same form; they were pyramids of several stories, the sides of which were placed exactly in the direction of the meridian and parallel of the place. The teocalli rose from the middle of a vast square enclosure surrounded by a wall. This enclosure, which one may compare to the *περίβολος* of the Greeks, contained gardens, fountains, habitations for the priests, and sometimes even magazines of arms; for each house of a Mexican god, like the ancient temple of Baal Berith, burned by Abimelech, was a place of strength. A great staircase led to the top of the truncated pyramid. On the summit of this platform were one or two chapels in the form of towers, which contained colossal idols of the divinity to whom the teocalli was dedicated. This part of the edifice ought to be regarded as the most essential; it was the *ναος*, or rather the *σηκος* of Grecian temples. It was there that the priests kept up the sacred fire. By the peculiar arrangement of the edifice, as we have just shown, the sacrificer could be seen by a great mass of people at the same time. One saw from a distance the procession of the teopixqui, as it ascended or descended the staircase of the pyramid. The interior of the edifice served as a sepulchre for the kings and principal personages of Mexico. It is impossible to read the descriptions which Herodotus and Diodorus Siculus have left of the temple of Jupiter Belus, without being struck with the features of resemblance which the Babylonian monument presents when compared with the teocallis of Anahuac.

When the Mexicans, or Aztèques, one of the seven tribes of the Anahuatlacs (bordering people), arrived in the year 1190, in the equinoctial region of New Spain, they found there the pyramidal monuments of Teotihuacan, Cholula or Chołollan, and Papantla, already erected. They attributed these great works to the Toltèques, a powerful and civilised nation that inhabited Mexico 500 years before. They made use of hieroglyphical writing, and had a year and a chronology more accurate than most of the people of the ancient continent. The Aztèques did not know for a certainty if other tribes had inhabited the country of Anahuac before the Toltèques. In regarding these houses of the god of the Téotihuacan and Chalollan as the work of the latter people, they assigned to them the highest antiquity of which they could form an idea. It might, however, be possible that they were erected before the invasion of the Toltèques, that is, about the year 648 of the common era. We should not be astonished that the history of any American people did not commence before the seventh century, and that the history of the Toltèques should be also as uncertain as that of the Pelasgians or Ausonians. The deeply read M. Schlœzer has proved almost to evidence that the history of the north of Europe does not ascend beyond the tenth century, an epoch when the Mexican plane already presented a civilisation much further advanced than that of Denmark, Sweden, or Russia.

The teocalli of Mexico was dedicated to Tezcatlipoca, the first of the Azteque divinities after Teotl, who was the supreme and invisible Being, and to Huitzilopochtli, the god of War. It was erected by the Aztèques after the model of the pyramids of Teotihuacan, only six years before the discovery of America by Christopher Columbus. This truncated pyramid, called by Cortez the principal temple, had a base 97 metres long, and about 54 metres high. It is not surprising that a building of these dimensions should have been destroyed in so short a time after the siege of Mexico. In Egypt there remains scarcely any vestige of the enormous pyramids that rose from the middle of Lake Mœris, and which Herodotus says were ornamented with colossal statues. The pyramids of Porsenna, of which the description appears somewhat fabulous, had statues, according to Varro, more than 80 metres high; these also have disappeared from Etruria.

But if the European conquerors have overthrown the Aztèque teocallis, they have not equally succeeded in destroying the more ancient monuments, those which are attributed to the Toltèque nation. We shall now give a short description of these monuments, remarkable for their form and magnitude.

The group of Teotihuacan pyramids stands in the valley of Mexico, eight leagues distant and north-east of the capital, on the plain called Micoatl, or path of the dead. One still observes two great pyramids dedicated to the sun (tonatiuh) and the moon (metzitli), and surrounded by some hundreds of small pyramids forming streets running exactly from north to south and from east to west. One of the two great teocallis has 55, the other 44 metres perpendicular elevation. The base of the first is 208 metres long; whence it results that the Tonatiuh Yztaqual, from the measurements of M. Oteyza, made in 1803, is more elevated than Mycerinus, or the third of the great pyramids of Djizeh in Egypt, and the length of its base is nearly that of Cephrenes. The small pyramids that surround the great houses of the sun and moon have scarcely 9 or 10 metres of elevation. According to the tradition of the natives, they served as sepulchres for the chiefs of the tribes. Around those of Cheops and Mycerinus in Egypt are also seen eight small pyramids placed symmetrically and parallel to the sides of the great ones. The two teocallis of Teotihuacan had four principal stories; each of these was subdivided in small steps, of which the edges may still be distinguished. The middle is clay mixed with small stones; it is covered with a thick wall of porous amygdaloid. This construction recalls to mind one of the Egyptian pyramids at Saccarah, which has six stories, and which, according to Pococke, is a mass of stone and yellow mortar, covered externally with rough stones. At the top of the great Mexican teocallis were placed two colossal statues of the sun and moon. They were of stone and covered with plates of gold; these plates were carried away by the soldiers of Cortez. While the Bishop Zumaraga, a Franciscan monk, undertook to destroy all that related to the religion, history, or antiquities of the indigenous people of America, he also broke the idols in the plain of Micoatl. There may still be seen the remains of a staircase, formed of large hewn stones, which formerly led to the platform of the teocalli.

To the east of the group of pyramids of Teotihuacan, in descending the Cordilleras near the Gulf of Mexico, in a thick forest called Tajin, rises the pyramid of Papantla. Its discovery was accidentally made by some Spanish hunters about thirty years ago; for the Indians contrive to conceal from the whites every object of ancient veneration. The form of this teocalli, which has six, or perhaps seven, stories, is more tapering than that of any of the other monuments of this kind. Its height is about 18 metres, while the length of its base is only 25; it is consequently lower by almost one half than the pyramid of Caius Cestius at Rome, which is 33 metres high. This little edifice is constructed of hewn stones of an extraordinary size, very finely and regularly cut. Three staircases lead to the top. The coating of these stories is ornamented with hieroglyphical sculpture, and small niches are symmetrically disposed. The number of these niches appear to allude to the 318 signs simple, and composed of the days of Cempohualilhuitl, or calendar civil of the Tolteques.

The greatest, the most ancient, and most celebrated of all the pyramidal monuments of Anahuac is the teocalli of Cholula. At this day it is called the mountain made by the hands of man. When seen at a distance, one is tempted to take it for a natural hill covered with vegetation.

Cortez described Cholula as being more beautiful than any city in Spain, and well fortified. From a mosque (teocalli) he reckoned more than 400 towers. Humboldt reckoned the number of inhabitants, when he visited it, at 16,000. Since then Bullock has estimated them at 6000 only.

The plane of Cholula is 2200 metres above the level of the sea. At a distance is seen the summit of the volcanic Orizaba covered with snow. This colossal mountain is 5285 metres in height, from the sea.

The teocalli of Cholula has four platforms of equal height, and its sides appear to have been placed with great exactness opposite the cardinal points of the compass; but as the angles are not very well defined, it is difficult to discover with correctness their exact original direction. This pyramidal monument has a more extended base than any other edifice of the same description found in the old continent. I have measured it with care, and am satisfied that its perpendicular height is not more than fifty-four metres, and that each side of its base is 439 metres in length.

Bernal Diaz del Castillo, a private soldier in the expedition of Cortez, amused himself in counting the number of steps in the staircases, which led to the platforms of the different teocallis; he found 114 in the great temple of Tenochtitlam, 117 in that of Tescuco, and 120 at Cholula. The base of the pyramid at Cholula is twice as large as that of Cheops, in Egypt, but its height is very little greater than that of Mycerinus.

In comparing the dimensions of the temple of the sun, at Teotihuacan, with those of the pyramid at Cholula, one sees that the people who constructed these remarkable monuments had the intention of making them all of the same height, but with bases of which the lengths should be in the proportion of one to two. As to the proportion between the base and height, one finds it very different in different monuments. In the three great pyramids of Djizeh, their heights are to their bases as 1 : 1·7; in the pyramid of Papantla, covered with hieroglyphics, this proportion is as 1 : 1·4; in the great pyramid of Teotihuacan, as 1 : 3·7; and in that of Cholula as 1 : 7·8. This last monument is built with unburned bricks alternating with layers of clay. The Indians of Cholula assured me that the interior is hollow, and that while Cortez occupied their town, their ancestors had concealed within it a number of warriors, with the intention of making a sudden attack on the Spaniards; but the materials of which the teocalli is constructed, and the silence of contemporary historians, render this assertion but little probable. However it cannot be doubted but that there were in the interior of this pyramid, as in other teocallis, considerable cavities which served for sepulchres; the discovery of them was owing to accident seven or eight years ago; the route from Puebla to Mexico, which formerly passed by the north of the pyramid, was changed, and in forming the new road they cut through the first platform, so that an eighth part of it remains isolated, like a heap of bricks. In making this cut they found in the interior a square house, formed of stones and supported by props of cypress; it contained two bodies, idols formed of basalt, and a great number of vases skilfully painted and enamelled. No care was taken to preserve these objects; but it is said to have been carefully ascertained that this chamber had no outlet. In supposing this pyramid not to have been built by the Tolteques, the first inhabitants of Cholula, but by prisoners made by the Cholulains, one might believe that these were the bodies of unfortunate slaves that had been caused to perish intentionally in the interior of the teocalli. We examined the ruins of this subterraneous chamber, and observed a particular arrangement of bricks, tending to diminish the pressure on the roof. The natives being ignorant of the arch, placed very large bricks horizontally, so that the upper course should pass beyond the lower; hence resulted an assemblage of steps, which supplied in a measure the Gothic arch. Similar vestiges of this rude substitute for the arch have been found in several Egyptian edifices.

It would be interesting to excavate a gallery through the centre of the teocalli of Cholula, to examine its internal construction; and it is astonishing that the desire to discover hidden treasures has not already caused an attempt to be made. During my travels in Peru, in visiting the vast ruins of the city of Chimù, near Mansiche, I entered the interior of the famous Huaca of Toledo, the tomb of a Peruvian prince, in which Garci Gutierez of Toledo discovered, while digging a gallery, in 1576, more than the value of five millions of francs (about 208,333*l.* sterling), in solid gold; this is proved by accounts preserved in the town-hall of Truxillo.

The great teocalli of Cholula, called also the mountain of unburned bricks (Tlalchihualtepec), had on its summit an altar dedicated to Quetzalcoatl, the god of the air. This Quetzalcoatl (a name signifying serpent covered with green feathers, from coatl, serpent, and quetzalli, green feather) is without doubt the being the most mysterious of all the

Mexican mythology: this was a white man with a beard like the Bochica of the Muyscas, of whom we have already spoken: he was chief priest to Tula, the lawgiver, the chief of a religious sect who, like the Sonyasis and the Buddhists of Hindostan, imposed upon themselves penances the most cruel; he introduced the custom of piercing the lips and ears, and wounding the rest of the body with thorns of the aloe, or the prickles of the cactus, and introduced reeds into the wounds to cause the blood to flow more freely. In a Mexican drawing, at the Vatican, I have seen a figure representing Quetzalcoatl assuaging by his penitence the anger of the gods, when, 13,060 years after the creation of the world (I give the chronology very vaguely stated by Father Rios), there was a great famine in the province of Culan; the saint retired towards Tlaxapuchicalco, near the volcanic Catcitepetl (talking mountain), where he marched with naked feet over the leaves of the aloe armed with thorns. This reminds one of the Rishi, hermits of the Ganges, the pious austerity of whom the Pouranas celebrate.

The reign of Quetzalcoatl was the golden age of the people of Anahuca: then all the animals, and even men, lived in peace, the earth produced without culture the richest harvests, the air was filled with a multitude of birds admired for their songs and beauty of their plumage; but this reign, like that of Saturn, and the happiness of the world, was not of long duration; the great spirit Tezcatlipoca, the Brahma of the people of Anahuac, offered to Quetzalcoatl a draught, which, in rendering him immortal, inspired him with the desire to travel, and particularly with an irresistible wish to visit a remote country, which tradition called Tlapallan. The analogy of this name with that of Huehuetlapallan, the country of the Tolteques, appears not to have been accidental; but how can one conceive that this white man, priest of Tula, should direct his course, as we shall soon see, to the south-east, towards the plains of Cholula, thence to the eastern coast of Mexico, to arrive at a northern country, whence his ancestors departed in the year 596 of our era.

Quetzalcoatl, in traversing the territory of Cholula, acceded to the entreaties of the inhabitants, who offered him the reins of government; he remained during twenty years among them, taught them the fusion of metals, instituted great fasts of twenty-four days, and regulated the intercalations of the Tolteque year; he exhorted them to peace; he desired they should make no other offerings to the divinity than the first fruits of the seasons. From Cholula, Quetzalcoatl passed to the mouth of the river Goasacoalco, whence he disappeared after having announced to the Cholulains that he should return hereafter to govern them again, and renew their happiness."

The descendants of this saint the unfortunate Montezuma believed he recognised in the companions in arms of Cortez:—"We know by our books," said he, in his first interview with the Spanish general, "that myself and all those who inhabit this country are not the original inhabitants, but that we were strangers that came from a great distance. We know also that the chief who brought our ancestors returned for a time to his native country, and when he returned here to seek those who were established, he found them married with the women of this country, having a numerous posterity, and living in cities which they had built; our people would not obey their ancient chief, and he returned alone. We have always believed that his descendants would come some day to take possession of this country. Considering that you come from that part where the sun was born, and that, as you assure me, you have known us for a long time, I can no longer doubt that the king who sent you is our natural chief."

The marvellous account which the Abbé Clarvigero gives of the more than oriental pomp of the barbaric Sultan of Tenochtitlan, his luxurious living, magnificent palaces, and extensive menageries may be compared with the following extract from the "Journal des Débats," which states that Layard's Assyrian discoveries confirm all that ancient authors tell us of the luxury indulged in by the most magnificent of the Asiatic sovereigns; and if already we knew, by the testimony of Lucian, that a number of wild beasts were kept in the Assyrian temples, we now learn from Layard that the great king furnished his menagerie with rare animals from different countries, either for utility or curiosity, such as the elephant, the rhinoceros, the camel with two humps, from Bactriana, the large kind of monkey called the sylvan, &c.

Among the numerous varieties of the feathered race which enliven the forests of Guatimala, Juarros says the quetzal holds the first rank for its plumage, which is of an exquisite emerald green: the tail feathers, which are very long, are favourite ornaments with the natives, and were formerly sent as a valuable present to the Sultans of Tenochtitlan. Great care was taken not to kill the birds; and they were released after being despoiled of their feathers. The birds, themselves, adds Juarros, as if they knew the high estimation their feathers were held in, build their nests with two openings, that, by entering one, and quitting them by the other, their plumes may not be deranged. This most beautiful bird is peculiar to this kingdom.

Manrique witnessed at Arracan a splendid ceremony of the idol Paragri, eleven palms high, made of silver, and trampling under foot a bronze serpent, covered with green scales.

The Indian god of the visible heavens is called Indra, or the King, and Divespetir, Lord of the Sky. He has the character of the Roman Genius, or Chief of the Good Spirits. His weapon is Vajra, or the thunderbolt. He is the regent of winds and showers; and though the east is peculiarly under his care, yet his Olympus is Meru, or the North Pole, allegorically represented as a mountain of gold and gems. He is the prince of the beneficent genii. (*Jones.*)

The Parsis historians in the Persian Chronicles, says Volney, relate that the reign of Djem-Chid was glorious, when God, to punish him for exacting adoration, excited against him Zohâk.

Zohâk overturned Djem-Chid, who disappeared and travelled 100 years over the whole earth. Zohâk, when king, became a cruel tyrant; he invented various tortures, among others, that of crucifying and flaying alive: he had several surnames, among them one was Quas-lohoub, that is to say, the Quaisi of the glittering arms; another name was Ajdehâc and Már, that is to say, serpent, because he had on his shoulders two serpents attached to two ulcers, which the devil had produced there by two kisses.

We shall next quote the historical authority of an empire that has been from a remote period aristocratically exclusive, where we find the mythological antiquity of the serpent.

It is stated in the *Magasin Pittoresque*, from manuscripts in the King's Library at Paris, that Fo-hi civilised China 3254 years before our era, and reigned 115 years. He had the body of a dragon, the head of an ox, according to some; others say he had the body of a serpent and the head of Kilin. It is easy here to distinguish an Indian type. Again, others say he had a long head, fine eyes, irregular teeth, lips of the dragon, a white beard that reached to the earth; his height was 9 feet 1 inch; he belonged to heaven, and departed for the east. He was adorned with all the virtues, and he united whatever there was of the highest or lowest. Here we find half the body that of a dragon or serpent, the beard white, reaching to the ground, and Fo-hi's departure easterly. The name of Quetzalcoatl signified a serpent covered with green feathers; he was a white man with a beard; he also disappeared, and was thought to have gone northerly, though he departed from the east coast of Mexico. He promised to return. The reign of Quetzalcoatl was the golden age of Anachua. He taught them how to fuse the metals, and desired they would make no further offerings to the divinity than the first fruits of the seasons.

"The pyramid of Belus was a temple and a tomb. In like manner, the tumulus of Calisto in Arcadia, described by Pausanias as a cone made by the hands of man, but covered with vegetation, had on its top a temple of Diana. The teocallis were also both temples and tombs; and the plain in which are built the houses of the sun and moon at Teotihuacan is called the Path of the Dead. The group of pyramids at Djizeh and Saccarah in Egypt, the triangular pyramid of the queen of the Scythians, mentioned by Diodorus, the fourteen Etruscan pyramids, which are said to have been enclosed in the labyrinth of king Porsenna at Clusium, the tumulus of Alyattes at Lydia, the sepulchres of the Scandinavian king Gormus, and his queen Daneboda, the tumuli found in Virginia, Canada, and Peru, in which numerous galleries built with stone communicate with each other by shafts, and extend through the interior of these artificial hills, also the pagoda of Tanjore, although pyramidal, and formed of many stories, wants the temple on the top, and therefore, like all other pagodas in Hindostan, is said to have nothing in common with the Mexican temples.

The platform of the pyramid of Cholula, upon which I made a great number of astronomical observations, measures 4200 square metres. A small chapel dedicated to Notre-Dame de los Remedios, and surrounded with cypress, has replaced the temple of the god of the Air, or the Mexican Indra: an ecclesiastic of Indian race daily celebrates mass on the summit of this ancient monument.

At the time of Cortez, Cholula was regarded as a holy city; nowhere was there to be found a greater number of teocallis, more priests and religious orders, more magnificence in the worship, more austerity among the fasting and penitent.

We have before noticed the striking analogy observable between the Mexican teocallis and the temple of Bel or Belus, at Babylon. This analogy had already occurred to M. Zoega, though he was only able to procure very incomplete descriptions of the group of pyramids at Teotihuacan. According to Herodotus, who visited Babylon, and saw the temple of Belus, this pyramidal monument had eight stages: its height was a stade; the length of its base equalled its height; the area included by the exterior wall equalled four square stades. The pyramid was constructed with bricks and asphalt; at the top there was a temple (*vaos*), and another near the base; the first, according to Herodotus, was without statues; there was only a table of gold, and a bed, upon which reposed a woman chosen by the god Belus. Diodorus Siculus, on the contrary, asserts that this higher temple had an altar and three statues, to which he gave, after the idea imbibed from the Greek worship, the names of Jupiter, Juno, and Rhea; but these statues and monuments neither existed at the time of Diodorus nor Strabo. In the Mexican teocallis one distinguishes, as in the temple of Bel, the naos inferior to that which is found upon the platform of the pyramid; this destination is clearly indicated in the letters of Cortez, in the "History of the Conquest," written by Bernal Diaz, who resided many months in the palace of the king Axajacatl, and, consequently, opposite the teocalli of Huitzilopochtli.

No ancient author, neither Herodotus, Strabo, Diodorus, Pausanias, Arrian, nor Quintus Curtius intimated that the temple of Belus was placed according to the four cardinal points of the compass, as are the Egyptian and Mexican pyramids. Pliny merely observes that Belus was regarded as the inventor of astronomy. Diodorus reports that the temple at Babylon served the Chaldæans as an observatory: "One understands," says he, "that this erection was of an extraordinary height, and that the Chaldæans there made their observations of the stars, so that their risings and sittings could be very accurately noted from the elevation of the building." The Mexican priests also observed the position of the stars from the tops of the teocallis, and announced to the people, by the sound of the horn, the hours of the night. These teocallis have been erected in the interval between the epoch of Mahomet and the reign of Ferdinand and Isabella; and one cannot regard without astonishment that these American edifices, of which the form is almost identical with that of one of the most ancient monuments on the banks of the Euphrates, should belong to a period so near our own."

Having quoted the descriptions and measurements of different American teocallis, we shall state the results of our calculations in succession, and draw all the teocallis on the same scale, so that their relative magnitudes may be compared. The internal and external pyramids of each teocalli will be similar. The side of the base of the internal pyramid will equal the side of the base of the lowest terrace, and the apex will be in the centre of the top platform. The side of the base of the external pyramid will equal the base of the circumscribing triangle, and height to apex equal height of triangle.

Each teocalli has two pyramids, and the number of terraces represented.

Belus and Cheops' pyramids are both teocallis, or terraced pyramids; their internal and external pyramids are drawn on the same scale. The eight terraces of Belus are represented, but not those of Cheops, the number being about 208. The pyramids of Mycerinus and of Cheops' Daughter are similar to Cheops'.

Fig. 69. Teocalli of Cholula.
„ 70. „ Sun.
„ 71. „ Moon.
„ 72. „ Mexitli.
„ 73. „ Pachachamac.
„ 74. „ Belus.
„ 75. „ Cheops.
„ 76. Pyramid of Mycerinus
„ 77. „ Cheops' Daughter.
„ 78. Silbury Hill.

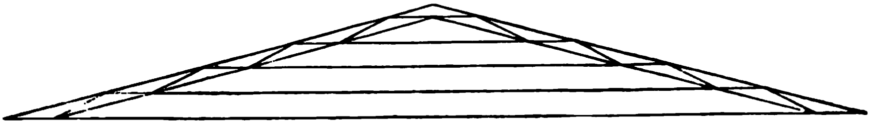


Fig. 69.

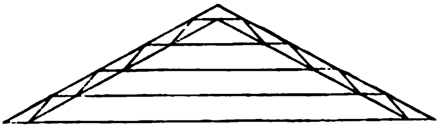


Fig. 70.

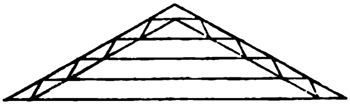


Fig. 71.

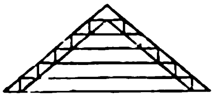


Fig. 72.

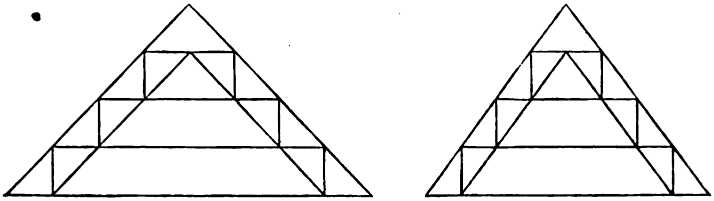
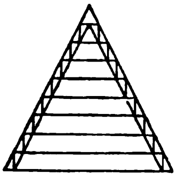


Fig. 73.



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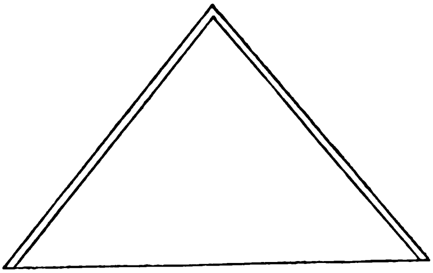

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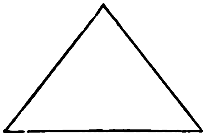

Fig. 76.

Fig. 77.

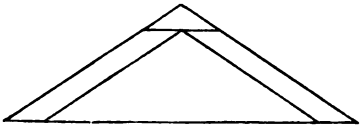

Fig. 78.

Side of base of lowest terrace of Cholula
$=439$ metres $=1245$ units.

Height to platform $=$ 54 metres
$=153$ units,

$\frac{5}{8}$ stade $=3{\cdot}75$ plethrons $=151{\cdot}875$ units,
$\frac{5}{8}$ stade $+\frac{5}{8}$ unit $=152$, &c. units,
5 stades $+$ 5 units $=1220$ units.

Internal Pyramid.

Height × area base
$=152$, &c. $\times 1220^2$, &c. $=2$ circumference.
Pyramid $=\frac{2}{3}$ circumference $=240$ degrees.

External Pyramid.

Height × area base
$=172$, &c. $\times 1374^3=\frac{3}{10}$ distance of moon.
Pyramid $=\frac{1}{10}$ distance of moon.
$\frac{1}{2}\,1374=687=\frac{1}{2}$ side of base,
if $=684=2\times 342$,
342 being Babylonian numbers.

Cube of side of base $=(2\times 684)^3$
$=22$ 6 circumference.

Cube of perimeter $=(8\times 684)^3=1446$ circumference
distance of Mercury $=1440$.

But, as has been stated, the distance assigned, 1440 circumference, is less than the calculated distance.

Thus the distance of Mercury expressed in Babylonian numbers, which are derived from $3^5=243$, will be

$$=(16\times 342)^3$$
$$=(8\times 684)^3$$

$684^2=$ circumference of earth in stades,
$684^2\times 243=$,, units.

Both these pyramids will be similar. The apex of the less pyramid will be in the centre of the top platform. The apex of the greater pyramid will be 21 units above the top platform, if the teocalli or terraced pyramid were cased as the pyramid of Cheops is said to have been, and the cube of perimeter of its base will = distance of Mercury.

Both pyramids will have height to side of base as 1 : 8.

The side of the top platform will accord with that of Humboldt, 184 units; but the side of base of the lowest terrace or side of base of less pyramid will

$$=1220 \text{ units.}$$

Humboldt's $=1245$ units.

The height to platform accords with that of Humboldt.

Cube of height to platform

$$=(\tfrac{1}{8} \text{ side of base})^3=(\tfrac{1}{8}\ 1220)^3$$

$$=152{\cdot}5^3=\frac{1}{8^3}\times 16=\frac{1}{32} \text{ circumference;}$$

$$2 \text{ cubes}=\tfrac{1}{16} \text{ circumference}=22{\cdot}5 \text{ degrees.}$$

Twice cube of height to platform : cube of side of base of external pyramid

:: 22·5 degrees : 22·6 circumference
:: degree : circumference.

Cube of side of base of external pyramid = 720 times cube of height to platform.

Cube of perimeter $= 720\times 4^3=46080$ times cube of height to platform.

Cube of 4 times height of external pyramid

$$=(4\times 172)^3= 688^3=\tfrac{3}{10} \text{ distance of moon}$$
$$(10\times 688)^3=\tfrac{3000}{10}=300.$$

Cube of 40 times height

$$=300 \text{ distance of moon}$$
$$=\text{diameter of orbit of Mercury.}$$

Cube of 40 times height : cube of 4 times side of base :: diameter of orbit of Mercury : distance of Mercury :: 2 : 1.

All the terraces are of equal height;

$\tfrac{1}{2}$ side of lowest terrace = 610 units,
$\tfrac{1}{4}$ 610 = 152, &c. = height to platform,
$\tfrac{1}{4}$ 152, &c. = 38, &c. = height of a terrace.

$$610^3 \quad = 2 \text{ circumference},$$

$$152^3, \&c. = \frac{2}{4^2} = \frac{2}{64} = \frac{1}{32},$$

$$38^3, \&c. = \frac{1}{32} \times \frac{1}{4^2} = \frac{1}{2048}.$$

Side of base of lowest terrace

$$= 2 \times 610 = 1220.$$

Cube of side $= 1220^3 = 2 \times 8 = 16$ circumference.

Cube of perimeter $= 16 \times 4^3 = 1024$.

Content of internal pyramid $= \frac{2}{3}$ circumference.

Content of external pyramid $= \frac{1}{10}$ distance of moon.

Cube of perimeter of base = distance of Mercury.

When the teocalli of Cholula is compared with other pyramids, it is made = circumference of earth; for this estimate was made before we knew that a teocalli represented two pyramids.

In all the Mexican teocallis we find the measurement of only one side of the base stated. Humboldt supposes that they were intended to have the sides as 2 : 1.

First we calculated the teocallis having the sides as 2 : 1; but afterwards by making the base equal to the square of the given side.

Should the sides of the base be as 2 : 1, the content of a teocalli will only equal half of what has been calculated.

The cube of the perimeter of the base of the teocalli of Cholula we make = distance of Mercury.

Though the sides should be as 2 : 1, still the cube of 4 times the greater side will = distance of Mercury; and the content of a pyramid having base = square of that side will be what has been computed.

Bullock remarks that at a distance the appearance which the teocalli of Cholula assumes is that of a natural conical hill, wooded and crowned with a small church; but, as the traveller approaches it, its pyramidal form becomes distinguishable, together with the four stories into which it is shaped, although covered with vegetation, the prickly pear, the nopal, and the cypress.

This descriptive view of the teocalli suggests the idea that the hanging gardens of Babylon might have been formed by planting trees and shrubs on the terraces of some old teocalli.

The tumulus of Calisto, in Arcadia, described by Pausanias as a cone made by the hands of man, but covered with vegetation, had on the top a temple of Diana.

The teocalli of the Sun has four terraces, and the height to the top platform = 55 metres = 180 feet English = 156 units.

The side of the base of lowest terrace = 208 metres = 682 feet = 590 units.

Height × area base

$$=156 \times 590^2 = \tfrac{1}{20} \text{ distance of moon.}$$

$$\text{Pyramid} = \tfrac{1}{3} \text{ of } \tfrac{1}{20} = \tfrac{1}{60},$$

or internal pyramid $= \frac{1}{60}$ distance of moon.

If height of external pyramid = 181 units,
and side of base = 685, &c.

Height × area base

$$=181 \times 685^2, \text{ \&c.} = \tfrac{3}{4} \text{ circumference,}$$

$$\text{Pyramid} = \tfrac{1}{3} \text{ of } \tfrac{3}{4} = \tfrac{1}{4}.$$

The two pyramids will be similar.

The apex of the less will be in the centre of the top platform.

The internal pyramid of Sun will $= \frac{1}{6}$ the external pyramid of Cholula.

The external pyramid of Sun will be to internal pyramid of Cholula

$$:: \tfrac{1}{4} : \tfrac{2}{3} \text{ circumference,}$$

$$:: 3 : 8.$$

The internal pyramid of Sun $= \frac{1}{6} = \frac{4}{24} = \frac{2^2}{24}$ the external pyramid of Cholula.

The external pyramid of Sun $= \frac{3}{8} = \frac{9}{24} = \frac{3^2}{24}$ the internal pyramid of Cholula.

Cube of twice perimeter of base of external pyramid of Sun

$=(8 \times 685)^3 = 5480^3 =$ distance of Mercury,

and $(8 \times 684)^3 =$ distance of Mercury.

or $(16 \times 342)^3 =$ distance of Mercury, in Babylonian numbers.

$(4 \times 181)^3 = 724^3 = \frac{10}{3}$ circumference.

3 cubes of 4 times height of external pyramid = 10 circumference.

$(3 \times 724)^3 = \frac{10}{3} \times 3^3 = 270$

10 cubes of 12 times height = 2700 circumference

= distance of Venus

100 cubes of 24 times height = 216000 circumference

= distance of Belus.

Internal : external pyramid

:: $\frac{1}{4}$ circumference : $\frac{1}{60}$ distance of moon

:: $\frac{1}{4}$ circumference : radius of earth

:: quadrantal arc : radius

:: circumference : 2 diameters.

Cube of height of internal pyramid

$= 156^3 = \frac{1}{30}$ circumference = 12 degrees

$3^5 = 243$

$(3 \times 342$ &c.$)^3 = 1028^3 =$ distance of moon

$(16 \times 342)^3 =$ distance of Mercury

Distance of moon : distance of Mercury nearly as $3^3 : 16^3$

$:: 1 : 151{\cdot}7$

$(2 \times 342)^2 \times 243 =$ circumference of earth.

Thus the circumference of earth, distance of moon, and distance of Mercury are expressed in Babylonian numbers.

The teocalli of the moon is stated to be 11 metres = 36 feet = 31 units lower than the teocalli of the sun, and the base much smaller.

$\therefore$ height will $= 156 - 31 = 125$ units.

No measurement of the base or top platform is given.

If the teocallis of the sun and moon were similar

then $156 : 125 :: 590 : 472$ &c.

or $156 : 590 :: 125 : 472$ &c.

say as $123 : 470$.

Height × area of base of internal pyramid $= 123 \times 470^2$
$= \frac{1}{40}$ distance of moon
pyramid $= \frac{1}{120}$
Internal pyramid of sun $= \frac{1}{60}$
Pyramids are as 1 : 2.

Height × area base of external pyramid of moon will
$= 143 \times 545^2$ &c. $= \frac{3}{8}$ circumference
External pyramid $= \frac{1}{8}$
External pyramid of sun $= \frac{1}{4}$
Pyramids are as 1 : 2.

Cube of side of base of external pyramid of moon
$= 545^3$ &c.
$(8 \times 545$ &c.$)^3 = \frac{1}{2}$ distance of Mercury,

or cube of twice perimeter of base of external pyramid of sun = twice cube of twice perimeter of base of external pyramid of moon.

The cubes of the similar sides of these two pyramids will be as 1 : 2.

The cubes of the similar sides of the internal pyramids will be in the same ratio, and so will the pyramids themselves.

Cube of height of external pyramid of moon $= 143^3$ &c..

$$= \frac{1}{38{\cdot}4} \text{ circumference.}$$

Cube of height of external pyramid of sun $= 181^3$

$$= \frac{1}{19{\cdot}2} \text{ circumference.}$$

Cubes are as 1 : 2.

$$\frac{1}{19{\cdot}2} : 1 :: 1 : 19{\cdot}2 \text{ circumference.}$$

Cube of height of external pyramid of sun
: circumference :: circumference : twice distance of moon.
Cube of height of external pyramid of moon
: $\frac{1}{2}$ circumference : $\frac{1}{2}$ circumference : distance of moon.

Bullock, who visited these pyramids, says: — " On descending the mountain, the pyramids are seen in a plain at about five or six miles distance. As we approached them the square and perfect form of the largest became at every step more and more visibly distinct, and the terraces could now be counted. We soon arrived at the foot of the largest pyramid, and began to ascend. It was less difficult than we expected, though, the whole way up, lime and cement are mixed with fallen stones. The terraces are perfectly visible, particularly the second, which is about 38 feet wide, covered with a coat of red cement eight or ten inches thick, composed of small pebble-stones and lime. In many places, as you ascend, the nopal trees have destroyed the regularity of the steps, but nowhere injured the general figure of the square, which is as perfect in this respect as the great pyramid of Egypt. On reaching the summit, we found a flat surface of considerable size, but which had been much broken and disturbed."

The width of 38 feet for the terraces agrees with the width in the outline we have given of this teocalli with its two bases and two heights.

On the summit of the teocalli of the moon are the remains of an ancient building, 47 feet long and 14 wide; the walls are principally of unhewn stone, three feet thick and eight feet high. Forty-seven feet = 1 plethron.

This pyramid is more dilapidated than the greater pyramid.

Sides of ancient building
47 by 14 feet
= 40·63 12·1 units
$10 \times 40{\cdot}7 = 407$
407^3 &c. = $\frac{1}{16}$ distance of moon
$(2 \times 407$ &c.$)^3$ = $\frac{1}{2}$
Cube of 20 times greater side
= $\frac{1}{2}$ distance of moon
$10 \times 12{\cdot}3 = 123$
123^3 &c. = $\frac{1}{60}$ circumference
Cube of 10 times less side = $\frac{1}{60}$ circumference.

The teocalli, or great temple of Mexitli, occupied the present site of the great cathedral of Mexico. Humboldt mentions its four sides as having corresponded exactly with the cardinal points of the compass; its base was 97 metres, and height 37 metres; the point, terminated by a cupola, was 54 metres in height from the base, and its having had five stories, like many of the pyramids of Saccarah, particularly like that of Meidoum. It formed a pyramid so truncated, that when viewed at a distance it appeared like an enormous cube, upon which were placed small altars with cupolas made of wood; the point where these cupolas terminated was 54 metres above the base. The stair-case to the platform contained 120 steps.

Teocalli of Mexitli.

Side of base $=97$ metres $=318$ feet $=275$ units,
say $=279$.

Height to platform $=37$ metres $=121{\cdot}4$ feet $=105$ units,
say $=108$.

Height × area base of internal pyramid $=108$ &c. $\times 279^2$
$=\frac{3}{40}$ circumference.

Pyramid $=\frac{1}{3}$ of $\frac{3}{40}$ $=\frac{1}{40}$.

Cube of height $=108^3=\frac{1}{90}$ circumference.

Cube of side of base $=279^3=\frac{1}{50}$ distance of moon.

If height of external pyramid $=130$ units,
and side of base $=336$,

then height × area base
$=130\times 336^2=\frac{1}{150}$ distance of moon.

pyramid $=\frac{1}{3}$ of $\frac{1}{150}$ $=\frac{1}{450}$ „

Cube of height $=129^3$ &c. $=\frac{1}{500}$ distance of the moon.

Cube of side of base $=336^3=\frac{1}{3}$ circumference.

Cube of height of external pyramid : cube of side of base of internal pyramid

$:: \frac{1}{500} : \frac{1}{50}$ distance of the moon,
$:: 1 : 10$.

Cube of height of internal pyramid : cube of side of base of external pyramid

$$:: \tfrac{1}{90} : \tfrac{1}{3} \text{ circumference},$$
$$:: 1 : 30.$$

The two pyramids will be similar.

If the height of the five terraces be equal, the height of each will $=21{\cdot}6$ units = difference of height of the two pyramids.

Cube of side of base of external pyramid $=336^3=\frac{1}{3}$ circumference $=120$ degrees.

The number of steps to the platform were 120.

The Mexitli teocalli is said to have been built after the model of the pyramids of Teotihuacan, only six years before Columbus discovered America. The Cathedral of Mexico stands on the site of the teocalli. This teocalli may be passed over as unimportant, if it were a modern structure, and as no traces of it remain.

Tower of Belus. (*Fig.* 67.)

If the height = side of base of internal pyramid = 242 &c. units,

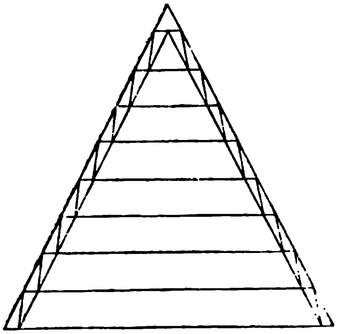

Fig. 67.

then cube of side of base $= 242^3$ &c. $= \frac{1}{8}$ circumference,
pyramid $= \frac{1}{3}$ of $\frac{1}{8}$ $\qquad = \frac{1}{24}$ „ .

If the height of external pyramid = side of base = 262 &c. units,

Cube of side will $= 262^3$ &c.
$= \frac{1}{60}$ distance of the moon.
= radius of the earth.

Pyramid $= \frac{1}{3}$ of $\frac{1}{60} = \frac{1}{180}$ distance of the moon.
Cubes of the sides are as

Radius : $\frac{1}{8}$ circumference of the earth.

Cubes of twice the sides are as

8 Radii : circumference.
4 diameters : circumference of the earth.

The two pyramids are similar.

360 external pyramids $= \frac{360}{180} = 2$ distance of the moon.
= diameter orbit of the moon.

360 internal pyramids $\frac{360}{24} = 15$ circumference.

External pyramid of Belus : external pyramid of Cheops,
:: $\frac{1}{180}$: $\frac{1}{18}$ distance of the moon,
:: 1 : 10 ,, .

Internal pyramid of Belus : internal pyramid of Cheops
:: $\frac{1}{24}$: $\frac{1}{2}$ circumference,
:: 1 : 12 ,, .

If the internal pyramid of the tower of Belus $= \frac{1}{24}$ circumference, and the external pyramid $= \frac{1}{180}$ distance of the moon, the sides of the terraces will be inclined as in *Fig.* 67., and not perpendicular as in *Fig.* 56. The top of the tower in *Fig.* 67. forms the outline of the Royal tent.

Cube of Babylon $= 120^3$ stades,
$= 29160^3$ units.

Twice height of external pyramid of tower
$= 2 \times 262$ &c. $= 524$ &c.

Section of cube to the height of 524 units

$$= \frac{524}{29160} = \frac{1}{55{\cdot}6} \text{ cube.}$$

Cube of Babylon = distance of Belus.
,, = 22500 distance of the moon.
distance of the earth = 400 ,, .

$$= \frac{400}{22500} = \frac{1}{56{\cdot}25} \text{ cube of Babylon.}$$

So distance of the earth will nearly equal a section of the cube of Babylon having the height = twice height of external pyramid of tower.

Distance of Mercury = $\frac{1}{150}$ distance of Belus.
" = $\frac{1}{150}$ cube of Babylon,
and $\frac{1}{150}$ of 29160 = 194.

So distance of Mercury = section of cube having height of 194 units,
= $\frac{4}{5}$ of 242 &c. = $\frac{4}{5}$ stade,
= $\frac{4}{5}$ height of internal pyramid.

Distance of Venus = $\frac{1}{80}$ distance of Belus,
= $\frac{1}{80}$ cube of Babylon,
and $\frac{1}{80}$ of 29160=364·25 units = $\frac{3}{2}$ stade.

So distance of Venus = section of cube having height
= $\frac{3}{2}$ stade,
= $\frac{3}{2}$ height of internal pyramid.

The teocallis and pyramids are drawn on the same scale, so an estimate may be formed of their relative magnitudes. To form a conception of their real magnitudes, their dimensions may be compared with some of the public buildings in London.

Waterloo Bridge over the Thames is built with granite; the length = 1280 feet.

The side of the base of the internal pyramid of Cholula = 1410 feet.

Side of base of the external pyramid = 1589 feet.

So that the side of the base of the internal pyramid would exceed the length of the bridge by 130 feet; and side of base of the external pyramid by 309 feet.

The height of each pyramid = $\frac{1}{8}$ side of base.

The square area of Lincoln's Inn Fields = about that of the base of Cheops' pyramid, and height = $\frac{5}{8}$ side of base.

The dimensions of St. Paul's Cathedral from east to west, within the walls, are stated at about 510 feet; and the line from north to south, within the portico doors, at 282 feet.

281 feet = 1 stade = height = side of base of the tower of Belus.

Humboldt says, "Another monument well worthy the attention of the traveller is the intrenched military station of Xochicalco. This is an isolated hill, 117 metres high, surrounded by fosses, and divided by the hand of man into 5 stories or terraces; the sides of the terraces being formed of masonry. The whole forms a truncated pyramid, having the four sides placed exactly according to the four cardinal points. The platform of this extraordinary monument nearly equals 9000 square metres; on the top is seen the ruins of a small square edifice, that served, no doubt, as the last resort of the besieged.

"The terraces have about 20 metres of perpendicular elevation. They contract towards the top, as in the teocallis or Aztec pyramids, the summit of which is ornamented with an altar. All the terraces are inclined towards the south-east; probably to facilitate the flow of water during the rains, which are very abundant in this region. The hill is surrounded by a fosse pretty deep and very broad: the whole entrenchment has a circumference of about 4000 metres. The magnitude of these dimensions ought not to surprise us: on the ridge of the Cordilleras of Peru, and on heights almost equal to that of the Peak of Teneriffe, M. Bonpland and myself have seen monuments still more considerable. Lines of defence and entrenchments of extraordinary length are found in the plains of Canada. The whole of these American works resemble those that are daily discovered in the eastern part of Asia. Nations of the Mongol race, especially those that are more advanced in civilisation, have built walls that separate whole provinces.

"The summit of the hill of Xochicalco presents an oblong platform, which from north to south has 72 metres, and from east to west 86 metres in length. This platform is surrounded by a wall of hewn stone, having a height exceeding 2 metres, that served as a defence to the attacked.

"In the centre of this spacious place of arms is found the remains of a pyramidal monument that had five terraces; the form resembling that of a teocalli. The first terrace only has been preserved; the proprietors of a neighbouring sugar manufactory having been barbarous enough to destroy this pyramid, by tearing away the stones to construct their furnaces. The Indians of Tetlama assert that the five terraces still existed in 1750; and from the dimensions of the first step or terrace (*gradin*) it may be supposed that the whole edifice had an elevation of 20 metres. The sides are placed exactly according to the four cardinal points. The base of this edifice has a length of 20·7 metres, and a breadth of 17·4 metres. What is very remarkable, no vestige of a staircase leading to the top of the pyramid has been discovered, though it is asserted that a stone seat or chair (*ximoilalli*), ornamented with hieroglyphics, had been found.

"Travellers who have examined this work of the native Americans have not been able sufficiently to admire the cutting and polishing of the stones, which are all of the form of parallelopipedons; the care with which they have united them without any cement being interposed, and the execution of the reliefs with which the terraces or steps are adorned, each figure occupying many stones, and their forms not interrupted by the joints of the stones; so that one might suppose the reliefs had been sculptured after the edifice had been built.

"Among the hieroglyphical ornaments of the pyramid of Xochicalco we distinguish the heads of crocodiles spouting water, and figures of men sitting cross-legged, according to the custom of several nations of Asia.

"The fosse that surrounds the hill, the coating of the terraces, the great number of subterraneous apartments cut in the north side of the rock, the wall that defends the approach to the platform,—all concur to give to the monument of Xochicalco the character of a military monument. The natives designate to this day the ruins of the pyramid that rises in the middle of the platform by a name equivalent to that of citadel. The great analogy in form remarkable between this presumed citadel and the houses of the Aztec gods, the teocallis, makes me suppose that the hill of Xochicalco was nothing else than a fortified temple. The pyramid of Mexitli, or the great temple of Tenachtitlan, contained also an arsenal in its enclosure, and served, during the siege, as a stronghold, sometimes to the Mexicans and sometimes to the Spaniards. The sacred writings of the Hebrews inform us that, from the highest antiquity, the temples of Asia, as, for instance, those of Baal-Berith, at Sichem in Canaan, were at the same time edifices consecrated to worship and entrenchments into which the inhabitants of the city might fly to shelter themselves against the attacks of the enemy. In short, nothing can be more natural to men than to fortify the places in which they preserved the tutelary deities of the country; nothing more confiding, when public affairs were endangered, than to take refuge at the foot of their altars, and combat under their immediate protection. Among the people where the temples had preserved one of the forms the most ancient, that of the pyramid of Belus, the construction of the edifice might answer the double purpose of worship and defence. In the Greek temples the wall alone that formed the περίβολος afforded an asylum to the besieged."

Humboldt's description of the hill of Xochicalco, in his "Monuments de Peuple Indigènes de l'Amerique," differs from that given in his "Essai Politique." One account makes the area of the platform 9000 square metres; the other makes the sides 72 by 86 metres, which = 6192 square metres.

No dimensions of the base are mentioned.

We may remark that 117 metres, the height to the platform, = 383·8 feet = 331·9 units,

and 331^3 &c. = $\frac{1}{30}$ distance of the moon,
or cube of height = $\frac{1}{30}$ distance of the moon,
= diameter of the earth.

Sides of platform are 72 by 86 metres

= 203 by 243 units (1 stade)

204^3 &c. = $\frac{3}{40}$ circumference
242^3 &c, = $\frac{1}{8}$ = $\frac{5}{40}$

Sum of cubes of 2 sides

$$= \frac{3}{40} + \frac{5}{40} = \frac{8}{40} = \frac{1}{5} \text{ circumference} = 72 \text{ degrees.}$$

The height to the platform of the teocalli of Cholula $= 152^3$ &c.

Cube of the heights of the two teocallis will be

$$\text{as } 153^3 \text{ \&c.} : 331^3 :: 1 :: 10.$$

It appears that Humboldt had never seen the hill of Xochicalco, and has given the measurement of M. Alzate.

If the teocallis of Cholula and Xochicalco were similar, their contents would be as 1 : 10.

The external pyramid of Cholula $= \frac{1}{10}$ distance of moon.

So the external pyramid of Xochicalco would = distance of the moon.

The external pyramid of Cheops $= \frac{1}{18}$ distance of the moon, $= \frac{1}{18}$ part of the external pyramid of Xochicalco.

The circumference of the fosse is about 4000 metres.

The French measured $\frac{1}{4}$ circumference of the earth passing through the poles. A ten-millionth part of this quadrant was made a standard of length and called a metre; being equal to 39·371 English inches.

Circumference of fosse = 4000 metres,

$$\tfrac{1}{4} = 1000$$

$$\tfrac{1}{4} \text{ circumference} = \frac{1000}{10000000} = \frac{1}{10000}$$

= one ten-thousandth part of the quadrant from the equator to the pole.

∴ cirumference of the fosse will = one ten-thousandth part of the circumference of the earth passing through the poles.

Hence the measurement of the earth's circumference made at a very remote period by an unknown race, who constructed the great teocalli of Xochicalco, accords with the measurement lately made by the French, if the circumference of the fosse = 4000 metres.

Pyramidal Monument.

Sides 20·7 by 17·4 metres,
= 58·7 by 49·26 units.

$588^3 = \frac{3}{16}$ distance of the moon.
494^3 &c. $= \frac{1}{9}$ " " "

Supposed height 20 metres.

Height × area of base $= 588 \times 588 \times 494 = \frac{3}{2}$ circumfer.
Pyramid $= \frac{1}{3}$ of $\frac{3}{2} = \frac{1}{2}$ "

Pyramid of 10 times the dimensions of the internal pyramid of the small teocalli $= \frac{1}{2}$ circumference,
= internal pyramid of Cheops.

Cube of 10 times greater side $= \frac{3}{16}$ distance of the moon.
Cube of 10 times less side $= \frac{1}{9}$ " " "
Cube of 20 times greater side $= \frac{24}{16} = \frac{3}{2}$ " "
Pyramid $= \frac{1}{3}$ of $\frac{3}{2} = \frac{1}{2}$ " " "

Or pyramid having height = side of base = 20 times the greater side of base of small pyramid $= \frac{1}{2}$ distance of the moon.

Should the fosse form a square, side would = 1000 metres.

Side of base of teocalli of Cholula = 439 metres.

Side of base of a similar teocalli having content = 10 times content of the teocalli of Cholula will = 946 metres.

So that a similar teocalli of 10 times the content of that of Cholula will have a square base less than the square formed by the fosse.

Small pyramid $= \frac{1}{2000}$ circumference,
= 5 times circumference of fosse,

$$5 \times 4000 = 20000 \text{ metres.}$$

The teocalli has 5 terraces.

The pyramid of Pachacamac in Peru is thus described in the recent narrative of the United States' exploring expedition:—

"The Temple of Pachacamac, or Castle, as it is called by the Indians, is on the summit of a hill, with three terraces; the view of it from the north is somewhat like that of the pyramid of Cholula, given by Humboldt, except that the flanks were perpendicular. The whole height of the hill is 250 feet, that of the mason-work 80; the form is rectangular, the base being 500 by 400 feet. At the south-eastern extremity the three distinct terraces are not so perceptible, and the declivity is more gentle. The walls, where great strength was required to support the earth, were built of unhewn square blocks of rock; these were cased with sun-dried bricks (adobes), which were covered with a coating of clay or plaster, and stained or painted of a reddish colour. A range of square brick pilasters projected from the uppermost wall, facing the sea, evidently belonging originally to the interior of a large apartment. These pilasters gave it the aspect of an Egyptian structure. In no other Peruvian antiquities have pilasters been seen by us. On one of the northern terraces were also remains of apartments; here the brick appeared more friable, owing to a greater proportion of sand; where they retained their shape their dimensions were nine inches in width by six inches deep, varying in height from nine inches to two feet; and they were laid so as to break joint, though not always in a workmanlike manner. The remains of the town occupy some undulating ground, of less elevation, a quarter of a mile to the northward. This also forms a rectangle, one-fifth by one-third of a mile in size: through the middle runs lengthwise a straight street, twenty feet in width. The walls of some of the ruins are thirty feet high, and cross each other at right angles. The buildings were apparently connected together, except where the streets intervened. The larger areas were again divided by thinner partitions, and one of them was observed to contain four rectangular pits, the plastering of which appeared quite fresh. No traces of doors or windows towards the streets could be discovered, nor indeed anywhere else. The walls were exclusively of sun-dried brick, and their direction north east and south-west, the same as those of the temple, which fronted the sea. Some graves were observed to the southward of the temple, but the principal burying-ground was between the temple and town. Some of the graves were rectangular pits, lined with a dry wall of stone, and covered with layers of reeds and canes, on which the earth was filled in to the depth of a foot or more, so as to be even with the surface.

The skulls brought from this place were of various characters; the majority of them presented the vertical elevation, or raised occiput, the usual characteristic of the ancient Peruvians, while others had the forehead and top of the head depressed. Eight of these were obtained, and are now deposited at Washington. The bodies were found enveloped in cloth of various qualities, and a variety in its colours still existed. Various utensils and other articles were found, which seemed to denote the occupation of the individual: wooden needles and weaving utensils; netting made in the usual style; a sling; cordage of different kinds; a sort of coarse basket; fragments of pottery, and plated stirrups. They also found various vegetable substances: husks of Indian corn, with ears of two varieties, one with the grain slightly pointed, the other, the short and black variety, which is still very commonly cultivated; cotton-seeds; small bunches of wool; gourd-shells, with a square hole cut out, precisely as is done at present. These furnished evidence of the style of the articles manufactured before the arrival of the Spaniards, and of the cultivation of the vegetable products; when to these we add the native tuberous roots (among them the potato) cultivated in the mountains, and the animals found domesticated, viz., the llama, dog, and Guinea-pig, and the knowledge of at least one metal, we may judge what has since been acquired."

Teocalli of Pachacamac.

Height to platform 250 feet = 216 units.
Sides of base of lowest terrace = 500 by 400 feet
= 432 by 345·5 units.

Height × area of base of internal pyramid $= 220 \times 342$ &c. $\times\ 432 = \frac{3}{100}$ distance of the moon
pyramid $= \frac{1}{100}$.

External Pyramid.

Height × area of base $= 294 \times 454$ &c. $\times\ 574 = \frac{2}{3}$ circumference
pyramid $= \frac{2}{9}$.

Cube of sum of 2 sides $= (454 + 574)^3 = 1028^3 =$ distance of the moon.

Cube of perimeter = 8 times distance of moon.

Internal Pyramid.

$$\begin{array}{lrr} \text{Sides are 342 \&c. by} & 432 & \\ \end{array}$$

$$\begin{array}{rr} 342 & 432 \\ \underline{\quad 3} & \underline{\quad 3} \\ 1028 & 1296 \end{array}$$

Cube of 3 times less side $= 1028^3 =$ distance of the moon.

Cube of 3 times greater side $= 1296^3 = 6^{12} =$ diameter of the orbit of the moon.

The 2 pyramids are similar, and the sides of the terraces will be perpendicular to base.

Lowest terrace $= \frac{1}{3}$ height × area of base of internal pyramid = internal pyramid.

$$\begin{array}{rr} \text{Height of the 3 terraces} = & 220 \\ \frac{1}{3} = & \underline{73 \cdot 3} \\ & 293 \cdot 3. \end{array}$$

The 3 terraces are as 1^2, 2^2, 3^2, their heights being equal.

The 3 pyramids rising from the base of the 1, 2, 3 terraces will be as 1^3, 2^3, 3^3.

$$\begin{array}{lll} \text{3rd pyramid} = \frac{1}{100} & = \frac{27}{2700} & \text{distance of the moon} \\ \text{2nd} \quad ,, \quad = \frac{8}{27} \text{ of } \frac{1}{100} & = \frac{8}{2700} & \\ \text{1st} \quad ,, \quad = \frac{1}{27} \text{ of } \frac{1}{100} & = \frac{1}{2700}. & \end{array}$$

$$\begin{array}{llll} \text{3rd terrace} = \text{3rd pyramid} & = \frac{1}{100} & = \frac{9}{900} & = \frac{27}{2700} \\ \text{2nd} \quad ,, \quad = \frac{4}{9} \text{ of 3rd terrace} & & = \frac{4}{900} & = \frac{12}{2700} \\ \text{1st} \quad ,, \quad = \frac{1}{9} \quad ,, & & = \frac{1}{900} & = \frac{3}{2700} \end{array}$$

$$\left.\begin{array}{ll} \text{1st terrace} & = \frac{3}{2700} \\ \text{1st pyramid} & = \frac{1}{2700} \end{array}\right\} \text{difference} = \frac{2}{2700}$$

$$\left.\begin{array}{ll} \text{2nd terrace} & = \frac{12}{2700} \\ \text{2nd pyramid} & = \frac{8}{2700} \end{array}\right\} \quad ,, \quad = \frac{4}{2700}$$

$$\left.\begin{array}{ll} \text{3rd terrace} & = \frac{27}{2700} \\ \text{3rd pyramid} & = \frac{27}{2700} \end{array}\right\} \quad ,, \quad = 0.$$

1st pyramid $= \frac{1}{3}$ of 1st terrace
2nd ,, $= \frac{2}{3}$ of 2nd ,,
3rd ,, $= \frac{3}{3}$ of 3rd ,,

The 3 sections of the internal pyramid, made at equal distances, are as

$$1^3\ ,\quad 2^3-1^3\ ,\quad 3^3-2^3$$
$$1\ ,\quad 7\ ,\quad 19$$
$$=\tfrac{1}{2700}\ ,\quad \tfrac{7}{2700}\ ,\quad \tfrac{19}{2700}.$$

Sum of 3 sections $=\frac{27}{2700}=$ whole pyramid.

3rd terrace − 3rd section $=\frac{8}{2700}$
2nd „ − 2nd „ $=\frac{5}{2700}$
1st „ − 1st „ $=\frac{2}{2700}$.

Common difference $=\frac{3}{2700}$
Sum of differences $=\frac{15}{2700}$
And 3rd terrace − 2nd terrace $=\frac{15}{2700}$.

Therefore 3rd terrace − 2nd terrace
= sum of 3 terraces − sum of 3 sections
= „ − internal pyramid
= „ − 3rd terrace
= sum of 1st and 2nd terrace $=\frac{15}{2700}$ distance of the moon.

Sides of the base of lowest terrace are 342 by 432.
These are Babylonian numbers derived from $3^5 = 243$.

$(342 \times 2)^2 \times 243 =$ circumference of the earth
(342 &c. × 3)^3 = distance of the moon
$(432 \times 3)^3 =$ diameter of the orbit of the moon
(342, &c. × 16)^3 = distance of Mercury
$(432 \times 16)^3 =$ diameter of the orbit of Mercury.

Cubes of the sides of the base of the lowest terrace are as 342^3 &c : 432^3 :: 1 : 2.

(342 &c. × 2^4)^3 = distance of Mercury.
$(432 \times 2^4)^3 =$ diameter of the orbit of Mercury.

The cube of the sum of 3^5 when transposed by changing the places of the first and last numbers and multiplying by 2^4 = distance of Mercury.

The cube of the sum of 3^5 when transposed by placing the first number the last and multiplying by 2^4 = diameter of the orbit of Mercury.

The cube of 3 times the first transposed numbers = distance of the moon.

The cube of 3 times the last transposed numbers = diameter of the orbit of the moon.

The square of twice the first transposed numbers multiplied by 3^5 = circumference of the earth.

Circumference of the earth $= (342 \times 2)^2 \times 243$

Diameter of the orbit of Belus = 432000 circumference

$= 432 \times 10^3$ „

$\therefore (342 \times 2)^2 \times 243 \times 432 \times 10^3 =$ diameter of orbit of Belus;

or, $4 \times 342^2 \times 243 \times 432 \times 10^3 =$ „ „ „

so, $2 \times 342^2 \times 243 \times 432 \times 10^3 =$ distance of Belus.

Cube of height to platform $= 216^3$

$= \frac{1}{4}$ cube of less side of base $= \frac{1}{4}(342)^3$

$= \frac{1}{8}$ cube of greater side of base $= \frac{1}{8}(432)^3$

Internal pyramid $= \frac{1}{100}$ distance of the moon.

Height = 220 units,

and 221^3 &c. $= \frac{1}{100}$ distance of the moon.

The rectangular enclosure of the town $= \frac{1}{3}$ by $\frac{1}{5}$ of a mile.

$$
\begin{aligned}
\text{a mile} &= 18{\cdot}79 \text{ stades}\\
\tfrac{1}{3} &= 6{\cdot}\dot{2}\dot{6}\\
\tfrac{1}{5} &= \underline{3{\cdot}76}\\
\tfrac{1}{3}+\tfrac{1}{5} &= 10{\cdot}02\\
&\phantom{=10{\cdot}0}\underline{2}\\
\text{perimeter} &= 20{\cdot}04.
\end{aligned}
$$

One side of the rectangular enclosure = 6·26 stades = 1521 units, say = 1538; then $1538^3 =$ 32 circumference

$= \frac{1}{45}$ distance of Mercury,

or 45 cubes = 1440 circumference = distance of Mercury from the Sun.

The two sides of the rectangle = 10 stades = 2430 units, and one side = 1538, so the other side will = 2430 − 1538 = 892,

and $898^3 = \frac{2}{3}$ distance of Moon,

or 3 cubes = 2 „

$1\frac{1}{2}$ „ = 1.

Thus the two sides will = 1538 + 898 = 2426, and $\frac{1}{2}$ perimeter = 10 stades = 2430 units.

Perimeter of the walls of Pachacamac $= 20$ stades $= \frac{1}{6}$ 120 $= \frac{1}{6}$ the side of the square enclosure of Babylon.

Cube of 20 stades $= \frac{1}{216}$ the cube of 120
$= \frac{1}{216}$ cube of Babylon
$= \frac{1}{216}$ distance of Belus
$= \frac{1}{216}$ of 216000 circumference
$=$ 1000 circumference.

Thus the cube of the perimeter of the walls of Pachacamac $= \frac{1}{216}$ the cube of Babylon, $= \frac{1}{216}$ the distance of Belus,
$= 1000$ times the circumference of the earth,
$=$ more than 100 times the distance of the moon from the earth.

$1\frac{1}{2}$ cube of 898 $=$ distance of moon from earth.

Distance of earth from sun $= 400$ times the distance of moon from earth,

$$= 400 \times 1\tfrac{1}{2} = 600 \text{ times } 898^3.$$

Distance of Mercury from sun $= 150$ times the distance of moon from earth,
$= 150 \times 1\frac{1}{2} = 225 = 15^2$ times 898^3.
or 153^3 &c. $= \frac{1}{300}$ distance of the moon,
$(10 \times 153, \text{\&c.})^3 = \frac{1000}{300} = \frac{10}{3}$.
Cube of side $= \frac{10}{3}$.
45 cubes $= 150$ distance of the moon,
$=$ distance of Mercury,
120 cubes $=$ distance of the earth,
or 15 cubes of twice side $=$ distance of the earth.

We find in "Tschudi's Travels in Peru," that prior to the Spanish conquest, the valley of Lurin was one of the most populous parts of the coast of Peru. The whole of the broad valley was then called Pachacamac, because near the sea-shore and northward of the river, there was a temple sacred to the "Creator of the Earth."

Pachacamac was the greatest deity of the Yuncas, who did not worship the sun till after their subjugation by the Incas. The temple of Pachacamac was then dedicated to the sun by the Incas, who destroyed the idols which the

Yuncas had worshipped, and appointed to the service of the temple a certain number of virgins of royal descent. In the year 1534, Pizarro invaded the village of Lurin; his troops destroyed the temple, and the Virgins of the Sun were dishonoured and murdered.

The ruins of the temple of Pachacamac are among the most interesting objects on the coast of Peru. They are situated on a hill about 558 feet high. The summit of the hill is overlaid with a solid mass of brickwork about thirty feet in height. On this artificial ridge stood the temple, enclosed by high walls, rising in the form of an amphitheatre. It is now a mass of ruins; all that remains of it being some niches, the walls of which present faint traces of red and yellow painting. At the foot and on the sides of the hill are scattered ruins, which were formerly the walls of habitations. The whole was encircled by a wall eight feet in breadth, and it was probably of considerable height, for some of the parts now standing are twelve feet high, though the average height does not exceed three or four feet. The mania for digging for treasures every year makes encroachments on these vestiges of a bygone age, whose monuments are well deserving of a more careful preservation."

De la Vega adds that the name by which the Peruvians called the devil was Capay, which they never pronounced but they spit, and showed other signs of detestation. Their principal sacrifice to the sun were lambs, but they offered also all sorts of cattle, fowls, and corn, and even their best and finest clothes, all which they burned in the place of incense, rendering their thanks and praises to him, for having sustained and nourished all those things, for the use and support of mankind; they had, also, their drink-offerings, made from maize; and when they first drank after their meals (for they never drank while they were eating), they dipped the tip of their finger into the cup, and lifting up their eyes with great reverence to heaven, gave the sun thanks for their liquor, before they presumed to take a draught of it; and here he takes an opportunity to assure us, that the Incas always detested human sacrifices, and would not suffer any such in the countries under their dominion, as they had heard that the Mexicans, and some other countries did. He admits that the ancient Peruvians sacrificed men to their gods.

The oracle at Rimac was consulted before the introduction of the worship of the sun by the Incas. "The valley of Rimac," says De la Vega, "lies four leagues to the northwards of Pachacamac, and received its name from a certain idol of the figure of a man, that spoke, and answered questions like the oracle of Apollo at Delphos. The idol was seated in a magnificent temple, to which the great lords of Peru either went in person, or inquired by their ambassadors, of all the important affairs relating to their provinces; and the Incas themselves held this image in great veneration, and consulted it after they conquered that part of the country.

De la Vega, who was descended from the Incas, makes a remarkable concession in relation to the Peruvians worshipping Pachacamac, the almighty invisible God, before the Incas introduced the worship of the sun. The royal historian assures us the Peruvians acknowledged one almighty God, maker of heaven and earth, whom they called Pacha Camac, Pacha in their language signifying the universe, and Camac the soul. Pacha Camac, therefore, signified him who animated the world. They worshipped him in their hearts as the unknown God. This doctrine was more ancient than the time of the Incas, and dispersed through all the kingdoms, both before and after the conquest. They believed that he was invisible, and therefore built no temples to him, except one in the valley of Pacha Camac, dedicated to the Unknown God, which was standing when the Spaniards arrived in Peru; neither did they offer him any sacrifices, as they did to the sun, but showed, however, the profound veneration they had for him, by bowing their heads, lifting up their eyes, and by other outward gestures, whenever his sacred name was mentioned. Though he was seldom worshipped, because they knew so little of him, or in what manner he ought to be adored.

De la Vega describes the principal rites and ceremonies in the religion of the Incas. He informs us they had four grand festivals annually, besides those they celebrated every moon. The first of their great feasts, called Raymi, was held in the month of June, immediately after the summer solstice, which they did not only keep in honour of the sun, that blessed all creatures with its heat and light, but in commemoration of their first Inca, Manca Capac, and Coya Mana Oclo, his wife and sister, whom the Inca looked upon as their first parents, descended immediately from the sun, and sent by him into the world to reform and polish mankind.

They fasted three days, as a preparative to the feast, eating nothing but unbaked maize and herbs and drinking water. The morning being come, the Inca, accompanied by his brethren and near relations, drew up in order, according to their seniority, went in procession at break of day to the market place, in Cusco, barefoot, where they remained looking attentively towards the east in expectation of the rising sun, which no sooner appeared than they fell down and adored the glorious luminary with the most profound veneration, acknowledging him to be their god and father.

The king rising upon his feet (while the rest remained in a posture of devotion), took two great gold cups in his hands, filled with their common beverage made of Indian corn, and invited all the Incas, his relations, to partake with him and pledge him in that liquor. The Caracas and nobility drank of another cup of the same kind of liquor, prepared by the wives of the sun; but this was not esteemed so sacred as that consecrated by the Inca.

The Inca offered the vases or golden bowls, with which he performed the ceremony of drinking, and the rest of the royal family delivered theirs into the hands of the priests. Then the priests went out into the court and received from the Caracas and governors of the respective provinces their offerings, consisting of gold and silver vessels, and the figures of all sorts of animals cast of the same metals.

These offerings being made, great droves of sheep and lambs were brought; out of which the priests chose a black lamb, and having killed and opened it, made their prognostics and divinations thereupon relating to peace or war, and other events, from the entrails of the beast; always turning the head of the animal to the east when they killed it.

After the first lamb, the rest of the cattle provided were sacrificed, and their hearts offered to the sun; and their carcases were flayed and burnt, with fire lighted by the sun's rays, contracted by a piece of crystal, or something like a burning-glass. They never make use of common fire on these occasions, unless the sun was obscured. Some of the fire was carried to the temple of the sun, and to the cloister of the select virgins, to be preserved the following year without extinction.

The sacrifice being over, they returned to the market-place, where the rest of the cattle and provisions were dressed and eaten by the guests; the priests distributing them first to the Incas and then to the Caracas and their people in their order; and after they were done eating, great quantities of liquor were brought in.

It should have been observed, that the people fell down on their knees and elbows when they adored the sun, covering their faces with their hands; and it is remarkable that the Peruvians expressed their veneration for the temple, and other holy places, by putting off their shoes, as the Chinese, the people of the East Indies, and other Asiatics do, though at the greatest distance from them, and not by uncovering their heads, as the Europeans do at divine service.

The nuns of Cusco were all of the whole blood of the Incas, dedicated to the sun, and called the wives of the sun. The select virgins in the other provinces were either taken out of such families as the Incas had adopted, and given the privilege to bear the name of Incas, or out of the families of the Caracas and nobility residing in the respective provinces, or such as were eminent for their beauty and accomplishments: these were dedicated to the Inca, and called his wives.

As to the notions the Peruvians had of a future state, it is evident they believed the soul survived the body, by the

Incas constantly declaring they should go to rest, or into a state of happiness provided for them by their god and father the sun, when they left this world.

Manco Capac not only taught all his subjects to adore his father (the sun), but instructed them also in the rules of morality and civility, directing them to lay aside their prejudices to each other, and to do as they would be done by. He ordained that murder, adultery, and robbery should be punished with death; that no man should have but one wife; and that in marriage they should confine themselves to their respective tribes.

Besides the worship of the sun, they paid some kind of adoration to the images of several animals and vegetables that had a place in their temples. These were the images brought from the conquered countries, where the people adored all manner of creatures, animate or inanimate; for whenever a province was subdued, their gods were immediately removed to the temple of the sun at Cusco, where the conquered people were permitted to pay their devotions to them, for some time at least, for which there might be several political reasons assigned.

"The bodies of the Incas were embalmed and placed in the temple of the sun, where divine honours were paid them, but their hearts and bowels were solemnly interred in a country place of the Incas, about two or three leagues from Cusco, where magnificent tombs were erected, and great quantities of gold and silver plate and other treasures buried with them; and at the death of the Incas and Caracas, or great lords, their principal wives, favourites, and servants either killed themselves, or made interest to be buried alive with them in the same tomb, that they might accompany them to the other world," says De la Vega, "and renew their immortal services in the other life, which, as their religion taught them, was a corporeal, and not a spiritual state." And here he corrects the errors of those historians who relate, that these people were killed or sacrificed by the successors of the deceased prince, which he seems to abhor; and observes further, that there was no manner of occasion for any law or force to compel them to follow their benefactors or masters to the other world; for when these were dead, they crowded after them so fast, that the magistrates were forced sometimes to interpose, and by persuasion, or by authority, to put a stop to such self-murders, representing that the deceased had no need of more attendants, or that it might be time enough to offer him their service when death should take them out of this world in a natural way.

What the form or dimensions of the Temple of the Sun were, neither De la Vega nor any other writer pretend to describe; but relate, that amongst all their buildings, none were comparable to this temple. It was enriched with the greatest treasures, every one of the Incas or emperors adding something to it, and perfecting what his predecessor had omitted.

The image of the sun was of a round form, consisting of one plate of gold, twice as thick as the plates that covered the walls. On each side of this image were placed the several bodies of the deceased Incas, so embalmed, it is said, that they seemed to be alive. These were seated on thrones of gold, supported by pedestals of the same metal, all of them looking to the west, except the Inca Haana Capac, the eldest of the sun's children, who sat opposite to it.

Besides the chapel that contained the sun, there were five others of a pyramidal form, — the first being dedicated to the moon, deemed the sister and wife of the sun. The doors and walls thereof were covered with silver, and here was the image of the moon, of a round form, with a woman's face in the middle of it. She was called Mama Quilca, or Mother Moon, being esteemed the mother of their Incas; but no sacrifice was offered to her as to the sun.

Next to this chapel was that of Venus, called Chasca, the Pleiades, and all the other stars. Venus was much esteemed as an attendant on the sun, and the rest were deemed maids of honour to the moon. This chapel had its walls and doors plated with silver, like that of the moon; the ceiling representing the sky, adorned with stars of different magnitudes.

The third chapel was dedicated to thunder and lightning, which they did not esteem as gods, but as servants of the sun; and they were not represented by any image or picture. This chapel, however, was sealed and wainscotted with gold plates like that of the sun.

The fourth chapel was dedicated to Iris, or the rainbow, as owing its origin to the sun. This chapel was also covered with gold, and had a representation of the rainbow on one side of it.

The fifth apartment was for the use of the high priest, and the rest of the priests, who were all of the royal blood; not intended for eating or sleeping in, but was the place where they gave audience to the sun's votaries, and consulted concerning their sacrifices. This was also adorned with gold from the top to the bottom, like the chapel of the sun.

Though there were no other image worshipped in this temple but that of the sun, yet they had the figures of men, women and children, and all manner of birds, beasts, and other animals of wrought gold, placed in it for ornament.

The Indians not only adorned themselves, their houses, and temples with gold, but buried it with them when they died. They also buried and concealed gold from the Spaniards; but never purchased houses or lands with it, or esteemed it the sinews of war, as the Europeans do.

It has been observed that the Burmese at the present day make use of their gold for ornamenting their temples, but employ none as a medium of circulation or commerce.

Diodorus Siculus relates that the Egyptians worshipped the sun under the name of Osiris, as they did the moon by the name of the goddess Isis.

Techo, the Jesuit, relates that the natives of La Plata, which is contiguous to Peru, worship the sun, moon, and stars; and in some part of the country the Jesuits relate that they worshipped trees, stones, rivers, and animals, and almost everything animate and inanimate. One of the objects of their adoration was a great serpent.

The Hindoos record two races of their early monarchs, and claim for them a supernatural descent — one from Surya, the sun; the other from Indú, the moon. These solar and lunar kings are said to have, between them, ruled the countries of India for, as Jones calculates, thirty-two generations. Another dynasty, sprung from the lunar branch, is said to have eclipsed them both. This was the line of the kings of Magadha, found by the Greeks in the provinces of the Ganges. Chandragupta, who is said to have usurped their power, is believed to be the Sandracottus who received the ambassadors of Seleucus, and whose seat of government was at Palibothra.

Wilkes, in the United States exploring expedition, remarks that at the Tonga Islands, though it is not known that any person is actually worshipped, as elsewhere, there are two high chiefs, whose official titles are Tuitonga and Veati, and a woman called Tamaha, who are believed to be descended from the gods, and are treated with reverence on that account by all, not excepting the king, who regards them as his superiors in rank. In New Zealand the great warrior-chief, Hongi, claimed for himself the title of a god, and was so called by his followers. At the Society Islands, Tamatoa, the last heathen king of Raiatea, was worshipped as a divinity. At the Marquesas there are, on every island, several men who are termed *atua*, or gods, who receive the same adoration, and are believed to possess the same powers as other deities. In the Sandwich Islands the reverence shown to some of the chiefs borders on religious worship. At the Depeyster's group, the westernmost cluster of Polynesia, we were visited by a chief, who announced himself as the *atua* or god of the islands, and was acknowledged as such by the other natives.

This singular feature in the religious system of the Polynesians, appearing at so many distant and unconnected points, must have originated in some ancient custom, or some tenet of their primitive creed, coeval, perhaps, with the formation of their present state of society. There is certainly no improbability in the supposition that the law-giver, whose decrees have come down to us in the form of the tabu system, was a character of this sort — a king, invested by his subjects with the attributes of divinity. It is worthy of remark, that in all cases in which we know of living men having been thus deified, they were chiefs of high rank, and not ordinary priests (tufuna), or persons performing the sacerdotal functions.

But of all the qualities that distinguish this race, there is none which exerts a more powerful influence than their superstition; or, perhaps, it would be more just to say, their strong religious feeling. When we compare them with the natives of Australia, who, though not altogether without the idea of a god, hardly allow this idea to influence their conduct, we are especially struck with the earnest devotional tendencies of this people, among whom the whole system of public polity, and the regulation of their daily actions, have reference to the supposed sanction of a supernatural power; who not only have a pantheon surpassing, in the number of divinities and the variety of their attributes, those of India and Greece, but to whom every striking and natural phenomenon, every appearance calculated to inspire wonder and fear, — nay, often the most minute, harmless, and insignificant objects, seem invested with supernatural attributes, and worthy of adoration. It is not the mere grossness of idolatry, for many of them have no images, and those who have look upon them simply as representations of their deities; but it is a constant, profound, absorbing sense of the ever-present activity of divine agency, which constitutes the peculiarity of this element in the moral organisation of this people.

Yet, this religious feeling is wholly independent of morality, to which the Polynesians lay no manner of claim. They expose their children, sacrifice them to idols, bury their parents alive, indulge in the grossest licentiousness, lie and steal beyond example, — yet they are devout.

Father Leander, of the order of bare-footed Carmelites, says, in describing Balbec, that by following the road by the cavern, to the extent of 50 paces, an ample area of a spherical figure presents itself, surrounded by majestic columns of granite, some of them of a single piece, and others formed of two pieces, the whole of them of so large a dimension that two men can with difficulty girt them. They are of the Ionic order of architecture, and are placed on bases of the same stone, at such distances from each other that a coach and six might commodiously turn between them. They support a flat tower or roof, which projects a cornice with figures wrought with matchless workmanship; these rise above the capitals with so nice a union, that the eye, however perfect it may be, cannot distinguish the part in which they are joined. At the present time the greater part of this colonnade is destroyed, the western part alone remaining perfect and upright. This fabric has an elevation of 500 feet, and is 400 feet in length:

$$\begin{aligned} 500 \text{ feet} &= 432 \text{ units} \\ 400 \quad &= 345{\cdot}5 \end{aligned}$$

$$(3 \times 342, \&c.)^3 = 1028^3 = \text{distance of moon}$$

$$(3 \times 432)^3 = 1296^3 = 2 \text{ distance of moon}$$

$$(2 \times 342)^3 \times 243 = \text{circumference.}$$

In the year 1773, two monks, Fathers Gracès and Font, after a journey of nine days from the presidency of Horcasitas, arrived at a fine open plain at the distance of a league from the south bank of the river Gila. There they found the ruins of an ancient Aztec city covering an extent of about a square league; in the midst rose an edifice called Casa Grande. This great house accords exactly with the cardinal points, and has from north to south a length of 136 metres, and from east to west 84 metres. A wall with towers surrounds this edifice. Vestiges of an artificial canal for conducting the water from the Gila to the city were found:

$$\begin{aligned} 136 \text{ metres} &= 446 \text{ feet} = 385 \text{ units} \\ 84 \quad &= 275 \quad\quad = 238 \end{aligned}$$

$$385^3 = \tfrac{1}{2} \text{ circumference}$$

$$238^3 = \tfrac{1}{81} \text{ distance of moon.}$$

Cube of sum of 2 sides

$$= (385 + 238)^3 = 623^3 = \tfrac{2}{9} \text{ distance of moon}$$

$$(3 \times 623)^3 = \tfrac{2}{9} \times 3^3 = 6$$
$$(5 \times 3 \times 623)^3 = 6 \times 5^3 = 750.$$

10 cubes of 15 times sum of 2 sides

= 7500 distance of moon = distance of Saturn,
20 cubes ,, = ,, Uranus,
60 cubes ,, = ,, Belus.

9 cubes of sum of 2 sides

= 2 distance of moon = diameter of orbit.

Cube of perimeter $= \frac{16}{9} = \frac{4^2}{3^2}$ distance of moon.

Cube of less side : cube of 2 sides

:: $238^3 : 623^3$
:: $\frac{1}{81} : \frac{2}{9}$ distance of moon
:: 1 : 18

$385^3 = \frac{1}{2}$ circumference
$(60 \times 385)^2 = \frac{1}{2} \times 60^3 = \frac{1}{2}$ 216000 circumference.

2 cubes of 60 times greater side

= 216000 circumference = distance of Belus
$(9 \times 238)^3 = \frac{1}{81} \times 9^3 = 9$ distance of moon.

Cube of 9 times less side
= 9 times distance of moon.

The ruins of the ancient city covered about a square league.

Taking a league at three English miles, a side would = 3 × 18·79 = 56·37 stades.

If side = 60 stades,

then cube of side = cube of 60 stades = $\frac{1}{8}$ cube of Babylon.

Cube of twice the side, or of 2 × 60 stades would = cube of 120 stades = cube of Babylon = distance of Belus.

"At Mal-Amir," says de Bode, "in the middle of the plain, rises an immense artificial mound, the dimensions of which are certainly not less imposing than those at Shush and Babylon. It is surrounded by broken and uneven ground; but a luxuriant carpet of green grass conceals its structure from the inquisitive eye. Its external form and appearance resembling the Susian and Babylonian mounds, and the circumstance of cuneatic inscriptions being found in its vicinity, bespeak the high antiquity of the place, and afford a strong argument in favour of the existence here, in former times, of a considerable fort, corroborating my impression that Mal-Amir is the site of the Uxian town besieged by Alexander."

If the triangle and pyramid of Belus be divided like *Fig.* 68., then the several sections will represent the Babylonian broad arrow. The straight lines intercepted by the two apices of each double set of triangles are equal. The areas of all the single triangles are equal, and therefore of each double set. The content of each of the differential solid sections, the difference between the 1st and 2nd, the 2nd and 3rd pyramid, &c., are also equal. The two opposite triangles which form the arrow head are similar and equal. Each of these two triangles, though equal to each of the other triangles in every set, are similar only to each other. Or the arrow heads, though dissimilar to each other, have their breadths, areas and contents, equal.

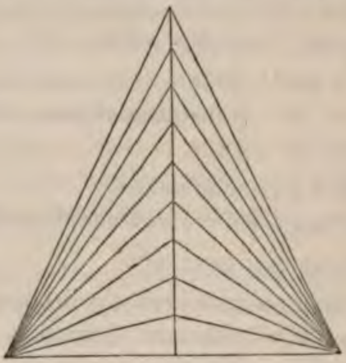

Fig. 68.

Thus when the heights of the triangles and pyramids vary as 1, 2, 3, &c., while their bases are equal or common, then

Height $\propto$ 1, 2, 3, 4, 5, 6, &c.
Difference ∞ 1, 1, 1, 1, 1, 1, &c.

Areas $\propto$ 1, 2, 3, 4, 5, 6, &c.
Difference $\propto$ 1, 1, 1, 1, 1, 1, &c.

Solids $\propto$ 1, 2, 3, 4, 5, 6, &c.
Difference $\propto$ 1, 1, 1, 1, 1, 1, &c.

No arches were found in the teocallis, but, as a substitute, large bricks were placed horizontally, so that the upper course passed beyond the lower, which supplied in a measure the gothic arch, like those found in several Egyptian edifices; whence it is inferred that the inhabitants of both countries were ignorant of the method of constructing arches. We should say that the arch was not found because it was not admissible in the obeliscal style of architecture, since the curve never appears in the construction of the obeliscal series of squares; but the overlapping of bricks is said to have supplied in a measure the gothic arch. This is the obeliscal arch, if it may be so called. The framework or mould for such an arch would be the obeliscal series of squares, where the sections, as they increase from the apex, project laterally beyond each other. These projections have their sides bounded by vertical and horizontal straight lines, so that each layer of bricks would be necessarily placed horizontally, and the upper course project beyond the lower as the sides of the arch approached the apex.

It has been seen that such obeliscal or parabolic and hyperbolic arches, which symbolise the laws of gravitation, can be constructed in a variety of ways.

Druidical Remains in England.

Those in Cumberland.—About three miles south-west of Castle Sowerby is a stupendous mountain, called Carrock Fell, being 803 yards above the level of the sea, and 520 yards above the surrounding meadows. The whole of this mountain is a ridge of horrid precipices, abounding with chasms, not to be fathomed by the eye. Close under it, for nearly two miles, is a winding path, just wide enough for a horse to pass singly, and everywhere intercepted by enormous stones, which have fallen from the summit of the mountain. In the year 1740, a cavern was discovered at the end of it, which has never been explored: near which is a remarkable pool of water, called Black Hole, 150 yards in circumference, and in some places 65, and in others 45, fathoms deep. The eastern end of Carrock Fell, for upwards of a mile in length, is almost covered with masses of granite of various sizes, some of them not less than 300 tons in weight; and on the highest part is a singular monument of antiquity, of which the following description is given in the history of Cumberland.

The summit of this huge fell is of an oval form. Round its circumference is a range or enclosure of stones, which seem to be incontestably the work of men's hands. The stones of the sides of the enclosed area are about eight yards perpendicular below the ridge of the mountain, but at the ends not more than four. In some places, however, the height is six feet, in others three only, or even less; this variation is probably owing to a practice continued from age to age of rolling some of the stones down the sides of the mountain for amusement, or rather from a desire of witnessing the effects of their increasing velocity. The stones are in general from one to two or three, and even four hundred-weight; but many of them are considerably smaller. From the few stones that may be found within the area, it would seem that the whole range has been formed by the stones obtained in the enclosed space, which is nearly destitute of vegetation.

The direction of the ridge of the top of the fell in its transverse diameter is nearly east and west; and in this direction within the surrounding pile of stones it measures 252 yards: the conjugate diameter is 122 yards, and the content of the space enclosed is about three acres and a half. The entrances are four, one opposite each point of the compass; those on the west and south sides are four yards in width; that on the east appears to have been originally of the same dimensions, but is now about six yards wide; the width of the northern entrance is eight yards. Besides these on the north-west quarter there is a large aperture or passage twelve yards in width; which, if the nature of the ground is attended to, and the apparent want of stones in this part considered, seems never to have been completed.

At the distance of 66 yards from the east end of this range, on the summit of the hill, stands an insulated pile of stones, appearing at a little distance like the frustum of a cone. Its base is about 11 yards in diameter, and its perpendicular height 7 yards. On clambering to the top, the interior is found to be funnel-shaped; the upper part or top of the funnel being five yards diameter; but as the hollow gradually slopes downwards, the width at the bottom is little more than two feet: the largest stones appear to weigh about $1\frac{1}{2}$ cwts.

The crowned head of Old Carrock is by no means perfectly uniform, the end to the westward being about 15 yards higher than the middle of the oval. On the highest point is a fragment of rock projecting about three yards above the surface of the ground, having stones heaped up against two of its sides, and at a distance assuming the appearance of the one just described, though of twice its magnitude. Both these piles seem to be coeval with the surrounding range, but there are other smaller heaps that are evidently of modern contrivance, and appear to have been erected, speaking locally, as ornaments to the mountain. The name given to this monument by the country people is the Sunken Kirks.

Transverse diameter = 252 yards = 756 feet = 653·6 units
Conjugate diameter = 122 „ = 366 „ = 317 „
$20 \times 653 = 13040$
Distance of Jupiter = 13040^3

$30 \times 319 = 9570$
Diameter of orbit of earth = 9560^3
Sum of diameters = $653 + 319 = 972$

$30 \times 972 = 29160$
Distance of Belus = 29160^3

Cube of 20 times greater diameter
= distance of Jupiter.

Cube of 30 times less diameter
= diameter of orbit of earth.

Cube of 30 times sum of 2 diameters
= distance of Belus.

Mean of 2 diameters $= \frac{1}{2}(653+317) = 485$
$485^3 =$ circumference.

Cube of mean = circumference

$$(2\times485)^3 = 8$$
$$(30\times2\times485)^3 = 8 \times 30^3 = 216000 \text{ circumference.}$$

Cube of 30 times sum of 2 diameters
= distance of Belus.

These diameters are within the pile of stones: the breadth of the pile is not stated.

Circumference of circle having diameter 655 &c.
= 2056 units
$\frac{1}{2} = 1028$

$1028^3 =$ distance of moon
$(2\times1028)^3 = 8$

Cube of $\frac{1}{2}$ circumference = distance of moon
Cube of circumference = 8 „ „
If diameter of a circle = 648 units
diameter$^3 = 648^3 =$ cube of Cheops $= \frac{1}{4}$ distance of moon,
circumference^3 will $= \frac{31}{4}$.
But if diameter = 655 &c. units
cube of circumference will $= \frac{32}{4} = 8$ distance of moon.

If diameter of a circle = 1
circumference = 3·1415 &c.
square of circumference = 9·869 &c.
cube of circumference = 31·004 &c.

Cube of diameter : cube of circumference :: 1 : 31
Square of diameter : square of circumference
:: 1 : 9·869 &c.

Circumference of earth : distance of moon
:: 1 : 9·55 &c.

Internal diameters are 653 and 317 units.

Cylinder having height = 321
and diameter of base = 657
will $= 321 \times 657^2 \times {\cdot}7854$
$= \frac{1}{10}$ or $\frac{3}{30}$ distance of moon.
Spheroid $= \frac{2}{30}$ „
Cone $= \frac{1}{30}$ „

Cylinder of 10 times dimension will $= \frac{1000}{10}$
= 100 distance of moon.

Cylinder of 20 times dimension will $= 100 \times 2^3$
= 800 distance of moon.

= diameter of orbit of earth.

Less internal diameter = 317 units.
Circumference of circle of diameter 317=996
$996^3 = \frac{70}{8}$ circumference
$(2 \times 996)^3 = 70$
Cube of twice circumference of circle
= 70 times circumference of earth.

Sum of 2 circumferences $= 2056 + 996 = 3052$ units.
$2 \times 3052 = 6104$

610^3 &c. = 2 circumference
$(10 \times 610$ &c.$)^3 = 2000$
Cube of 2 sum of 2 circumferences
(or of 4 times mean)
= 2000 times circumference of earth.

The frustum of the cone of stones has a diameter of 11 yards=33 feet=28·58 units; circumference will=89·6 units.

$898^5 = \frac{2}{3}$ distance of moon.

3 cubes of 10 times circumference = diameter of orbit of moon.

If diameter=28·3 units.
$(10 \times 28{\cdot}3)^3 = \frac{1}{5}$ circumference.
$(10 \times 10 \times 28{\cdot}3)^3 = \frac{1000}{5} = 200$ circumference.

Cube of 100 times diameter = 200 circumference of earth; therefore, cube of 40 times circumference = 400.

Cylinder having height = diameter of base = 27, &c., units,

= $\frac{3}{60}$ degree.
= 3 minutes.
Sphere - = $\frac{2}{3}$ - = 2 ,,
Cone - - = $\frac{1}{3}$ - = 1 ,,

Cone = 1 minute = 1 geographical mile.

$$(10 \times 89{\cdot}8)^3 = \tfrac{2}{3} \text{ distance of moon.}$$
$$(3 \times 10 \times 89{\cdot}8)^3 = \tfrac{2}{3} \times 3^3 = 18.$$
$$(5 \times 3 \times 10 \times 89{\cdot}8)^3 = 18 \times 5^3 = 2250.$$

10 cubes of 150 times circumference = 22,500 distance of moon

= distance of Belus.
$\frac{1}{15}$ cube - - = ,, Mercury;

or, $\frac{1}{15}$ cube of 150 times circumference = 150 times distance of moon = distance of Mercury.

Should diameter of cone = 29·16 units,
circumference will = 91·6.

$1000 \times 29{\cdot}16$ = 29160
Distance of Belus = 29160^3
$60 \times 91{\cdot}6$ = 5496
Distance of Mercury = 5490^3.

Cube of 1000 times diameter = distance of Belus.

Cube of 60 times circumference = distance of Mercury.

Mean distance of Mercury may be between 5460^3 and 5490^3.

"There is a conical hill, called Tagsher, in Western Barbary; near which, as I learned from the kaid, are some curious ruins. He described them as being those of a large castle, built of extraordinary materials, every stone of which being of such a size that no hundred men of modern times could move it; some of them, he said, were as much as twenty feet square, and about fifteen feet high.

He described the entrance as having been blocked up by earth and sand, except in one place through which he entered and proceeded some distance under ground; the passage becoming at last so narrow that he could not advance further, although by light he perceived it was of yet greater extent. At a short distance from the building lay a flat stone, which he lifted up, and found beneath it a pit, that, by his description, was of an inverted conical form: it was empty." —(*Hay.*)

At the village of Salkeld, on the summit of a hill, is a large and perfect Druidical monument, called by the country people Long Meg and her Daughters. A circle of about eighty yards in diameter is formed by massy stones, most of which remain standing upright. These are sixty-seven in number, of various qualities, unhewn or untouched with any tool, and seem by their form to have been gathered from the surface of the earth. Some are of blue and gray limestone, some of granite, and some of flints. Many of such of them as are standing measure from twelve to fifteen feet in girt, and ten feet high; others are of an inferior size. At the southern side of the circle, at the distance of eighty-five feet from its nearest member, is placed an upright stone, naturally of a square form, being of red freestone, with which the country about Penrith abounds. This stone, placed with one of its angles towards the circle, is nearly fifteen feet in girt, and eighteen feet high, each angle of its square answering to a cardinal point. In that part of the circle most contiguous to the column, four large stones are placed in a square form, as if they had constructed or supported the altar; and towards the east, west, and north, two large stones are placed, at greater distances from each other than any of the rest, as if they had formed the entrances into this mystic round. What creates astonishment to the spectator is, that no such stones, nor any quarry or bed of stones, are to be found within a great distance of this place; and how such massy bodies could be moved, in an age when we may suppose the mechanical powers were little known, is not easily to be determined.

Diameter of circle = about 80 yards = 240 feet = 208 units.

Cylinder having height = diameter of base = 208 nuits will

$= \quad 208^3$, &c. $\times \cdot 7854 = \frac{1}{16}$ circumference.
$\frac{2}{3} =$ Inscribed sphere $= \frac{1}{24}$,,
$\frac{1}{3} =$ Inscribed cone $= \frac{1}{48}$,,

If diameter = 238, &c., feet, circumference = 749 feet = 648 units = side of base of pyramid of Cheops.

Then cube of circumference $= 648^3 = \frac{1}{4}$ distance of moon.

4 cubes = distance of moon from earth.

If diameter = 208 units,
208^3, &c. $= \frac{8}{100}$ circumference.
$(10 \times 208$, &c.$)^3 = \frac{8000}{100} \quad = 80.$
$(5 \times 10 \times 208$, &c.$)^3 = 80 \times 5^3 = 1000$ circumference.
Cube of 50 times diameter = 1000 circumference.

$(6 \times 5 \times 10 \times 208$, &c.$)^3 = 1000 \times 6^3 = 216{,}000.$
Cube of 300 times diameter = 216,000 circumference.
= distance of Belus.
Pyramid - - = ,, Uranus.

Should diameter = 209 units,
Circumference will = 657, &c.
657^3, &c. $= \frac{10}{4}$ circumference.
$(4 \times 657)^3 = \frac{10}{4} \times 4^3 = 160.$

9 cubes of 4 times circumference of circle
= 1440 circumference of earth
= distance of Mercury.
$(3 \times 4 \times 657)^3 = 160 \times 3^3 = 4320$ circumference.
100 cubes of 12 times circumference of circle,
= 432,000 circumference of earth
= diameter of orbit of Belus.

Circumference of circle, diameter 208 = 653,
$20 \times 653 = 13{,}060.$
Distance of Jupiter $= 13{,}040^3$.
Cube of 20 times circumference = distance of Jupiter.
Cube of 50 times diameter $= \frac{1}{2}$ distance of Jupiter.

Cube of 10 diameter $= (10 \times 1)^3 = 10^3$
Cube of 4 circumference $= (4 \times 3 \cdot 1416)^3 = 12 \cdot 5664^3$.

$$10^3=1000$$
$$12{\cdot}5^3\ \&c.=2000$$

Cubes are as 1 : 2.
Cube of 10 diameter $=\frac{1}{2}$ cube of 4 circumference.
Cube of diameter $=\frac{1}{2}$ „ $\frac{4}{10}$ „
Cube of circumference $=2$ „ $\frac{10}{4}$ diameter.

The cube of 10 times diameter being $=\frac{1}{2}$ cube of 4 times circumference must only be regarded as an approximation. Neither is the cube denoting planetary distances to be otherwise regarded.

We first used the cube for planetary distances in a rough manner only, not having tables for the higher numbers, and not then expecting to make so general a use of these expressions. When accurate measurements of ancient monuments have been made, the expressions of planetary distances ought also to be corrected.

The same observations will apply to

$\frac{10}{4}$ diameter $=2{\cdot}5$
$\frac{4}{10}$ circumference $=1{\cdot}256$ &c.
$\frac{10}{4}$ diameter : $\frac{4}{10}$ „ :: 2 : 1.

Cubes are as 8 : 1.
Cube of 10 diameter $=\frac{1}{2}$ cube of 4 circumference
Cube of $\frac{4}{10}$ circumference $=\frac{1}{8}$ „ $\frac{10}{4}$ diameter.

Several Druidical circles, and other remains of antiquity, are to be seen in the neighbourhood of Black-comb; the most remarkable of which is the Druidical temple called Sunken Kirk, situated in the level part of a wet meadow, about a mile east from this mountain. It is a circle of large stones, and is thus described by Gough: — "At the entrance are four large stones, two placed on each side at the distance of 6 feet; the largest, on the left-hand side, is 5 feet 6 inches in circumference. Through this you enter into a circular area, 29 yards by 30. The entrance is nearly south-east: on the north or right-hand side is a huge stone, of a conical form, its height nearly 9 feet. Opposite the entrance is another large stone, which has once been erect, but has now fallen within the area; its length is 8 feet. To the left hand, to the south-west, is one, in height 7 feet, in circumference 11 feet 9 inches. The altar probably stood in the middle, as there are some stones still to be seen, though sunk deep in the earth. The circle is nearly complete, except on the western side, where some stones are wanting; the large stones are 31 or 32 in number. The outward part of the circle, upon the sloping ground, is surrounded with a buttress, or rude pavement of small stones, raised about half a yard from the surface of the earth. The situation and aspect of the Druidical temple near Keswick is, in every respect, similar to this, except the rectangular recess formed by ten large stones, which is peculiar to that at Keswick; but upon the whole (I think), the preference will be given to this, as the stones appear much larger, and the circle more entire."

If diameter = 30 yards = 90 feet, circumference = 281 feet = 1 stade = 243 units. Transpose 2 and 3, or read the figures backwards, and 342 is expressed, which, multiplied by 2, and raised to the power of $2 = \overline{684}^2$, and $\overline{684}^2 \times 243 =$ circumference of the earth.

Thus by means of a circle, having a circumference of 1 stade, the Druids could show that the circumference of the earth equalled $\overline{684}^2$ stades, or $\overline{684}^2 \times 243$ units.

Or circumference of circle : circumference of the earth $:: 1 : \overline{684}^2$.

Circumference = 1 stade = 243 units; 243 transposed, by placing 3 the first, = 324; and $324 \times 2 = 648 =$ side of base of pyramid of Cheops; the cube of which $= 648^3 = \frac{1}{4}$ distance of the moon.

4 cubes $= 4 \times 648^3 =$ distance of the moon from the earth.

Cube of circumference of circle $= 243^3 = \frac{1}{8}$ circumference of the earth.

Cube of twice circumference of circle = circumference of the earth.

Cube of 120 times circumference of circle = cube of 120 stades = cube of Babylon = distance of Belus.

The circumference of circle at Black-comb = the height of the tower of Belus.

Diameter 29 yards = 75·2 units.

Cylinder having height = diameter of base = 74 units will = 1 degree = $\frac{1}{360}$ circumference

Sphere = $\frac{2}{3} = \frac{2}{3}$ „

Cone = $\frac{1}{3} = \frac{1}{3}$ „

Circumference of circle, diameter 75·2 units = 236 &c., should circumference = 239.

$$40 \times 239 = 9560$$

diameter of the orbit of the earth = 9560^3.

Cube of 40 times circumference = diameter of the orbit of the earth.

Cube of 3 times 40 times circumference of 243 units = distance of Belus

Sphere = „ Neptune

Pyramid = „ Uranus

= diameter of the orbit of Saturn.

Diameter of circle = 29 yards = 87 feet = 75·2 units

circumference = 236 „

$$235^3 \text{ \&c.} = \tfrac{6}{500} \text{ distance of the moon}$$

$$(10 \times 235 \text{ \&c.})^3 = \tfrac{6000}{500} = 12$$

$$(5 \times 10 \times 235 \text{ \&c.})^3 = 12 \times 5^3 = 1500.$$

5 cubes of 50 times circumference = 7500 distance of moon

= distance of Uranus

15 cubes „ „ = „ Belus.

Diameter of circle = 30 yards = 90 feet = 77·8 units

circumference = 244 „

$$243^3 = \tfrac{1}{8} \text{ circumference}$$

$$(2 \times 243)^3 = 1 \quad „$$

$$(10 \times 2 \times 243)^3 = 1000.$$

Cube of 20 times circumference of circle

= 1000 times circumference of the earth

= $\frac{1}{36}$ distance of Saturn

= $\frac{1}{72}$ „ Uranus

= $\frac{1}{216}$ „ Belus.

$(30 \times 2 \times 243)^3 = 1 \times 30^3 = 27000.$

Cube of 60 times circumference of circle
= 27000 times circumference of the earth
= 10 times distance of Venus.

$(60 \times 2 \times 243)^3 = 1 \times 60^3 = 216000.$

Cube of 120 times circumference of circle
= 216000 times circumference of the earth
= distance of Belus.

Cube of twice circumference of circle = circumference of the earth.

Diameter of circle = 29 yards = 87 feet = 75·22 units
if = 75·84
circumference = 238·35.

$100 \times 75{\cdot}84 = 7584$
distance of the earth = 7584^3.
$40 \times 238{\cdot}5 = 9540$
diameter of the orbit of the earth = 9540^3.

Cube of 100 times diameter = distance of the earth
Cube of 40 times circumference = diameter of the orbit of the earth.

The cubes are as 1 : 2.

There is a Druidical circle on the summit of a bold and commanding eminence called Castle-Rigg, about a mile and a half on the old road, leading from Keswick, over the hills to Penrith. Castle-Rigg is the centre-point of three valleys that dart immediately under it from the eye, and whose mountains form part of an amphitheatre which is completed by those of Borrowdale on the west, and by the precipices of Skiddaw and Saddleback close on the north. Such seclusion and sublimity were indeed well suited to the dark and wild mysteries of the Druids.

The circle at present consists of about forty stones, of different sizes, all, or most of them, of dark granite; the highest about seven feet, several about four, and others considerably less. The form may with more propriety be called an oval, being 35 yards in one direction, and 33 yards in another, in which respect it assimilates exactly to that of Rollick, in Oxfordshire; but what distinguishes this from all other Druidical remains of a similar kind is the rectangular enclosure on the eastward side of the circle, including a space of about eight feet by four.

Diameters = 35 by 33 yards,
= 105 by 99 feet,
= 90·75 by 85·6 units.

Circumference of circle, diameter 90 = 283 &c. units,
" " " " 85 = 267 "

283^3 &c. $= \frac{1}{5}$ circumference,
266^3 &c, $= \frac{1}{6}$ "

Cube of circumference of greater diameter
$= \frac{1}{5}$ circumference of the earth.
Cube of circumference of less diameter
$= \frac{1}{6}$ circumference of the earth.
Cube of 5 times greater circumference
$= \frac{1}{5} \times 5^3 = 25 = 5^2$ circumference.
Cube of 6 times less circumference
$= \frac{1}{6} \times 6^3 = 36 = 6^2$ circumference.
Cube of 60 times greater circumference
$= \frac{1}{5} \times 60^3 = \frac{1}{5}\, 216000$ circumfer.
$= \frac{1}{5}$ distance of Belus.
Cube of 60 times less circumference
$= \frac{1}{6} \times 60^3 = \frac{1}{6}\, 216000$ circumfer.
$= \frac{1}{6}$ distance of Belus,
= distance of Saturn.
Sum of 2 circumferences = 283 + 266 = 549 units,
mean = 279 &c.
279^3 &c. $= \frac{1}{50}$ distance of the moon,
$= \frac{60}{50} = \frac{6}{5}$ radius of the earth.
Cube of mean $= \frac{6}{5}$ radius of the earth.
Cube of 10 times mean, or of 5 times sum
$= (10 \times 279 \text{ \&c.})^3 = \frac{1000}{50} = 20$ distance of the moon.

20 cubes of 10 times mean,
or of 5 times sum,
$= 20 \times 20 = 400$ distance of the moon,
= distance of the earth,
30 cubes = distance of Mars.

Sum of 2 diameters $= 90 + 85 = 175$ units.
175^3 &c. $= \frac{1}{200}$ distance of the moon.
$(10 \times 175 \text{ \&c.})^3 = \frac{1000}{200} = 5.$
Cube of 10 times sum of 2 diameters
= 5 times distance of the moon.

Cube of 20 times sum of 2 diameters
= 40 times distance of the moon,
$= \frac{1}{10}$ distance of the earth.
Or, 10 cubes = 400 distance of moon = distance of earth.

At West Kennet, in Wiltshire, there is a kind of walk about a mile long, which was once enclosed with large stones: on one side the enclosure is broken down in many places, and the stones taken away; but the other side is almost entire. On the brow of the hill near this walk is a round trench, enclosing two circles of stones, one within another: the stones are about 5 feet in height; the diameter of the outer circle 120 feet, and of the inner, 45 feet. At the distance of about 240 feet from this trench have been found great quantities of human bones, supposed to have been those of the Saxons and Danes who were slain at the battle of Kennet, in 1006.

Diameter of the outer circle = 120 feet,
= 104 units, say = 106.

Cylinder having height = diameter of base will
$= 106^3$ &c. $\times \cdot 7854$,
= 3 degrees.

Inscribed sphere $= \frac{2}{3}$ of $3 = 2$.
Inscribed cone $= \frac{1}{3}$ of $3 = 1$.

Diameter of inner circle = 45 feet.
Circumference = 141 feet $= \frac{1}{2}$ stade $= 121{\cdot}5$ units.
Twice circumference $= 2 \times 121{\cdot}5 = 243 = 3^5$.
243 transposed = 342.

$342 \times 2 = 684$.
684^2 = circumference in stades.
$684^2 \times 243$ = circumference in units.

Circumference of circle $= \frac{1}{2}$ stade.
2 circumference $=$ 1 stade.
$=$ side of tower of Belus.
Cube $= \frac{1}{8}$ circumference of the earth.

Cube of 4 times circumference = cube of 2 sides of tower = cube of side of square enclosure of the tower = circumference of the earth.

Diameter = 45 feet = 38·9 units.

Cylinder having height = diameter of base = 39 &c. units will $= \frac{1}{2400}$ circumference = 9 minutes.
Sphere $= \frac{2}{3} = \frac{1}{3600}$ „ $= 6$ „
Cone $= \frac{1}{3} = \frac{1}{7200}$ „ $= 3$ „

1 minute = 1 geographical mile.

Or, cylinder having height = diameter of base = 37 units will $= \frac{1}{8}$ degree.
Sphere $= \frac{1}{12}$ „
Cone $= \frac{1}{24}$ „

Diameter of outer circle = 104 units.

104^3 &c. $= \frac{1}{100}$ circumference.
$(10 \times 104$ &c.$)^3 = \frac{1000}{100} = 10$ „
or, 1044^3 = 10 circumference,
and 1028^3 = distance of the moon.

The transverse and conjugate diameters of the Druidical circles are often stated as differing from each other.

If one diameter of the outer circle of West Kennet = 104·4 units, cube of 10 times diameter = 10 times circumference of the earth.

If the other diameter = 102·8 units, cube of 10 times this diameter = distance of the moon.

Diameter of outer circle = 120 feet = 103·75 units.
Circumference = 326 „

$40 \times 326 = 13040$.
Distance of Jupiter $= 13040^3$

Cube of 40 times circumference = distance of Jupiter.
Cube of 100 times diameter $= \frac{1}{2}$ distance of Jupiter.

Diameter of inner circle = 45 feet = 38·9 units.
Circumference = 122 ,,

$300 \times 122 = 36600.$

Diameter of orbit of Belus $= 36600^3$.

Cube of 300 times circumference = diameter of orbit of Belus.

$122^3 = \frac{1}{600}$ distance of moon.
$(10 \times 122)^3 = \frac{1000}{600} = \frac{10}{6}.$
$(6 \times 10 \times 122)^3 = \frac{10}{6} \times 6^3 = 360.$

Cube of 60 times circumference = 360 distance of moon.
$(5 \times 6 \times 10 \times 122)^3 = 360 \times 5^3 = 45000.$

Cube of 300 times circumference = 45000 dist. of moon,
= diam. of orbit of Belus.

Cube of circumference $= \frac{1}{600}$ distance of the moon,
$= \frac{60}{600} = \frac{1}{10}$ radius of the earth.

Cube of 10 times circumference $= \frac{1000}{10} = 100$ radius of the earth.

Cube of $\frac{10}{4} \times 300$ times diameter = distance of Belus.

Diameter of outer circle = 120 feet = 103·75 units,
Circumference = 326 ,,

If circumference of a circle = 324 &c. units,
324^3 &c. $= \frac{3}{10}$ circumference,
$(10 \times 324$ &c.$)^3 = \frac{3000}{10} = 300.$

Cube of 10 times circumference = 300 circumf. of earth.
$(4 \times 10 \times 324$ &c.$)^3 = 300 \times 4^3 = 19200.$

Cube of 40 times circumference = 19200 circumference.
Distance of Jupiter = 19636 ,,

If circumference of a circle = 324 units,

$40 \times 324 = 12960$

$1296^3 = 6^{12}$ = diameter of orbit of the moon.
$(10 \times 1296)^3 = 1000$ diameters
= 2000 distance of the moon.

Distance of Jupiter = 2045 ,,

Cube of 40 times circumference = 2000 distance of moon;
but if circumference = 326,
the cube of 40 times circumference = distance of Jupiter;
$\therefore$ cube of 100 times diameter = $\frac{1}{2}$ distance of Jupiter.

The walk is about a mile in length.

1 mile = 18·79 stades = 4566 units,
if = 4770 „
Then, cube of length = $4770^3 = \frac{1}{4}$ distance of earth.
Cube of 2 length = $9540^3 = \frac{8}{4} = 2$ „
= diameter of orbit of the earth.

If = 4345,
$2 \times 4345 = 8690$.
Cube of twice length = 8690^3 = distance of Mars.

The two great Druidical temples of Avebury and Stonehenge are both in Wiltshire.

The mound at Avebury, according to Stuckeley, is in a situation that seems to leave no doubt that it was one of the component parts of the grand temple.

This artificial mound of earth is conical, and called Silbury hill; it is the largest tumulus in Europe, and one worthy of comparison with those mentioned by Homer, Herodotus, and other ancient writers.

The circumference of the hill, as near the base as possible, measures 2027 feet; the sloping height, 316 feet; the perpendicular height, 170 feet; and the diameter of the top 120 feet. This artificial hill covers the space of 5 acres and 34 perches. A proof that this wonderful work was raised before the Roman-British period is furnished by the Roman road from Bath to London, which is straight for some distance, till it reaches the hill, where it diverges to the south to avoid it, and then again continues its direct course. Many barrows are found in the neighbourhood, one of which the Roman road just mentioned has cut through. Other Druidical remains are found around Avebury, including circles, cromlechs, and stones erect, confirming the impression that this place must have been the greatest and most important of the kind in Britain. No marks of tools are anywhere visible on the stones of Avebury; they were set up in their rude natural grandeur. Two circles of stones, not concentric, are enclosed by a great circle of stones; a very deep circular trench was dug without these stones. The inner slope of this bank measures 80 feet, and circumference at the top was 4442 feet; the area thus enclosed was about 28 acres.

Silbury Hill. (*Fig.* 79.)

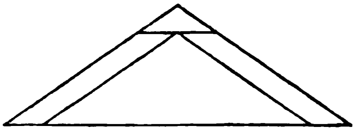

Fig. 79.

Height to platform	= 170 feet	= 147 units.
Circumference of base	= 2027 „	= 1828 „ .
Diameter „	= 645 „	= 557 „ .
Diameter of platform	= 120 „	= 104 „ .
Say, originally		= 115 „ .

Height to apex of external cone, according to these data, will = 186 units.

Height × area base,

$$= 186 \text{ \&c.} \times 557^2 \times \cdot 7854 = \tfrac{2}{5} \text{ circumference.}$$

External cone $= \frac{1}{3}$ of $\frac{2}{5} = \frac{2}{15}$ „ .

The internal cone will have the apex in the centre of the platform, height to apex = height to platform, and diameter of base = diameter of base of external cone, less diameter of platform $= 557 - 115 = 442$ units.

Height × area of base

$$= 148 \times 442^2 \times \cdot 7854 = \tfrac{1}{5} \text{ circumference.}$$

Internal cone $\frac{1}{3}$ of $\frac{1}{5} = \frac{1}{15}$ „ .

The two cones will be as $\frac{1}{15} : \frac{2}{15}$ circumference.
$:: 1 : 2$,, .

and the cones will be similar.

Cube of height to apex of external cone $= 186^3$ &c. $= \frac{6}{1000}$ distance of the moon.

Cube of height to apex of internal cone = cube of height to platform $= 148^3 = \frac{3}{1000}$ distance of the moon.

The cubes of the heights of the two cones are as $\frac{3}{1000} : \frac{6}{1000}$ distance of the moon, as $1 : 2$.

External cone less internal cone

= difference of cones,
= the sides of the hollow cone,
= the hollow cone,
= the internal cone,
= $\frac{1}{15}$ circumference = 24 degrees,

which is the reciprocal of the tower of Belus.

For the tower $= \frac{1}{24}$ circumference = 15 degrees.

The tower $= \frac{15}{24} = \frac{5}{8}$ internal cone.

Inclined side of internal cone = 266 units.

Cube of side $= 266^3$&c. $= \frac{1}{6}$ circumference.

The inclined side of internal cone will equal sloping side of hill.

Inclined side of internal cone = 266 units = 308 feet.

Sloping side by measurement = 316 feet.

Inclined side of external cone = sloping side of hill continued to apex = 335 &c. units,

and 335^3&c. $= \frac{1}{3}$ circumference.

Cubes of the inclined sides of the two cones are as

$\frac{1}{6} : \frac{1}{3}$ circumference,
as $1 : 2$.

The cubes of the diameters of their bases are in the same ratio.

Cube of 3 times circumference of base of external cone

$= (3 \times 1828)^3 = 5484^3$,
= distance of Mercury.

Cube of 3 times circumference of base of internal cone $= \frac{1}{2}$ distance of Mercury.

Cube of 10 times height of internal cone $= (10 \times 148\,\&c.)^3$ $= \frac{3000}{1000} = 3$ distance of moon.

50 cubes = 150 distance of moon.
= distance of Mercury.

Cube of 10 times height of external cone

$= (10 \times 186\,\&c.)^3 = \frac{6000}{1000} = 6$ distance of moon.
25 cubes = 150 distance of the moon.
= distance of Mercury.

A conical hill, having diameter of base = 2317 units, and height to apex = 773 units, will = distance of the moon.

The internal similar cone, having diameter of base = 1091 units, and height to apex = 364 units, will = circumference of the earth. *Fig.* 80.

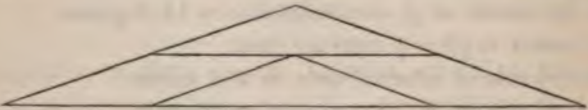

Fig. 80.

The apex of the internal cone will be in the centre of the platform of the truncated cone.

As there are many large mounds, both in Asia and America, with circular or rectangular bases, possibly one may be found to represent the distance of the moon combined with the circumference of the earth.

If the mound be circular the diameter of base should = 2317 units = 9·17 stades
$= \frac{1}{2}$ mile English nearly.

Height to platform = 364 units
= 421 feet English.

Many circles, like the Druidical, are surrounded by sloping entrenchments, or raised embankments, probably to represent the frustum of a cone, which would require less labour than the construction of an artificial mound, though in this case advantage would likely be taken of a natural hill, by forming it into the required dimensions.

If the height and sides of base were reduced to $\frac{1}{4}$ the dimensions of the supposed conical mound, the contents would be reduced to $\frac{1}{64}$ the supposed contents; but the proportion of the distance of the moon to the circumference of the earth would remain.

At Mount Barkal, in Upper Nubia, lat. 18° 25′, there was once a city: the remains prove it to have been an ancient establishment of priests, who possessed a kindred worship to that of Egypt. The temples lay between the mountain and the Nile.

It is not said whether the sides of Mount Barkal are circular or rectangular.

The height corresponds to the height of the supposed mound, or truncated cone, the circumference of which would = $1\frac{1}{2}$ mile.

The peculiar form of Mount Barkal, says Ruppel, must have fixed attention in all ages. From the wide plain there arises up perpendicularly on all sides a mass of sandstone, nearly 400 feet high, and about 25 minutes in circuit. The unusual shape of the mountain must have become still further an object of curiosity, from the phenomena with which it is connected. The clouds, attracted from all around to this isolated mass, descend in fruitful showers; and hence we need hardly wonder if, in ancient times, it was believed that the gods here paid visits to man, and held communion with him. Temple rose after temple; and who can say how far many a devotee came to ask advice of the oracle?

The circuit of 25 minutes would be about $1\frac{1}{2}$ mile.

The sides of Mount Barkal are perpendicular.

Height to platform × diameter of base of cone

$$= 364 \times 2317^2,$$

which will lie between 10 circumference
and distance of moon 9·55 ,,

So by a slight reduction of base and height we shall have a solid square terrace, having a height of 421 feet = distance of moon.

Perpendicular height of Mount Barkal = about 400 feet.

Fig. 81. If the square terrace = distance of the moon, and if, upon the platform, a cone be made similar and equal to the cone at the base, then we shall have a square terrace = distance of the moon, and cone on the platform = circumference of the earth.

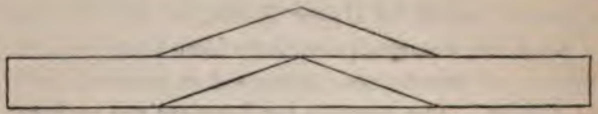

Fig. 81.

The height of the cone will equal height of terrace.

The Assyrian mound of Kóyunjik, at Nineveh, is 2563 yards in length, nearly = $1\frac{1}{2}$ mile English.

If a square mound or terrace had the side = 2563 yards
= 6638 units
and height = 12 &c.
the content would = distance of the moon.

The mound of Kóyunjik is bounded by a ditch, which, like the rampart, encircles the whole ruins.

Layard, in some remarks on his recent researches at Nineveh, states, that the date of the ruins discovered was still a mystery, but there could be no doubt of their extreme antiquity. He would afford one proof of it; the earliest buildings in Nineveh were buried, and the earth which had accumulated over them had been used as a burial-place by a nation who had lived 700 years before Christ. Probably the buildings dated from 1200 years before Christ. The rooms were lined with slabs of marble, covered with bas-reliefs, which were joined together by double dovetails of iron. The doorways were flanked by winged figures of greater height than the slabs; on all these figures was the mark of blood, as if thrown against them, and allowed to trickle down. The walls were of sun-dried bricks, and where these showed above the sculptured slabs, up to the ceiling, they were covered with plaster and painted. The beams, where they remained, were found to be of mulberry. That the slabs should have been preserved so long puzzled many. In truth, however, the bricks being simply dried in the sun, in falling had returned to earth, and had thus buried the tablets and protected them. The buildings were provided with a complete system of sewerage. Each room had a drain connected with a main sewer. In the midst of these ruins he discovered a small chamber formed of bricks regularly arched. The bas-reliefs sent to England by him were, in many cases, found in positions showing that they had been taken from other buildings and re-used — the sculptured face of the slab being turned to the wall, and the back re-worked.

The small chamber is perfectly vaulted with unburnt bricks, the diameter of the arch being 13 or 14 feet, and the form semicircular.

Another curious fact mentioned was the existence of cramps of iron, of a dovetailed form at each end, which had been used to connect the slabs of the internal walls.

The "Journal de Constantinople" publishes an extract from a letter written by Layard from Mousoul. "My excavation has so far succeeded," he says, "that I have penetrated to the interior of eight chambers, and found four pairs of winged bulls of gigantic forms. These blocks of marble are covered with sculptures of perfect workmanship, but so injured by fire that it is impossible to take their impression. Among the bas-reliefs which have more particularly attracted my notice, is one that represents a mountainous country. Another has also mountains covered with pines and firs. In a third there are vines — in a fourth a sea-horse. In one is seen the sea ploughed by many vessels — in others cities, which, bathed by the waters of a river, and shadowed by palm-trees, represent, perhaps, the ancient Babylon. The palace brought to light appears to have occupied a considerable extent of ground, and would require large sums of money for its due examination. An artist should be sent out to draw these bas-reliefs, which differ essentially in style and execution from those of Khorsabad. The palace where these discoveries have been made is better known to travellers than Nimroud, and would certainly interest them more. Major Rawlinson makes sensible progress in his reading of the cuneiform characters. It seems certain that the first palace explored at Nimroud was reared by Ninus; that the obelisk records the exploits of that one of his sons who built the central palace; and that thirty years of his reign were employed in the embellishment of these monuments. They treat of the conquest of India and other countries — as also of the principal acts of certain other monarchs, ancestors of Ninus.

At Nineveh, Botta has laid open fifteen rooms of what appears to have been a vast palace, some of which are 160 feet long, and the walls covered with sculpture and inscriptions, the latter historical, and the former illustrating sieges, naval combats, triumphs, &c. The characters employed resemble those of Persepolis, at Ecbatana (Hamadan), and Van. The sculpture is admirably executed, original in design, and said to be much superior to the figures on the monuments of the Egyptians, and show a remarkable knowledge of anatomy and the human face, great intelligence, and harmony of composition. The ornaments, robes, &c., are executed with extraordinary minuteness, and the objects, such as vases, drinking-cups, are extremely elegant; the bracelets, ear-rings, &c., show the most exquisite taste. Botta is inclined to place the sculpture and inscriptions in the period when Nineveh was destroyed by Cyaxares.

$$\dot{1}60 \text{ feet} = 138 \text{ \&c. units}$$
$$139^3 \text{ \&c.} = \tfrac{1}{400} \text{ dist. of moon}$$
$$(20 \times 139)^3 = \tfrac{1}{400} \times 20^3 = 20$$
$$\text{cube of 20 times length} = 20 \text{ dist. of moon}$$
$$\text{20 cubes of 20 times length} = 400 \text{ dist. of moon}$$
$$= 20^2 \text{ distance of moon } = \text{dist. of earth.}$$

There are curious traces of a large rectangular enclosure south of Medinet-Abou, Thebes, and bordering very near on the enclosure of the temples. "This rectangle, according to Heeren, is about 6,392 feet in length, and 3,196 in breadth, comprising an area of 2,269,870 square yards, which is about seven times as much as the Champ de Mars, at Paris, and consequently offered room enough for the exercises and manœuvres of a large army. The whole had an enclosure, which is indicated by elevations of earth, between which may still be distinguished the entrances, which have been counted to the number of thirty-nine; there may, however, have been as many as fifty or more. The principal entrance was on the east side, where a wider opening is seen. The whole enclosure shows distinctly that it was once adorned with the splendid architecture of triumphal monuments. Probably this extensive circus lay out of the city, but still close to it. A similar one of smaller dimensions is seen to the east side of the river, nearly opposite to this on the west, and we may therefore, with some degree of certainty, determine from this double evidence the southern limits of the city. It is highly probable that these spacious enclosures were not merely intended for games, such as chariot races, but also for the mustering and exercising of armies, which, under Sesostris and other conquerors, here began their military expeditions, and returned hither triumphant after victory."

Sides of the rectangled enclosure are 6392 by 3196 feet
$= 5527$ „ 2763 units.

Supposing the height of the enclosing walls, which are indicated by the elevation of the earth, to have originally been 12 units,

then height × area base

$$= 12 \times 5527 \times 2763$$
$$= \tfrac{1}{10} \text{ dist. of moon.}$$

Or the content might have equalled circumference of earth.

10 times height of walls

$$= 10 \times 12 = 120 \text{ units}$$
$$= 120 \times 5527 \times 2763 = \text{dist. moon,}$$

or $120 \text{ \&c. } \times 5527 \times 2763 = 10 \text{ circumference}$

$$\tfrac{1}{2} \text{ stade} = 121{\cdot}5 \text{ units}$$

sum of 2 sides $= 5527 + 2763 = 8290$

$829^3 = 5$ circumference
$8290^3 = 5000$
$(2 \times 8290)^3 = 40000.$
Cube of sum of 2 sides $= 5000$ circumference,
Cube of perimeter $= 40000$
perimeter $= 16580$ units
dist. saturn $= 15990^3 = 36000$ circumference.

Avebury Circle.

If the circumference at the top of the mound of the Avebury circle $= 4442$ feet, diameter will
$= 1413$ feet $= 1222$ units
1220^3 units $= 16$ circumference.
Maurice says the diam. of the Avebury circle $= 1400$ feet, which will $= 1210$ units.

Suppose the diam. to equal 1202 units,
area of circle will $= 1202^2 \times \cdot 7854$.

If the area be made a stratum of the depth of unity, the circular stratum will $= \frac{1}{100}$ circumference.
5 stades $= 30$ plethrons $= 1215$ units $= 1405$ feet.

Cube of 1202 $= \frac{8}{5}$ distance of moon,
Inscribed cylinder $= 12$ circumference
„ sphere $= 8$
„ cone $= 4$
„ pyramid $= \frac{8}{15}$ distance of moon
$\frac{5}{8}$ cube of 1202 $=$ distance of moon
$\frac{5}{4}$ „ $= 2$ distance
$=$ diam. of orbit of moon

$\frac{5}{8}$ cylinder, diam. 8×1202, distance of earth
$\frac{5}{4}$ „ „ 2 distance of earth
$=$ diam. of orbit of earth,

or $(3 \times 1220)^3 = 16 \times 3^3 = 432$ circumference
$(10 \times 3 \times 1220)^3 = 432000$

Cube of 30 times diam. of circle
$= 432000$ times circumference of earth
$=$ diam. of orbit of Belus.

A cylinder having the height = diameter of base = 1202 units will

$= 1202 \times 1202^2 \times {\cdot}7854 = 1202^3 \times {\cdot}7854 = 12$ circumference.

Inscribed sphere $= \frac{2}{3}$ circumscribing cylinder
$= \frac{2}{3}\ 12 = 8$ circumference.

Inscribed cone $= \frac{1}{3}\ 12 = 4$ circumference.

Sphere diam. 1202 units $= 8$ circumference,
601 $= 1$ „
300 $= \frac{1}{8}$ „
150 $= \frac{1}{64}$ „
75 $= \frac{1}{512}$ „

Thus a sphere having a diameter = that of the circle (of stones) will = 8 times circumference.

A sphere having the diameter = radius of the circle of stones will = circumference.

Suppose the diameter of the circular trench having sloping sides to have equalled, originally,

$$1202 + 72 = 1274 \text{ units},$$

cylinder having height = diameter of base will $= 1274^3 \times {\cdot}7854$.

Inscribed sphere will

$$= \tfrac{2}{3} 1274^3 \times {\cdot}7854 = 9{\cdot}55 \text{ circumference}$$
$$= \text{distance of moon from earth.}$$

Call distance $= 30$ diameters earth
$= 30 \times 7926 = 237780$ miles
circumference $= 24899$ miles,
and $9{\cdot}55 \times 24899 = 237780$ miles.

Thus a sphere having the diameter = the diameter of the circular trench will = 9·55 circumference = distance of moon from earth.

The circumference at the top of the mound

$= 4442$ feet $= 3854$ units
384^3, &c. $= \frac{1}{2}$ circumference
$(10 \times 384, \&c.)^3 = \frac{1000}{2} = 500.$

Cube of circumference of circle = 500 circumference of earth.

Cube of 2 circumference $=4000$ circumference of earth.
2 circumference $=20 \times 384$, &c. $=7680$ nearly.

If circumference of a circle $=3790$ units,
the cube of twice circumference of circle would
$=7580^3=$ distance of earth.

Cube of circumference would
$=\frac{1}{8}$ distance of earth
$=\frac{400}{8}=50$ distance of moon.

3 cubes of circumference = distance of Mercury,
75 " " = " Saturn,
150 " " = " Uranus,
450 " " = " Belus.

The inner slope of the bank of the trench $=80$ feet.
Should the circumference of the outer circle $=4335$ units,
Cube of circumference will $=\frac{1}{8}$ distance of Mars.
Cube of 2 circumference $=1$ "
Cube of 2 circumference at top of mound $=4000$ circumference.
Cube of 3×2 circumference $=4000 \times 3^3=108000$.
Cube of 6 times circumference of circle $=108000$ circumference of earth
$=\frac{1}{2}$ distance of Belus.
Cube of 3×2 circumference $=3$ distance of Saturn.

Measured circumference $=4442$ feet,
diameter $=1413$ feet $=1222$ units,
$1220^3=16$ circumference,
$(30 \times 1220)^3=16 \times 30^3=432000$ circumference,
= diameter of orbit of Belus.
Cube of 30 times diameter = diameter of orbit of Belus.
Cube of 24 times diameter = distance of Belus.
For $30 : 24 :: 5 : 4$
$5^3 : 4^3 :: 125 : 64 :: 2 : 1$ nearly.

If diameter of circle $=1202$ units,
$\frac{1}{2}=601=$ side of base of the pyramid of Cephrenes, the cube of which
$=601^3=\frac{1}{8}$ distance of moon.

5 cubes $= 5 \times 601^3 =$ distance of moon.
Cube of diameter $= 1202^3 = \frac{8}{5}$ distance of moon
$(5 \times 1202)^3 = \frac{8}{5} \times 5^3 = 200.$

2 cubes of 5 times diameter $= 400$ distance of moon
$=$ distance of earth.

Near Avebury is a fallen cromlech; and various barrows are visible in different parts of the neighbourhood.

According to another description of Avebury, the remains originally consisted of one large circle of stones, 138 feet by 155, inclosing two smaller circles, and having two extensive avenues of upright stones.

Diameters are 138 by 155 feet
$= 119$ by 133 units
say $= 117$ by 131 „

Cylinder having height $= 117$ and diameter of base $= 131$, will
$= 117 \times 131^2 \times \cdot7854 = \frac{1}{60}$ circumference $= 6$ degrees
Spheroid $= \frac{2}{3}$ $= \frac{1}{90}$ „ $= 4$ „
Cone $= \frac{1}{3}$ $= \frac{1}{180}$ „ $= 2$ „

Cylinder having height $= 131$ and diameter of base $= 117$, will
$= 131 \times 117^2 \times \cdot7854$
$= \frac{1}{80}$ circumference $= 4\cdot5$ degrees
Spheroid $= \frac{2}{3} = \frac{1}{120}$ „ $= 3$ „
Cone $= \frac{1}{3} = \frac{1}{240}$ „ $= 1\cdot5$ „
Diameters are 138 by 155 feet
$= 119$ by 133 units.
If diameter $= 119$, circumference $= 374$ units
378^3, &c. $= \frac{1}{20}$ distance of moon.

Or cube of circumference $= \frac{1}{20}$ distance of moon,
cube of 10 circumference $= \frac{1000}{20} = 50.$
3 cubes $= 150$ distance of Mercury,
8 cubes $= 400$ „ earth.

Or 1 cube of 20 times circumference $=$ distance of earth.

If diameter $=133$, circumference $=417$ units

$414^3 = \frac{5}{8}$ circumference

$(2 \times 414)^3 = 5$,

or cube of 2 circumference of circle $= 5$ circumference of earth.

The diameters to these circumferences will be about 120 and 132 units,

121^3, &c. $= \frac{1}{600}$ distance of moon

$= \frac{60}{600} = \frac{1}{10}$ radius of earth

131^3, &c. $= \frac{1}{50}$ circumference.

If circumference of less circle $= 379{\cdot}2$ units

$20 \times 379{\cdot}2 = 7584$

distance of earth $= 7584^3$

cube of 20 times circumference = distance of earth,

and cube of 50 times diameter $= \frac{1}{2}$ distance of earth.

If circumference of greater circle $= 421{\cdot}2$, &c. units

$8 \times 421{\cdot}2$, &c. $= 3370$

$\frac{1}{8}$ distance of Venus $= 3370^3$.

Cube of 8 times circumference $= \frac{1}{8}$ distance of Venus.

Cube of 16 times circumference = distance of Venus.

Cube of $16 \times \frac{10}{4}$, or 40 times diameter $= \frac{1}{2}$ distance of Venus.

Sum of diameters $= 121$, &c. $+ 131$, &c. $= 253$ units

253^3, &c. $= \frac{3}{200}$ distance of the moon

$(10 \times 253, \text{\&c.})^3 = \frac{3000}{200} = 15$

10 cubes of 10 times sum $= 150$ distance of the moon

$=$ distance of Mercury.

De Ulloa states, that at about 50 toises north of the palace of the Incas of Quito, still called by the ancient name Callo, and fronting its entrance, is a mountain, the more singular as being in the midst of a plain; its height is between 25 and 30 toises, and so exactly, on every side, formed with the conical roundness of a sugar-loaf, that it seems to owe its form to industry; especially as the end of its slope on all sides forms exactly with the ground the same angle in every part. And what seems to confirm the opinion is, that guacas, or mausoleums, of prodigious magnitude, were greatly affected by the Indians in those times. Hence the common opinion that it is artificial, and that the earth was taken out of the breach north of it, where a little river now runs, does not seem improbable. But this is no more than conjecture, not being founded on any evident proof. In all appearance this eminence, now called Panecillo de Callo, served as a watch-tower, commanding an uninterrupted view of the country, in order to provide for the safety of the province on any sudden alarm of an invasion, of which they were under continual apprehensions, as appears from the account of their fortresses.

Taking the toise as equal to 6·44 feet English, we have 27 toises = 175 feet, which, if taken as the height of the conical hill at Callo, would make it nearly of the same height as the conical hill at Silbury, and also = the height of the teocalli of Cholula, or = $\frac{5}{8}$ stade.

Ulloa gives[*] the proportion of the French to the English foot as 846 to 811, and 6 French feet make 1 toise; so that

$$\begin{aligned} \tfrac{5}{8} \text{ stade will} &= 28{\cdot}12 \text{ toises,} \\ 1 \text{ stade} &= 45 \quad \text{,,} \end{aligned}$$

There is in Lydia a tomb of Alyattes, the father of Crœsus, which exceeds in magnitude, according to Herodotus, other monuments, with the exception of those of Egypt and Babylon. The base is formed of large stones, and the rest is terraced. There are five termini placed on the summit of the tomb, on which are inscribed letters indicating what portion of the work each party had accomplished, whence it appears from the measurements that the women had executed a larger portion than the men. The circuit of the tomb measures 6 stadia and 2 plethra, the length, thirteen plethra.

$$\begin{aligned} \text{Circuit} &= 6 \text{ stades and } 2 \text{ plethrons,} \\ &= 1539 \text{ units,} \\ \tfrac{1}{2} &= 769 \text{ \&c.} \\ 769^3 &= 4 \text{ circumference.} \\ (2 \times 769)^3 &= 32. \end{aligned}$$

Cube of circuit = 32 circumference,

$$(5 \times 2 \times 769)^3 = 32 \times 5^3 = 4000.$$

9 cubes of 5 times circuit = 36000 circumference,
,, ,, = distance of Saturn,
18 ,, ,, = ,, Uranus,
54 ,, ,, = ,, Belus.
2 cubes of 15 times circuit = ,, Belus.

Length = 13 plethrons = 526·5 units,
525^3 &c. = $\frac{4}{30}$ distance of the moon.
$(3 \times 525)^3 = \frac{4}{30} \times 3^3 = \frac{36}{10}$,
$(5 \times 3 \times 525)^3 = \frac{36}{10} \times 5^3 = 450$.

Cube of 15 times length = 450 distance of the moon.
,, ,, = 3 ,, Mercury.
50 cubes ,, ,, = 22500 ,, the moon.
,, ,, = distance of Belus.
$(10 \times 3 \times 525)^3 = \frac{36000}{10} = 3600$ dist. of moon.
$= 3750 - 150$,,

Cube of 30 times length = distance between Saturn and Mercury.

Circuit = 38 plethrons.
2 length = 26
2 breadth = $\overline{12}$
Breadth = 6 plethrons = 1 stade = 243 units.
$243^3 = \frac{1}{8}$ circumference,
$(2 \times 243)^3 = 1$ circumference,
or cube of twice breadth = circumference of the earth.

Cube of 120 times breadth = cube of Babylon = distance of Belus.
Cube of 12 times breadth = $\frac{1}{1000}$.
Cube of 20 times breadth = $\frac{1}{216}$.

In the environs of Sardis is a colossal tumulus, believed to be the tomb of Alyattes. It is a cone of earth 200 feet high. Leake regards it as one of the most remarkable antiquities in Asia. The base is now covered with earth, but the tomb still retains the conical form, and has the appearance of a natural hill.

Newbold describes Sardis, the ancient capital of Crœsus, as being now desolate,—scarcely a house remaining. The melancholy Gygæan lake,—the swampy plain of Hermus,—the thousand mounds forming the necropolis of the Lydian monarchs, among which rises conspicuous the famed tumulus of Alyattes,—produce a scene of gloomy solemnity. Massive ruins of buildings still remain, the walls of which are made of sculptured pieces of the Corinthian and Ionic columns that once formed portions of the ancient pagan temples. The Pactolus, famed for its golden sands, contains no gold; but the sparkling grains of mica with which the sand abounds, have probably originated the epithet.

Stonehenge stands in the middle of a flat area, near the summit of a hill. It is enclosed by a double circular bank and ditch, nearly thirty feet broad, after crossing which an ascent of nearly thirty yards leads to the work. The whole fabric was originally composed of two circles and two ovals. The outer circle is about 108 feet in diameter, consisting, when entire, of 60 stones, 30 uprights, and 30 imposts. 11 uprights have their 5 imposts on them by the grand entrance; these stones are from 13 to 20 feet high. The smaller circle is somewhat more than 8 feet from the inside of the outer one, and consisting of 40 smaller stones, the highest measuring about 6 feet, 19 only of which now remain, and only 11 standing. The walk between these two circles is 300 feet in circumference.

The "adytum," or cell, is an oval formed of 10 stones, from 16 to 22 feet high, in pairs, and with imposts above 30 feet high, rising in height as they go round, and each pair separate, and not connected as the outer pair; the highest 8 feet. Within these are 19 other smaller single stones, of which 6 only are standing. At the upper end of the adytum is the altar, a large slab of blue coarse marble, 20 inches thick, 16 feet long, and 4 feet broad; it is pressed down by the weight of the vast stones which have fallen upon it. The whole number of stones, uprights and imposts, comprehending the altar, is 140.

Another account makes the circumference of the surrounding ditch 369 yards.

According to another description of Stonehenge, the whole structure was composed of 140 stones, including those of the entrance, forming two circles and two ovals, respectively concentric. The whole is bounded by a circular ditch, originally 50 feet broad, the inside verge of which is 100 feet distant, all round, from the greater extremity of the greater circle of stones. The circle is nearly 108 feet in diameter; so that the diameter of the area wherein Stonehenge is situated, is about 408 feet. The vallum is placed inwards, and forms a circular terrace, through which was the entrance to the north-east by an avenue of more than 1700 feet in a straight line, bounded by two ditches, parallel to each other, about 70 feet asunder.

Avenue is more than 1700 feet, or 1470 units.

$$(148\ \&c.)^3 = \tfrac{3}{1000} \text{ distance of the moon}$$
$$(10 \times 148\ \&c.)^3 = 1482^3 = \tfrac{3000}{1000} = 3.$$

Cube of length = 3 distance of the moon.

$$(5 \times 10 \times 148\ \&c.)^3 = 3 \times 5^3 = 375.$$

10 cubes of 5 times length = 3750 distance of the moon
= distance of Saturn
20 ,, ,, = ,, Uranus
60 ,, ,, = ,, Belus.

Distance between the parallel ditches is about 70 feet, or 60 units.

Sum of 2 sides = 1482 + 60 = 1542.

$$153^3\ \&c. = \tfrac{1}{300} \text{ distance of the moon}$$
$$(10 \times 153\ \&c.)^3 = \tfrac{1000}{300} = \tfrac{10}{3}.$$

Cube of sum of 2 sides = $\frac{10}{3}$ distance of the moon.

$$(3 \times 10 \times 153\ \&c.)^3 = \tfrac{10}{3} \times 3^3 = 90$$
$$(5 \times 3 \times 10 \times 153\ \&c.)^3 = 90 \times 5^3 = 11250.$$

2 cubes of 15 times sum of 2 sides = 22500 distance of the moon = distance of Belus.

Cube of greater side : cube of sum of
2 sides :: 3 : $\frac{10}{3}$ :: 9 : 10.

or breadth = 60 units

$$(10 \times 60{\cdot}1)^3 = 601^3 = \tfrac{1}{5} \text{ distance of the moon}$$
$$(10 \times 10 \times 60{\cdot}1)^3 = \tfrac{1000}{5} = 200.$$

2 cubes of 100 times breadth = 400 distance of the moon
= distance of the earth.

Diameter of circumscribing circle
= 408 feet = 353 units
circumference = 1109 „
$\frac{1}{2}$ = 554 &c.
554^3 &c. = $\frac{3}{2}$ circumference
$(2 \times 554 \text{ \&c.})^3 = \frac{24}{2} = 12.$

Cube of circumference of circle = 12 times circumference of the earth.

Cube of twice circumference = $12 \times 8 = 96$.
15 cubes „ = 1440 circumference = distance of Mercury
40 cubes „ = 3840 circumference = distance of the earth
or 5 cubes of 4 circumference = „ „

Cube of 10 times circumference = 12000 circumference of the earth
= $\frac{1}{3}$ distance of Saturn
= $\frac{1}{6}$ „ Uranus
= $\frac{1}{18}$ „ Belus.

Diameter of great circle of stones = 108 feet = 93·37 units
circumference = 293 &c.
293^3 &c. = $\frac{2}{9}$ circumference
$(3 \times 293 \text{ \&c.})^3 = \frac{2}{9} \times 3^3 = 6.$

Cube of 3 times circumference of circle = 6 times circumference of the earth.

Cube of 6 times circumference of circle = 48 times circumference of the earth.

30 cubes = 1440 circumference = distance of Mercury
80 cubes = 3840 „ = „ the earth.

Cube of 30 times circumference = 6000 circumference of the earth
= $\frac{1}{6}$ distance of Saturn
= $\frac{1}{12}$ „ Uranus
= $\frac{1}{36}$ „ Belus

Should circumference $=291{\cdot}6$ units, cube of 100 times circumference would $=29160^3=$ distance of Belus
$=$ cube of Babylon.

Circumference of ditch $=369$ yards
$=1107$ feet $=957$ units

$$954^3=\tfrac{8}{10}\text{ distance of the moon}$$
$$(10\times 954)^3=\tfrac{8000}{10}=800.$$

Cube of 10 times circumference $=800$ distance of the moon
$=$ diameter of the orbit of the earth.

Cube of $10\times\frac{10}{4}$, or of 25 diameter $=$ distance of the earth nearly.

Cube of 5 times circumference $=100$ distance of the moon.

3 cubes $=300$ distance of the moon
$=$ diameter of the orbit of Mercury

4 cubes $=$ distance of the earth
75 ,, $=$,, Uranus
225 ,, $=$,, Belus.

Breadth of ditch $=50$ feet.

So that the diameter of the circle on the inside verge will $=408-100=308$ feet
circumference $=967$ feet $=836$ units,
and $828^3=5$ circumference,
or cube of circumference of circle $=5$ times circumference of the earth.

$$(4\times 828)^3=5\times 4^3=320\text{ circumference.}$$

12 cubes of 4 times circumference of the circle
$=3840$,, ,, earth
$=$ distance of the earth.

Cube of 10 times circumference of the circle $=5000$ circumference of the earth.

Cube of $10\times\frac{10}{4}$, or of 25 diameter, $=2500$ circumference of the earth.

If circumference $=841$ &c. units
8×841 &c. $=6730$
distance of Venus $=6730^3$.

Cube of 8 times circumference = distance of Venus.

Diameter of inner circle of stones is somewhat more than 92 feet, or 79·54 units,

circumference = 249·8
if = 254·4
$100 \times 254{\cdot}4 = 25440$
diameter of the orbit of Uranus = 25440^3.

Cube of 100 times circumference = diameter of the orbit of Uranus.

Cube of $100 \times \frac{10}{4}$ diameter, or of 250 diameter = distance of Uranus.

Circumference of ditch = 957 units
$10 \times 956 = 9560$
diameter of the orbit of the earth = 9560^3.

Cube of 10 times circumference = diameter of the orbit of the earth.

Cube of $10 \times \frac{10}{4}$, or of 25 times diameter = distance of the earth.

Twice circumference of inner circle of stones = $2 \times 243 =$ 486 units.

Cube of twice circumference of circle = 486^3 = circumference of the earth.

The numerals 486 transposed and squared = 684^2 = circumference of the earth in stades.

Sum of 2 diameters of circles of stones
$= 93{\cdot}37 + 79{\cdot}5 = 173$ units
circumference = 544
$546^3 = \frac{3}{20}$ distance of the moon
$(20 \times 546)^3 = \frac{3}{20} \times 20^3 = 1200$
Cube of 20 times circumference = 1200 distance of the moon
pyramid = $\frac{1}{3}$ = 400 ,, ,,
= distance of the earth.
$(10 \times 546)^3 = \frac{3000}{20} = 150$ distance of the moon
Cube of 10 times circumference = distance of Mercury
= 150 distance of the moon
150 cubes ,, ,, = distance of Belus
= 22500 distance of the moon.

Cube of circumference of greater circle of stones : cube of circumference of less $:: \frac{2}{9} : \frac{1}{8} :: 16 : 9 :: 4^2 : 3^2$.

If sum of 2 diameters $=$ 173 &c. units
50×173 &c. $= 6890$
distance of Mars $= 6890^3$
10 times circumference $= 5460$
distance of Mercury $= 5460^3$.

Cube of 50 times diameter = distance of Mars
" 10 " circumference = " Mercury
" 120 " circumference
of circle $= 243 =$ " Belus.

The outer circle, when entire, consisted of 60 stones, 30 uprights, and 30 imposts; 17 of the uprights remain standing, and 6 are lying on the ground, either whole or in pieces, and 1 leaning at the back of the temple, to the south-west, upon a stone of the inner circle; these 24 uprights and 8 imposts are all that remain of the outer circle. The upright stones are from 18 to 20 feet high, from 6 to 7 broad, and about 3 feet in thickness; and being placed at the distance of 3½ feet from each other were joined at the top by mortise and tenon to the imposts, or stones laid across like architraves, uniting the whole outer range in one continued circular line at the top. The outsides of the imposts were rounded a little to favour the circle, but within they were straight, and originally formed a polygon of 30 sides. At the upper end of the adytum, or cell, is the altar, a large slab of blue coarse marble, 20 inches thick, 16 feet long, and 4 broad: it is pressed down by the weight of vast stones that have fallen upon it.

At some distance round this famous monument are great numbers of sepulchres, or, as they are called, barrows, being covered with earth, and raised in a conical form. They extend to a considerable distance from the temple, but are so placed as to be all in view of it. Such as have been opened were found to contain either human skeletons or ashes of burnt bones, together with warlike instruments, and such things as the deceased used when alive.

From these sepulchres being within sight of the temple, as we have seen the small pyramids and sepulchral chambers erected near the great pyramidal temples, we may conclude that, like the Christians of the present age, the ancients thought it was most proper to bury their dead adjoining those places where they worshipped the Supreme Being. Indeed, all worship indicates a state of futurity, and they might reasonably imagine that no place was so proper for depositing the relics of their departed friends as the spot dedicated to the service of that Being with whom they hoped to live for ever. The sentiment is altogether natural; no objection can be made to it, while the depositories of the dead are detached from populous towns or cities; but no one can excuse the present mode of crowding corrupt bodies into vaults under churches, adjoining to the most public streets, where the noxious effluvia may be attended with the most fatal consequences to the living.

Close to the village of E'Mozòra, in Western Barbary, is the site of an heliacal temple, whereof, among numerous remains now prostrate, one stone, called vulgarly by the Moors Al Ootsed, or the peg, stands yet erect, and is of such large dimensions, that it would not discredit the stupendous structure on Salisbury Plain.—(*Hay.*)

The ancient *Sorbiodunum*, or Old Sarum, is about a mile north of Salisbury, and was one of the ten British cities admitted to the privileges of the Latin law. Of this once flourishing and celebrated place nothing now remains but its ruins. It is to this place the present city owes its origin. The name is supposed to be derived from a British compound word, signifying a dry situation; and the Saxons, who called this place *Searysbyrie*, seem to have a reference to the same circumstance; *searan*, in the Saxon language, signifying "to dry." Leland supposes Sorbiodunum to have been a British post prior to the arrival of the Romans, with whom it afterwards became a principal station, or *castra stativa*. Besides the evidence of the Itineraries, and the several roads of that people which here concentrate, the great number of

Roman coins found within the limits of its walls prove its occupation as a place of consequence by the Romans. According to the author of "Antiquitates Sarisburiensis," some of the Roman emperors actually resided at Old Sarum. Leland mentions this place as having been very ancient and exceedingly strong. It covers the summit of a high steep hill, which originally rose equally on all sides to a point. The area was nearly 2000 feet in diameter, surrounded by a fosse or ditch of great depth, and two ramparts, some remains of which are still to be seen. On the inner rampart, which was much the highest, stood a wall, nearly 12 feet thick, made of flint and chalk strongly cemented together, and cased with hewn stones, on the top of which was a parapet, with battlements quite round. Of this wall there are some remains still to be seen, particularly on the north-west side. In the centre of the whole rose the summit of the hill, on which stood a citadel or castle, surrounded with a deep entrenchment and very high rampart. In the area under it stood the city, which was divided into equal parts, north and south, by a meridian line. Near the middle of each division was a gate, which were the two grand entrances; these were directly opposite to each other, and each had a tower and a mole of great strength before it. Besides these, there were two other towers in every quarter, at equal distances, quite round the city; and opposite to them, in a straight line with the castle, were built the principal streets, intersected in the middle by one grand circular street. In the north-west angle stood the cathedral and episcopal palace; the former, according to Bishop Godwin, was consecrated in an evil hour; for the very next day the steeple was set on fire by lightning. The foundations of these buildings are still to be traced, but the site of the whole city has been ploughed over. Leland adds to his account, that "without each of the gates of Old Sarum was a fair suburb, and in the east suburb a parish church of St. John, and thereon a chapel, yet standing. There had been houses in time out of mind inhabited in the east suburb; but there is not one within or without the city. There was a parish church of the Holy

Rood, in Old Saresbyrie, and another over the gate, whereof some tokens remain."

About the time the West Saxon kingdom was established, King Kenric, or Cynric, resided here, after having defeated the Britons. This prince, about four years after, incorporated Wiltshire with Wessex. About the middle of the tenth century, in the reign of Edgar, a great council, or *witenagemote*, was summoned by that prince, when several laws were enacted for the better government of church and state. Soon afterwards (in the year 1003) it was plundered and burnt by Sweine, the Danish king, in revenge for the massacre committed by the English on his countrymen the preceding year. It was, however, rebuilt, and became so flourishing, that the bishop's see was removed thither from Sherborne, and the second of its bishops built a cathedral. William the Conqueror summoned all his states of the kingdom hither, to swear allegiance to him, and several of his successors often resided here.

In 1095, William II. held a great council, which impeached William, Earl de Ou, of high treason, for conspiring to raise Stephen, Earl of Albemarle, to the throne. His cruel punishment marks the barbarity of the age.—Henry I. held his court here in 1100, and again in 1106. In 1116, he ordered all the bishops, abbots, and barons, to meet here, to do homage to his son William, as his successor to the throne.

Here, in 1483, was executed Henry Stafford, Duke of Buckingham, who had exerted all his influence, and used every effort, to advance Richard III. to the throne.—James I. frequently visited Salisbury, as did Charles I. On one occasion, when the latter was here, in 1632, a boy only fifteen years of age was hanged, drawn, and quartered, for saying he would buy a pistol to kill the king.

We find the first prelude to its downfall was a quarrel that happened between King Stephen and Bishop Roger, the latter of whom espoused the cause of the Empress Maud, which enraged the king to such a degree, that he seized the castle, which belonged to the bishops, and placed a governor and garrison in it.

This was looked upon as a violation of the rights of the church, and occasioned frequent differences between the military and the monks and citizens, the issue of which was, that the bishop and canons determined to remove to some place where they might be less disturbed, having in vain applied to the king for redress of their grievances.

From the time that Stephen put a garrison into the castle, Old Sarum began to decay.

The removal of the city was first projected by Bishop Herbert, in the reign of Richard II.; but the king dying before it could be effected, and the turbulent reign of John ensuing, the plan could not be carried into execution until the reign of Henry III., when Bishop Richard Poore fixed upon the site of the present cathedral, and translated the episcopal see. The inhabitants of Old Sarum speedily followed, being intimidated by the insolence of the garrison, and at the same time suffering great inconvenience through the want of water. By degrees, Old Sarum was entirely deserted, and at present there is but one building left within the precincts of the ancient city. However, it is still called the borough of Old Sarum, and sent two members to Parliament, till the Reform Act of 1832, who were chosen by the proprietors of certain lands adjacent.

The area of the base of the conical hill is nearly 2000 feet, or 1730 units in diameter.

Diameter of external cone of Silbury will = 557 units, and content = $\frac{2}{15}$ circumference.

Diameter of external cone at Sarum = 1738 units.

If the conical hills at Silbury and Sarum were similar their contents would be as 1 : 30,

Then content of the conical hill at Sarum would

$$= \frac{2}{15} \times 30 = \frac{60}{15} = 4 \text{ circumference}$$
$$= 8 \text{ times pyramid of Cheops.}$$

The hill is surrounded by a fosse and two ramparts.

If the diameter of one of these circles should be about 1740 units, circumference would = 5466.

Cube of circumference would = 5466^3

= distance of Mercury.

The height of the steep hill, which originally rose equally on all sides to a point, is not stated.

The principal streets radiated from the castle and were intersected in the middle by one grand circular street.

Sarum appears to have been the Rome of Britain, the residence of her pontifical Druids, whose altars were overturned and religion extirpated by the Romans.

The throne of the Cæsars at Rome has since been supplanted by the hierarchal chair of St. Peter, where the sovereign pontiff by his supreme temporal and spiritual authority rules the Eternal City and states; as the Roman emperors previously ruled the destinies of kingdoms by military power.

The glory of Sarum gradually became extinct: the last ray was when, reduced to only one house, she retained the power of returning two members to parliament; among those whom towards the last she sent to commence their political career was Chatham, the father of Pitt.

At last Sarum, after having been the Rome of the Druids, and the Windsor of kings and emperors, who ruled by their will, was deprived of even a representative in the Commons of England, and is now forgotten.

It would seem more than probable that the great teocallis were originally constructed for religious purposes, and also as places of defence in time of danger.

The old ballad, alluding to Sarum, says —

"'Twas a Roman town, of strength and renown,
  As its stately ruins show.
Therein was a castle for men and arms,
  And a cloister for men of the gown."

The cathedral of Salisbury, or New Sarum, is a Gothic structure. From the centre of the roof, which is 116 feet high, rises a beautiful spire of freestone, the altitude of which is 410 feet from the ground, and is esteemed the highest in the kingdom; being nearly 70 feet higher than the top of St. Paul's, and just double the height of the Monument in London.

1 stade = 281 feet = height of tower of Belus
$1\frac{1}{2}$ „ = 421·5 „ = „
410 „ = height of the spire.

The singularity of there being in this cathedral 365 windows, &c. is explained in the following verses:—

> "As many days as in one year there be,
> As many windows in this church you see;
> As many marble pillars here appear
> As there are hours throughout the fleeting year;
> As many gates as moons one here does view;
> Strange tale to tell! yet not more strange than true."

Between Ashbourne and Buxton in Derbyshire is a circle of stones, or Druidical temple, called Arbe Lowes, 150 feet in diameter, surrounded by a large bank of earth, about 11 yards high in the slope, but higher towards the south or south-east, and formed by a large barrow; the ditch within is four yards in width, with two entrances, east and west.

Diameter = 150 feet = 129·6 units

$10 \times 129{\cdot}6 = 1296$

and 1296^3 = diameter of orbit of moon.

Cube of 10 times diameter = diameter of orbit of moon.

Cube of 4 times circumference of diameter 1296
= 2 diameter of orbit of moon.

Circumference = 407 units.

$90 \times 407 = 36630$

diameter of orbit of Belus $= 36630^3$.

Cube of 90 times circumference = diameter of orbit of Belus.

Cube of $90 \times \frac{10}{4}$ diameter, or of 225 diameter $= 29160^3$ = distance of Belus.

At Hathersage, in Derbyshire, above the church, at a place called Champ Green, is a circular area, 144 feet in diameter, encompassed with a high and pretty large mound of earth, round which is a deep ditch.

Diameter 144 feet = 124·5 units.
Circumference = 391, &c.
$60 \times 126{\cdot}4 = 7584$.
Distance of the earth $= 7584^3$.

Cube of 60 times diameter = distance of the earth.

If circumference = 399 units,
$30 \times 399 = 15960$.
Distance of Saturn $= 15960^3$.

Cube of 30 times circumference = distance of Saturn.

If circumference = 392 units,
$392^3 = \frac{1}{18}$ distance of the moon,
$(6 \times 392)^3 = \frac{1}{18} \times 6^3 = 12$,
$(5 \times 6 \times 392)^3 = 12 \times 5^3 = 1500$.

Cube of 6 times circumference = 12 distance of the moon.
Cube of 30 " = 1500 "
= 10 times distance of Mercury,
$= \frac{1}{15}$ distance of Belus.

On Stanton Moor, a rocky, uncultivated waste, about two miles in length, and one and a half broad, are numerous remains of antiquity, as rocking-stones, barrows, rock-basons, circles of erect stones, &c., which have generally been supposed of Druidical origin.

The following Druidical circles are also in Derbyshire. In a field north of Grand Tor, called Nine-stone Close, are the remains of a circle called Druidical, about 13 yards in diameter, now consisting of seven rude stones of various dimensions: one of them is about eight feet in height, and nine in circumference. Between seventy and eighty yards to the south are two other stones, of similar dimensions, standing erect.

Diameter 13 yards = 39 feet = 33·5 units.
If diameter = 33·2 units, circumference = 104, &c.
Diameter of circle to the power of 3 times $3 = 33{\cdot}2^9$
= diameter orbit of Belus.
Cube of circumference $= 104^3$, &c. $= \frac{1}{100}$ circumference.
Cube of 10 times circumference of circle $= \frac{1000}{100} = 10$
= 10 times circumference of earth.

Diameter = 13 yards = 33·5 units.
If = 33·65.
$100 \times 33{\cdot}65 = 3365$,
$\frac{1}{8}$ distance of Venus $= 3365^3$,
$200 \times 33{\cdot}65 = 6730$,
distance of Venus $= 6730^3$.

Cube of 200 times diameter = distance of Venus.

Cube of $200 \times \frac{4}{10}$ or of 80 times circumference
= diameter of orbit of Venus.

Circumference = 105, &c., if = 106, &c.
90×106, &c., $= 9540$.
Distance of the earth $= 9540^3$.

Cube of 90 times circumference = distance of the earth.

About a quarter of a mile west of the little valley which separates Hartle Moor from Stanton Moor is an ancient work, called Castle Ring, supposed to have been a British encampment. Its form is elliptical; its shortest diameter, from south-east to south-west, is 165 feet; its length, from north-east to south-west, 243. It was encompassed by a deep ditch and double vallum, but part of the latter has been levelled by the plough.

Greater diameter of Hartle Moor ellipse = 210 units = 243 feet.

Less diameter = 142 units = 165 feet.

211^3, &c. $= \frac{1}{12}$ circumference.
141^3, &c. $= \frac{1}{40}$ „

Circumference of circle diameter 210 = 659 units.

$658^3 = \frac{10}{4}$ circumference.
$(2 \times 658)^3 = \frac{80}{4} = 20$.

Cube of twice circumference of circle = 20 circumference of the earth.

Circumference of circle diameter 142 = 446 units.

449^3, &c. $= \frac{4}{5}$ circumference.
$(5 \times 449$, &c.$)^3 = \frac{4}{5} \times 5^3 = 100$.
$(3 \times 5 \times 449$, &c.$)^3 = 100 \times 3^3 = 2700$.

Thus cube of 5 times circumference = 100 circumference of the earth.

Cube of 15 times circumference of circle = 2700 circumference of the earth = distance of Venus.

80 cubes = distance of Belus,
or 10 cubes of 30 times circumference = distance of Belus.

Circumferences = 659 and 446,
if = 652 ,, 434.

$$30 \times 652 = 19560,$$
$$\text{Distance of Uranus} = 19560^3,$$
$$30 \times 434, \text{\&c.} = 13040.$$
$$\text{Distance of Jupiter} = 13040^3.$$

About half a mile north-east from the Router rocks, on Stanton Moor, is a Druidical circle, eleven yards in diameter, called the Nine Ladies, composed of the same number of rude stones, from three to four feet in height, and of different breadths. A single stone, named the King, stands at a distance of thirty-four yards.

$$\text{Diam.} = 11 \text{ yards} = 33 \text{ feet} = 28{\cdot}53 \text{ units.}$$
$$\text{say} = 27 \text{ \&c.}$$

Cylinder having height = diam. of base will

$$= 27^3, \text{\&c.} \times {\cdot}7854$$
$$= \tfrac{3}{60} \text{ degree} = 3 \text{ minutes}$$
$$\text{Sphere} = \tfrac{2}{3} = \tfrac{2}{60} \quad ,, \quad = 2 \quad ,,$$
$$\text{Cone} = \tfrac{1}{3} = \tfrac{1}{60} \quad ,, \quad = 1 \quad ,,$$

Near this circle are several cairns and barrows; most of which have been opened, and various remains of ancient customs discovered in them. Urns, with burnt bones, &c. have been found in these and some of the other barrows. Under one of the cairns human bones were found, together with a large blue glass bead.

$$\text{Cone} = 1 \text{ minute} = \frac{1}{360 \times 60} = \frac{1}{21600} \text{ circumference}$$
$$= 1 \text{ geographical mile.}$$
$$\text{Cube of Babylon} = 216000 \text{ circumference}$$
$$= 21600 \times 10.$$

Circumference $= 21600$ cones of Stanton Moor.

So cube of Babylon $= 21600^2 \times 10$ cones
$= 21600^2 \times 10$ miles,
or distance of Belus $= 10$ times the square of the earth's circumference when unity $= 1$ geographical mile.

Diameter $= 11$ yards $= 33$ feet $= 28{\cdot}53$ units.
Circumference $= 89{\cdot}53$.
$(10 \times 89{\cdot}8)^3 = \frac{2}{3}$ distance of the moon.
$(3 \times 10 \times 89{\cdot}8)^3 = \frac{2}{3} \times 3^3 = 18$.
$(5 \times 3 \times 10 \times 89{\cdot}8)^3 = 18 \times 5^3 \times 2250$.

Cube of 150 times circumference $= 2250$ distance of the moon
$= \frac{1}{10}$ distance of Belus.

3 cubes of 10 times circumference $= 2$ distance of the moon
$=$ diameter of the orbit of the moon.

Diameter $= 28{\cdot}53$ units.
$(10 \times 28{\cdot}4)^3 = \frac{2}{10}$ circumference.

10 cubes of 10 times the diameter of the circle
$=$ twice the circumference of the earth.

$28{\cdot}6^9 =$ distance of Neptune.

Should circumference $= 91$ units
$60 \times 91 = 5460$
Distance of Mercury $= 5460^3$.

Cube of 60 times circumference $=$ distance of Mercury.

On the top of Banbury Hill, in Berkshire, is a supposed Danish camp of a circular form, 200 yards in diameter, with a ditch of 20 yards wide.

Diameter 200 yards $= 600$ feet $= 519$ units.
Circumference $= 1630$.
$\frac{1}{2} = 815$.

$816^3 = \frac{1}{2}$ distance of moon.
$(2 \times 816)^3 = \frac{8}{2} = 4$.
Cube of circumference $= 4$ distance of the moon.

100 cubes = 400 dist. of moon = dist. of earth.
70 „ = 280 „ = „ Venus.

Cube of 10 times circumference = 4000 distance of moon,
= 10 times the distance of the earth.

Twice width of ditch = 40 yards = 120 feet = 103 units.

Diameter of outer circle will = 519 + 103 = 622 units, and circumference = 1952.

$\frac{1}{2} = 976.$

968^3, &c. = 8 circumference

$(2 \times 968, \&c.)^3 = 64$

Cube of circumference of circle = 64 circumference of the earth

$$(5 \times 2 \times 968, \&c.)^3 = 64 \times 5^3 = 8000.$$

9 cubes of 5 times circumference of the circle
= 72000 circumference of the earth
= distance of Saturn
18 „ = „ Uranus
54 „ = „ Belus
60 cubes of circumference = 64 × 60 = 3840 circumference
= distance of the earth.

Should circumference of less circle = 1596 units
10 × 1596 = 15960
Distance of Saturn = 15960^3.

If circumference of greater circle = 1956
10 × 1956 = 19560
Distance of Uranus = 19560^3.

Cube of 10 times less circumference = distance of Saturn
Cube of 10 times greater circumference = „ Uranus.

"Rath" is a Celtic word for "fort." It abounds in Scotland, but usually with a variety of pronunciations. Such forts are usually mere earth-works, forming a circle, or set of concentric circles, on plain ground, or cutting off the outer angles of a bank overhanging a rivulet. The enclosure is supposed to have contained temporary buildings for residence.

The celebrated hill of Tara, in the county of Meath, Ireland, is covered with a cluster of raths, and presents few other objects. From an indefinitely early period down to the sixth century it was a chief seat of the Irish kings, according to Wakeman. Shortly after the death of Dermot, the son of Fergus, in the year 563, the place was deserted, in consequence, as it is said, of a curse pronounced by St. Ruadan, or Rodanus, of Lorha, against that king and his palace. After thirteen centuries of ruin, the chief monuments for which the hill was at any time remarkable are distinctly to be traced. They consist for the most part of circular or oval enclosures and mounds, within or upon which the principal habitations of the ancient city undoubtedly stood. The rath called Rath Righ, or Cathair Crofinn, appears anciently to have been the most important work upon the hill, but it is now nearly levelled with the ground. It is of an oval form, measures in length from north to south about 850 feet, and appears in part to have been constructed of stone: within its enclosure are the ruins of the Forradh, and of Teach Cormac, or the house of Cormac. The mound of the Forradh is of considerable height, flat at the top, and encircled with two lines of earth, having a ditch between them. In its centre is a very remarkable pillar stone, which formerly stood upon, or rather by the side of a small mound, lying within the enclosure of Rath Righ, and called Dumhana-n-Giall, or the mound of the Hostages, but which was removed to its present site to mark the grave of some men slain in an encounter with the king's troops during the rising of 1798. It has been suggested by Petrie, that it is extremely probable that this monument is no other than the celebrated Lia Fail, or Stone of Destiny, upon which, for many ages, the monarchs of Ireland were crowned, and which is generally supposed to have been removed from Ireland to Scotland for the coronation of Fergus Mac Eark, a prince of the blood royal of Ireland, there having been a prophecy that in whatever country this famous stone was preserved, a king of the Scotic race should reign.

The Teach Cormac, lying on the south-east of the Forradh, with which is joined a common parapet, may be described as a double enclosure, the rings of which upon the western side became connected. Its diameter is about 140 feet ($\frac{1}{2}$ a stade $= 140\frac{1}{2}$ feet.)

Diameter of ellipse $= 850$ feet $= 734{\cdot}6$ units.

$734^3 = \frac{1}{3}$ distance of the moon.

$(3 \times 734)^3 = \frac{1}{3} \times 3^3 \quad = 9$ „

Cube of diameter $= \frac{1}{3}$ „

Cube of 3 diameter $= 9$ „

Circumference of circle of diameter $734 = 2206$ units.

221^3 &c. $= \frac{1}{100}$ distance of the moon.

$(10 \times 221$ &c.$)^3 = \frac{1000}{100} = 10.$

Cube of circumference $= 10$ distance of the moon.

Diameter of Teach Cormac $= \frac{1}{2}$ stade.

Cube of diameter $= \frac{1}{64}$ circumference.

Cube of circumference $= \frac{31}{64} = \frac{1}{2}$ circumference nearly.

Cube of 4 times diameter $=$ circumference.

Cube of 4 times circumference $= 31.$

Cube of 20 times circumference $= 31 \times 5^3 = 3875$ circumference.

distance of the earth $= 3840.$

This is the only measured Druidical monument in Ireland that we have met with.

We find Druidical monuments in Denmark, Sweden, and Norway, quoting from the French "Idolatrous Nations." It appears that the Laplanders in Denmark, natives of Finland, and the proper Laplanders in former times all worshipped Jumela as the Supreme Being, and likewise the Sun and Moon. Storjunkare is represented under the form of a large unpolished stone, such as is met with in the mountains; sometimes it is sculptured. This stone-god is frequently supplied with a numerous family; one of them is his wife, others his sons and daughters, and the rest his domestics. Rein-deers are sacrificed to Thoron, but to the Sun only young female deers. They have tutors and academies for the particular study of the black art. They stand in awe of their manes, or the souls of their dead, till they are actually transmigrated into new bodies; whence it is manifest that their notion, with respect to souls, is the same as that received among the Tartars and Scythians, who borrowed it from the eastern nations.

There is an ancient chapel, in ruins, situated between Revel and Nerva, where some devotees strip themselves naked and fall down on their knees, before a great stone, which stands in the middle of the chapel; they also dance round it, and offer oblations of fruits and other provisions.

This ceremony is a relic of that religious worship of the Goths which all the people in general of the north, the Germans, Gauls, &c., paid formerly to stones; and we are assured that this divine adoration of them was grounded on a notion, which was then established among all those idolaters, that some diminutive sprites, or imps of the devil, resided within those stones; nay, they carried the point still further, and were fully persuaded that those stones were oracles.

At this day the peasants in part of Brittany believe that at certain periods of the year, when the moon shines brightly, that hideous dwarfs, whom they call Cormandons, rise from their subterranean abodes, form an infernal ring about the dol-mens and men-hirs, and try to attract travellers by ringing gold upon the sacred stone.

We consider some of the single upright Druidical stones to be rough representations of the accurately proportioned and highly-finished obelisk of the Egyptians.

Indeed some of them assume the rough, square, tapering, truncated form, as already noticed. Others are sculptured.

In Scotland four or five ancient obelisks are still to be seen, called the Danish stones of Aberlemno, and are adorned with bas-reliefs of men on horseback, and many emblematical figures and hieroglyphics, not intelligible at this day. The stone near Forrest rises about 23 feet above the ground, and is supposed to be not less than 12 or 15 feet below, so that the whole height will be at least 35 feet, and its breadth is nearly 5 feet. A great variety of figures in relief are carved on it. Many Druidical monuments and temples are discernible in the northern parts of Scotland as well as in the isles. They are circular, and equally regular with those in England, but not on so large a scale.

The cromlech at Plas Newydd, in Anglesea, is formed by a massive irregular-shaped stone, supported laterally by other stones, which incline inwards from the base to the stone that forms the roof; the whole structure resembles an Egyptian propylon.

In the Druidical circle at Jersey, a large stone is represented as forming a projecting roof, which is supported laterally by other stones inclining inwards from the base to the top — like Kit's Cotty house, in Kent; one view of the last represents the external sides as but little inclined to the roof, which is nearly flat and projecting, like the top of a propylon, or the roof of the monolithic chapel at Butos, already described by Herodotus as having a single flat stone projecting over the sides of the chapel.

The tomb of Cyrus seems to have been formed like the Butos stone chapel, with a projecting roof, according to the "Antiquities of Persia." Kit's Cotty is called a Kist-vaen, or stone chest, which not only accords with our views, but it will be seen that the use made of the Kist-vaen may throw some light on the monolithic chambers or chest of the Egyptians.

Davies describes the probation of Taliesim, a Druidical noviciate. "I was first modelled in the form of a pure man in the hall of Ceridwin, who subjected me to penance. Though small within my ark, and modest in my deportment, I was great. A sanctuary carried me above the surface of the earth. Whilst I was enclosed within its ribs the sweet Awen rendered me complete." Whence Davies infers that the Kist-vaen is very probably the ark here referred to. The Kist-vaen, like other Druidical monuments, is found in different and remote parts of the world. There is one on the banks of the Jordan resembling Kit's Cotty.

The opinion of Clemens Alexandrinus is that columns were worshipped as the images of God. Herodian says the Phœnicians worshipped a great stone circular below, and ending with a sharpness above in the figure of a cone, and of a black colour. They report it to have fallen from heaven, and to be the image of the sun. The vertical section, or plane of an Egyptian obelisk revolving on its axis, would generate a solid answering this description, like the pointed minaret. This conical stone was called Elæogabalis.

M. Aurelius Antoninus, a Roman emperor, called Heliogabalus, because he had been a priest of that divinity in Phœnicia, obliged his subjects to pay adoration to the god Heliogabalus, which was no other than a large black stone, having the form of a cone, that he brought with him to Rome on his being elected emperor by the army; he built a temple to the god, and continued priest himself, commanding the Vestal fire, the palladium, and consecrated bucklers to be transported thither.

Mahomet destroyed other superstitions of the Arabs, but he was obliged to adopt their rooted veneration for the black stone, and transfer to Mecca the respect and reverence which he had designed for Jerusalem. — (*Pitts.*)

It appears that history can still trace among various other nations the worship of conical and pyramidal stones.

The Paphian Venus was the celestial Venus of the Assyrians, and represented, according to Tacitus, by a cone, but, according to Maximus Tyrius, by a white pyramid. The Paphian Venus, says Pausanias, was worshipped first by the Assyrians, afterwards by the Paphians and Phœnicians of Ascalon. The Cythereans acquired these rites from the Phœnicians. He also states that it was the custom of the Greeks, at an early period, to reverence the form of rude stones instead of statues; and adds, that several such existed at his time. At Pheræ there were thirty square stones, each called by the name of some deity. Mercury was frequently represented by a rude stone. The Apollo Carynus and Jupiter Milichius, in the forum of Sicyon, were worshipped under the form of small pyramids: the Diana Patroa, in the same place, under that of a column. The Hercules of Hyettus was a rude stone. The symbol of Cupid at Thespia was also a rude stone. According to Clemens Alexandrinus, the Delphic Apollo was once a column. Lactantius mentions the worship of Terminus under the form of a rude stone.

Hamilton describes one of the idols in the pagoda of Juggernaut as a huge black stone, of a pyramidal form. Maurice mentions a black stone, 50 cubits high, that stood before the gate of a temple erected to the sun by an ancient rajah. In the pagoda at Benares is an idol of a black stone. Boodh was represented by a huge column of black stone.

Hercules, Neptune, Cupid, Jupiter, Juno, and Diana, says Legrew, were first worshipped under the symbols of cones and pyramids; at a later period, these statues presented forms of transcendent beauty.

On the old coins of Apollonia, according to D'Ankerville, Apollo was represented by an obelisk a little different from those of Egypt. On the medal of the Chalcidians is an ancient representation of Neptune, in the form of a pyramid. On a medal of Ceos, Jupiter and Juno appear in the form of pyramids ornamented with draperies. Damascius, in his life of Isidorus, states, that many consecrated stones were to be seen near Heliopolis, in Syria; and adds, that they were dedicated to Gad, Jupiter, the Sun, and other deities.

Seetzen having assumed the character of a Mahometan, took a passage in a vessel from Suez, where there were a number of other pilgrims destined for Mecca. Before reaching Jidda, they came to a village called Rabog, where the ceremony took place of putting on the ehhran—the pilgrim's dress. Thus transformed into pilgrims, they began to cry aloud Lubbaik, Allahoumme Lubbaik, an ancient form of prayer which Seetzen suspects of being appropriated to Bacchus. At Mecca he found the holy temple composing a most majestic square, 300 feet by 200, and surrounded with a triple or quadruple row of columns. The houses of the town rose above it, and the surrounding mountains high above them, so that he felt as in the arena of a magnificent theatre. He had an opportunity of seeing the Kaaba encircled by more than a thousand pilgrims, Arabs from every province, Moors, Persians, Afghans, and natives of all the countries of the East. In their enthusiastic zeal to kiss the black stone, they rushed pell-mell in confused crowds, so as to cause an apprehension that some of them must have been suffocated. This religious tumult, with the multitude and various aspect of the groups, presented the most extraordinary spectacle he ever beheld.

TABLES

OF

SQUARES, CUBES, AND POWERS.

TABLE OF SQUARES,

From 1 to 1000.

Root or Numb.	Square.	Root or Numb.	Square.	Root or Numb.	Square.	Root or Numb.	Square.
1	1	37	1369	73	5329	109	11881
2	4	38	1444	74	5476	110	12100
3	9	39	1521	75	5625	111	12321
4	16	40	1600	76	5776	112	12544
5	25	41	1681	77	5929	113	12769
6	36	42	1764	78	6084	114	12996
7	49	43	1849	79	6241	115	13225
8	64	44	1936	80	6400	116	13456
9	81	45	2025	81	6561	117	13689
10	100	46	2116	82	6724	118	13924
11	121	47	2209	83	6889	119	14161
12	144	48	2304	84	7056	120	14400
13	169	49	2401	85	7225	121	14641
14	196	50	2500	86	7396	122	14884
15	225	51	2601	87	7569	123	15129
16	256	52	2704	88	7744	124	15376
17	289	53	2809	89	7921	125	15625
18	324	54	2916	90	8100	126	15876
19	361	55	3025	91	8281	127	16129
20	400	56	3136	92	8464	128	16384
21	441	57	3249	93	8649	129	16641
22	484	58	3364	94	8836	130	16900
23	529	59	3481	95	9025	131	17161
24	576	60	3600	96	9216	132	17424
25	625	61	3721	97	9409	133	17689
26	676	62	3844	98	9604	134	17956
27	729	63	3969	99	9801	135	18225
28	784	64	4096	100	10000	136	18496
29	841	65	4225	101	10201	137	18769
30	900	66	4356	102	10404	138	19044
31	961	67	4489	103	10609	139	19321
32	1024	68	4624	104	10816	140	19600
33	1089	69	4761	105	11025	141	19881
34	1156	70	4900	106	11236	142	20164
35	1225	71	5041	107	11449	143	20449
36	1296	72	5184	108	11664	144	20736

Root or Numb.	Square.	Root or Numb.	Square.	Root or Numb.	Square.	Root or Numb.	Square.
145	21025	191	36481	237	56169	283	80089
146	21316	192	36864	238	56644	284	80656
147	21609	193	37249	239	57121	285	81225
148	21904	194	37636	240	57600	286	81796
149	22201	195	38025	241	58081	287	82369
150	22500	196	38416	242	58564	288	82944
151	22801	197	38809	243	59049	289	83521
152	23104	198	39204	244	59536	290	84100
153	23409	199	39601	245	60025	291	84681
154	23716	200	40000	246	60516	292	85264
155	24025	201	40401	247	61009	293	85849
156	24336	202	40804	248	61504	294	86436
157	24649	203	41209	249	62001	295	87025
158	24964	204	41616	250	62500	296	87616
159	25281	205	42025	251	63001	297	88209
160	25600	206	42436	252	63504	298	88804
161	25921	207	42849	253	64009	299	89401
162	26244	208	43264	254	64516	300	90000
163	26569	209	43681	255	65025	301	90601
164	26896	210	44100	256	65536	302	91204
165	27225	211	44521	257	66049	303	91809
166	27556	212	44944	258	66564	304	92416
167	27889	213	45369	259	67081	305	93025
168	28224	214	45796	260	67600	306	93636
169	28561	215	46225	261	68121	307	94249
170	28900	216	46656	262	68644	308	94864
171	29241	217	47089	263	69169	309	95481
172	29584	218	47524	264	69696	310	96100
173	29929	219	47961	265	70225	311	96721
174	30276	220	48400	266	70756	312	97344
175	30625	221	48841	267	71289	313	97969
176	30976	222	49284	268	71824	314	98596
177	31329	223	49729	269	72361	315	99225
178	31684	224	50176	270	72900	316	99856
179	32041	225	50625	271	73441	317	100489
180	32400	226	51076	272	73984	318	101124
181	32761	227	51529	273	74529	319	101761
182	33124	228	51984	274	75076	320	102400
183	33489	229	52441	275	75625	321	103041
184	33856	230	52900	276	76176	322	103684
185	34225	231	53361	277	76729	323	104329
186	34596	232	53824	278	77284	324	104976
187	34969	233	54289	279	77841	325	105625
188	35344	234	54756	280	78400	326	106276
189	35721	235	55225	281	78961	327	106929
190	36100	236	55696	282	79524	328	107584

Root or Numb.	Square.	Root or Numb.	Square.	Root or Numb.	Square.	Root or Numb.	Square.
329	108241	375	140625	421	177241	467	218089
330	108900	376	141376	422	178084	468	219024
331	109561	377	142129	423	178929	469	219961
332	110224	378	142884	424	179776	470	220900
333	110889	379	143641	425	180625	471	221841
334	111556	380	144400	426	181476	472	222784
335	112225	381	145161	427	182329	473	223729
336	112896	382	145924	428	183184	474	224676
337	113569	383	146689	429	184041	475	225625
338	114244	384	147456	430	184900	476	226576
339	114921	385	148225	431	185761	477	227529
340	115600	386	148996	432	186624	478	228484
341	116281	387	149769	433	187489	479	229441
342	116964	388	150544	434	188356	480	230400
343	117649	389	151321	435	189225	481	231361
344	118336	390	152100	436	190096	482	232324
345	119025	391	152881	437	190969	483	233289
346	119716	392	153664	438	191844	484	234256
347	120409	393	154449	439	192721	485	235225
348	121104	394	155236	440	193600	486	236196
349	121801	395	156025	441	194481	487	237169
350	122500	396	156816	442	195364	488	238144
351	123201	397	157609	443	196249	489	239121
352	123904	398	158404	444	197136	490	240100
353	124609	399	159201	445	198025	491	241081
354	125316	400	160000	446	198916	492	242064
355	126025	401	160801	447	199809	493	243049
356	126736	402	161604	448	200704	494	244036
357	127449	403	162409	449	201601	495	245025
358	128164	404	163216	450	202500	496	246016
359	128881	405	164025	451	203401	497	247009
360	129600	406	164836	452	204304	498	248004
361	130321	407	165649	453	205209	499	249001
362	131044	408	166464	454	206116	500	250000
363	131769	409	167281	455	207025	501	251001
364	132496	410	168100	456	207936	502	252004
365	133225	411	168921	457	208849	503	253009
366	133956	412	169744	458	209764	504	254016
367	134689	413	170569	459	210681	505	255025
368	135424	414	171396	460	211600	506	256036
369	136161	415	172225	461	212521	507	257049
370	136900	416	173056	462	213444	508	258064
371	137641	417	173889	463	214369	509	259081
372	138384	418	174724	464	215296	510	260100
373	139129	419	175561	465	216225	511	261121
374	139876	420	176400	466	217156	512	262144

Root or Numb.	Square.	Root or Numb.	Square.	Root or Numb	Square.	Root or Numb.	Square.
513	263169	559	312481	605	366025	651	423801
514	264196	560	313600	606	367236	652	425104
515	265225	561	314721	607	368449	653	426409
516	266256	562	315844	608	369664	654	427716
517	267289	563	316969	609	370881	655	429025
518	268324	564	318096	610	372100	656	430336
519	269361	565	319225	611	373321	657	431649
520	270400	566	320356	612	374544	658	432964
521	271441	567	321489	613	375769	659	434281
522	272484	568	322624	614	376996	660	435600
523	273529	569	323761	615	378225	661	436921
524	274576	570	324900	616	379456	662	438244
525	275625	571	326041	617	380689	663	439569
526	276676	572	327184	618	381924	664	440896
527	277729	573	328329	619	383161	665	442225
528	278784	574	329476	620	384400	666	443556
529	279841	575	330625	621	385641	667	444889
530	280900	576	331776	622	386884	668	446224
531	281961	577	332929	623	388129	669	447561
532	283024	578	334084	624	389376	670	448900
533	284089	579	335241	625	390625	671	450241
534	285156	580	336400	626	391876	672	451584
535	286225	581	337561	627	393129	673	452929
536	287296	582	338724	628	394384	674	454276
537	288369	583	339889	629	395641	675	455625
538	289444	584	341056	630	396900	676	456976
539	290521	585	342225	631	398161	677	458329
540	291600	586	343396	632	399424	678	459684
541	292681	587	344569	633	400689	679	461041
542	293764	588	345744	634	401956	680	462400
543	294849	589	346921	635	403225	681	463761
544	295936	590	348100	636	404496	682	465124
545	297025	591	349281	637	405769	683	466489
546	298116	592	350464	638	407044	684	467856
547	299209	593	351649	639	408321	685	469225
548	300304	594	352836	640	409600	686	470596
549	301401	595	354025	641	410881	687	471969
550	302500	596	355216	642	412164	688	473344
551	303601	597	356409	643	413449	689	474721
552	304704	598	357604	644	414736	690	476100
553	305809	599	358801	645	416025	691	477481
554	306916	600	360000	646	417316	692	478864
555	308025	601	361201	647	418609	693	480249
556	309136	602	362404	648	419904	694	481636
557	310249	603	363609	649	421201	695	483025
558	311364	604	364816	650	422500	696	484416

Root or Numb.	Square.	Root or Numb.	Square.	Root or Numb.	Square.	Root or Numb.	Square.
697	485809	743	552049	789	622521	835	697225
698	487204	744	553536	790	624100	836	698896
699	488601	745	555025	791	625681	837	700569
700	490000	746	556516	792	627264	838	702244
701	491401	747	558009	793	628849	839	703921
702	492804	748	559504	794	630436	840	705600
703	494209	749	561001	795	632025	841	707281
704	495616	750	562500	796	633616	842	708964
705	497025	751	564001	797	635209	843	710649
706	498436	752	565504	798	636804	844	712336
707	499849	753	567009	799	638401	845	714025
708	501264	754	568516	800	640000	846	715716
709	502681	755	570025	801	641601	847	717409
710	504100	756	571536	802	643204	848	719104
711	505521	757	573049	803	644809	849	720801
712	506944	758	574564	804	646416	850	722500
713	508369	759	576081	805	648025	851	724201
714	509796	760	577660	806	649636	852	725904
715	511225	761	579121	807	651249	853	727609
716	512656	762	580644	808	652864	854	729316
717	514089	763	582169	809	654481	855	731025
718	515524	764	583696	810	656100	856	732736
719	516961	765	585225	811	657721	857	734449
720	518400	766	586756	812	659344	858	736164
721	519841	767	588289	813	660969	859	737881
722	521284	768	589824	814	662596	860	739600
723	522729	769	591361	815	664225	861	741321
724	524176	770	592900	816	665856	862	743044
725	525625	771	594441	817	667489	863	744769
726	527076	772	595984	818	669124	864	746496
727	528529	773	597529	819	670761	865	748225
728	529984	774	599076	820	672400	866	749956
729	531441	775	600625	821	674041	867	751689
730	532900	776	602176	822	675684	868	753424
731	534361	777	603729	823	677329	869	755161
732	535824	778	605284	824	678976	870	756900
733	537289	779	606841	825	680625	871	758641
734	538756	780	608400	826	682276	872	760384
735	540225	781	609961	827	683929	873	762129
736	541696	782	611524	828	685584	874	763876
737	543169	783	613089	829	687241	875	765625
738	544644	784	614656	830	688900	876	767376
739	546121	785	616225	831	690561	877	769129
740	547600	786	617796	832	692224	878	770884
741	549081	787	619369	833	693889	879	772641
742	550564	788	620944	834	695556	880	774400

Root or Numb.	Square.	Root or Numb.	Square.	Root or Numb.	Square.	Root or Numb.	Square.
881	776161	911	829921	941	885481	971	942841
882	777924	912	831744	942	887364	972	944784
883	779689	913	833569	943	889249	973	946729
884	781456	914	835396	944	891136	974	948676
885	783225	915	837225	945	893025	975	950625
886	784996	916	839056	946	894916	976	952576
887	786769	917	840889	947	896809	977	954529
888	788544	918	842724	948	898704	978	956484
889	790321	919	844561	949	900601	979	958441
890	792100	920	846400	950	902500	980	960400
891	793881	921	848241	951	904401	981	962361
892	795664	922	850084	952	906304	982	964324
893	797449	923	851929	953	908209	983	966289
894	799236	924	853776	954	910116	984	968256
895	801025	925	855625	955	912025	985	970225
896	802816	926	857476	956	913936	986	972196
897	804609	927	859329	957	915849	987	974169
898	806404	928	861184	958	917764	988	976144
899	808201	929	863041	959	919681	989	978121
900	810000	930	864900	960	921600	990	980100
901	811801	931	866761	961	923521	991	982081
902	813604	932	868624	962	925444	992	984064
903	815409	933	870489	963	927369	993	986049
904	817216	934	872356	964	929296	994	988036
905	819025	935	874225	965	931225	995	990025
906	820836	936	876096	966	933156	996	992016
907	822649	937	877969	967	935089	997	994009
908	824464	938	879844	968	937024	998	996004
909	826281	939	881721	969	938961	999	998001
910	828100	940	883600	970	940900	1000	1000000

TABLE OF CUBES,

From 1 to 1000.

Root or Number.	Cube.	Root or Number.	Cube.	Root or Number.	Cube.
1	1	37	50653	73	389017
2	8	38	54872	74	405224
3	27	39	59319	75	421875
4	64	40	64000	76	438976
5	125	41	68921	77	456533
6	216	42	74088	78	474552
7	343	43	79507	79	493039
8	512	44	85184	80	512000
9	729	45	91125	81	531441
10	1000	46	97336	82	551368
11	1331	47	103823	83	571787
12	1728	48	110592	84	592704
13	2197	49	117649	85	614125
14	2744	50	125000	86	636056
15	3375	51	132651	87	658503
16	4096	52	140608	88	681472
17	4913	53	148877	89	704969
18	5832	54	157464	90	729000
19	6859	55	166375	91	753571
20	8000	56	175616	92	778688
21	9261	57	185193	93	804357
22	10648	58	195112	94	830584
23	12167	59	205379	95	857375
24	13824	60	216000	96	884736
25	15625	61	226981	97	912673
26	17576	62	238328	98	941192
27	19683	63	250047	99	970299
28	21952	64	262144	100	1000000
29	24389	65	274625	101	1030301
30	27000	66	287496	102	1061208
31	29791	67	300763	103	1092727
32	32768	68	314432	104	1124864
33	35937	69	328509	105	1157625
34	39304	70	343000	106	1191016
35	42875	71	357911	107	1225043
36	46656	72	373248	108	1259712

Root or Number.	Cube.	Root or Number.	Cube.	Root or Number.	Cube.
109	1295029	155	3723875	201	8120601
110	1331000	156	3796416	202	8242408
111	1367631	157	3869893	203	8365427
112	1404928	158	3944312	204	8489664
113	1442897	159	4019679	205	8615125
114	1481544	160	4096000	206	8741816
115	1520875	161	4173281	207	8869743
116	1560896	162	4251528	208	8998912
117	1601613	163	4330747	209	9123329
118	1643032	164	4410944	210	9261000
119	1685159	165	4492125	211	9393931
120	1728000	166	4574296	212	9528128
121	1771561	167	4657463	213	9663597
122	1815848	168	4741632	214	9800344
123	1860867	169	4826809	215	9938375
124	1906624	170	4913000	216	10077696
125	1953125	171	5000211	217	10218313
126	2000376	172	5088448	218	10360232
127	2048383	173	5177717	219	10503459
128	2097152	174	5268024	220	10648000
129	2146689	175	5359375	221	10793861
130	2197000	176	5451776	222	10941048
131	2248091	177	5545233	223	11089567
132	2299968	178	5639752	224	11239424
133	2352637	179	5735339	225	11390625
134	2406104	180	5832000	226	11543176
135	2460375	181	5929741	227	11697083
136	2515456	182	6028568	228	11852352
137	2571353	183	6128487	229	12008989
138	2628072	184	6229504	230	12167000
139	2685619	185	6331625	231	12326391
140	2744000	186	6434856	232	12487168
141	2803221	187	6539203	233	12649337
142	2863288	188	6644672	234	12812904
143	2924207	189	6751269	235	12977875
144	2985984	190	6859000	236	13144256
145	3048625	191	6967871	237	13312053
146	3112136	192	7077888	238	13481272
147	3176523	193	7189057	239	13651919
148	3241792	194	7301384	240	13824000
149	3307949	195	7414875	241	13997521
150	3375000	196	7529536	242	14172488
151	3442951	197	7645373	243	14348907
152	3511808	198	7762392	244	14526784
153	3581577	199	7880599	245	14706125
154	3652264	200	8000000	246	14886936

Root or Number.	Cube.	Root or Number.	Cube.	Root or Number.	Cube.
247	15069223	293	25153757	339	38958219
248	15252992	294	25412184	340	39304000
249	15438249	295	25672375	341	39651821
250	15625000	296	25934336	342	40001688
251	15813251	297	26198073	343	40353607
252	16003008	298	26463592	344	40707584
253	16194277	299	26730899	345	41063625
254	16387064	300	27000000	346	41421736
255	16581375	301	27270901	347	41781923
256	16777216	302	27543608	348	42144192
257	16974593	303	27818127	349	42508549
258	17173512	304	28094464	350	42875000
259	17373979	305	28372625	351	43243551
260	17576000	306	28652616	352	43614208
261	17779581	307	28934443	353	43986977
262	17984728	308	29218112	354	44361864
263	18191447	309	29503629	355	44738875
264	18399744	310	29791000	356	45118016
265	18609625	311	30080231	357	45499293
266	18821096	312	30371328	358	45882712
267	19034163	313	30664297	359	46268279
268	19248832	314	30959144	360	46656000
269	19465109	315	31255875	361	47045881
270	19683000	316	31554496	362	47437928
271	19902511	317	31855013	363	47832147
272	20123648	318	32157432	364	48228544
273	20346417	319	32461759	365	48627125
274	20570824	320	32768000	366	49027896
275	20796875	321	33076161	367	49430863
276	21024576	322	33386248	368	49836032
277	21253933	323	33698267	369	50243409
278	21484952	324	34012224	370	50653000
279	21717639	325	34328125	371	51064811
280	21952000	326	34645976	372	51478848
281	22188041	327	34965783	373	51895117
282	22425768	328	35287552	374	52313624
283	22665187	329	35611289	375	52734375
284	22906304	330	35937000	376	53157376
285	23149125	331	36264691	377	53582633
286	23393656	332	36594368	378	54010152
287	23639903	333	36926037	379	54439939
288	23887872	334	37259704	380	54872000
289	24137569	335	37595375	381	55306341
290	24389000	336	37933056	382	55742968
291	24642171	337	38272753	383	56181887
292	24897088	338	38614472	384	56623104

Root or Number.	Cube.	Root or Number.	Cube.	Root or Number.	Cube.
385	57066625	431	80062991	477	108531333
386	57512456	432	80621568	478	109215352
387	57960603	433	81182737	479	109902239
388	58411072	434	81746504	480	110592000
389	58863869	435	82312875	481	111284641
390	59319000	436	82881856	482	111980168
391	59776471	437	83453453	483	112678587
392	60236288	438	84027672	484	113379904
393	60698457	439	84604519	485	114084125
394	61162984	440	85184000	486	114791256
395	61629875	441	85766121	487	115501303
396	62099136	442	86350888	488	116214272
397	62570773	443	86938307	489	116930169
398	63044792	444	87528384	490	117649000
399	63521199	445	88121125	491	118370771
400	64000000	446	88716536	492	119095488
401	64481201	447	89314623	493	119823157
402	64964808	448	89915392	494	120553784
403	65450827	449	90518849	495	121287375
404	65939264	450	91125000	496	122023936
405	66430125	451	91733851	497	122763473
406	66923416	452	92345408	498	123505992
407	67419143	453	92959677	499	124251499
408	67911312	454	93576664	500	125000000
409	68417929	455	94196375	501	125751501
410	68921000	456	94818816	502	126506008
411	69426531	457	95443993	503	127263527
412	69934528	458	96071912	504	128024064
413	70444997	459	96702579	505	128787625
414	70957944	460	97336000	506	129554216
415	71473375	461	97972181	507	130323843
416	71991296	462	98611128	508	131096512
417	72511713	463	99252847	509	131872229
418	73034632	464	99897344	510	132651000
419	73560059	465	100554625	511	133432831
420	74088000	466	101194696	512	134217728
421	74618461	467	101847563	513	135005697
422	75151448	468	102503232	514	135796744
423	75686967	469	103161709	515	136590875
424	76225024	470	103823000	516	137388096
425	76765625	471	104487111	517	138188413
426	77308776	472	105154048	518	138991832
427	77854483	473	105823817	519	139798359
428	78402752	474	106496424	520	140608000
429	78953589	475	107171875	521	141420761
430	79507000	476	107850176	522	142236648

Root or Number.	Cube.	Root or Number.	Cube.	Root or Number.	Cube.
523	143055667	569	184220009	615	232608375
524	143877824	570	185193000	616	233744896
525	144703125	571	186169411	617	234885113
526	145531576	572	187149248	618	236029032
527	146363183	573	188132517	619	237176659
528	147197952	574	189119224	620	238328000
529	148035889	575	190109375	621	239483061
530	148877000	576	191102976	622	240641848
531	149721291	577	192100033	623	241804367
532	150568768	578	193100552	624	242970624
533	151419437	579	194104539	625	244140625
534	152273304	580	195112000	626	245314376
535	153130375	581	196122941	627	246491883
536	153990656	582	197137368	628	247673152
537	154854153	583	198155287	629	248858189
538	155720872	584	199176704	630	250047000
539	156590819	585	200201625	631	251239591
540	157464000	586	201230056	632	252435968
541	158340421	587	202262003	633	253636137
542	159220088	588	203297472	634	254840104
543	160103007	589	204336469	635	256047875
544	160989184	590	205379000	636	257259456
545	161878625	591	206425071	637	258474853
546	162771336	592	207474688	638	259694072
547	163667323	593	208527857	639	260917119
548	164566592	594	209584584	640	262144000
549	165469149	595	210644875	641	263374721
550	166375000	596	211708736	642	264609288
551	167284151	597	212776173	643	265847707
552	168196608	598	213847192	644	267089984
553	169112377	599	214921799	645	268336125
554	170031464	600	216000000	646	269586136
555	170953875	601	217081801	647	270840023
556	171879616	602	218167208	648	272097792
557	172808693	603	219256227	649	273359449
558	173741112	604	220348864	650	274625000
559	174676879	605	221445125	651	275894451
560	175616000	606	222545016	652	277167808
561	176558481	607	223648543	653	278445077
562	177504328	608	224755712	654	279726264
563	178453547	609	225866529	655	281011375
564	179406144	610	226981000	656	282300416
565	180362125	611	228099131	657	283593393
566	181321496	612	229220928	658	284890312
567	182284263	613	230346397	659	286191179
568	183250432	614	231475544	660	287496000

Root or Number.	Cube.	Root or Number.	Cbe.	Root or Number.	Cube.
661	288804781	707	353393243	753	426957777
662	290117528	708	354894912	754	428661064
663	291434247	709	356400829	755	430368875
664	292754944	710	357911000	756	432081216
665	294079625	711	359425431	757	433798093
666	295408296	712	360944128	758	435519512
667	296740963	713	362467097	759	437245479
668	298077632	714	363994344	760	438976000
669	299418309	715	365525875	761	440711081
670	300763000	716	367061696	762	442450728
671	302111711	717	368601813	763	444194947
672	303464448	718	370146232	764	445943744
673	304821217	719	371694959	765	447697125
674	306182024	720	373248000	766	449455096
675	307546875	721	374805361	767	451217663
676	308915776	722	376367048	768	452984832
677	310288733	723	377933067	769	454756609
678	311665752	724	379503424	770	456533000
679	313046839	725	381078125	771	458314011
680	314432000	726	382657176	772	460099648
681	315821241	727	384240583	773	461889917
682	317214568	728	385828352	774	463684824
683	318611987	729	387420489	775	465484375
684	320013504	730	389017000	776	467288576
685	321419125	731	390617891	777	469097433
686	322828856	732	392223168	778	470910952
687	324242703	733	393832837	779	472729139
688	325660672	734	395446904	780	474552000
689	327082769	735	397065375	781	476379541
690	328509000	736	398688256	782	478211768
691	329939371	737	400315553	783	480048687
692	331373888	738	401947272	784	481890304
693	332812557	739	403583419	785	483736625
694	334255384	740	405224000	786	485587656
695	335702375	741	406869021	787	487443403
696	337153536	742	408518488	788	489303872
697	338608873	743	410172407	789	491169069
698	340068392	744	411830784	790	493039000
699	341532099	745	413493625	791	494913671
700	343000000	746	415160936	792	496793088
701	344472101	747	416832723	793	498677257
702	345948008	748	418508992	794	500566184
703	347428927	749	420189749	795	502459875
704	348913664	750	421875000	796	504358336
705	350402625	751	423564751	797	506261573
706	351895816	752	425259008	798	508169592

Root or Number.	Cube.	Root or Number.	Cube.	Root or Number.	Cube.
799	510082399	845	603351125	891	707347971
800	512000000	846	605495736	892	709732288
801	513922401	847	607645423	893	712121957
802	515849608	848	609800192	894	714516984
803	517781627	849	611960049	895	716917375
804	519718464	850	614125000	896	719323136
805	521660125	851	616295051	897	721734273
806	523606616	852	618470208	898	724150792
807	525557943	853	620650477	899	726572699
808	527514112	854	622835864	900	729000000
809	529475129	855	625026375	901	731432701
810	531441000	856	627222016	902	733870808
811	533411731	857	629422793	903	736314327
812	535387328	858	631628712	904	738763264
813	537366797	859	633839779	905	741217625
814	539353144	860	636056000	906	743677416
815	541343375	861	638277381	907	746142643
816	543338496	862	640503928	908	748613312
817	545338513	863	642735647	909	751089429
818	547343432	864	644972544	910	753571000
819	549353259	865	647214625	911	756058031
820	551368000	866	649461896	912	758550528
821	553387661	867	651714363	913	761048497
822	555412248	868	653972032	914	763551944
823	557441767	869	656234909	915	766060875
824	559476224	870	658503000	916	768575296
825	561515625	871	660776311	917	771095213
826	563559976	872	663054848	918	773620632
827	565609283	873	665338617	919	776151559
828	567663552	874	667627624	920	778688000
829	569722789	875	669921875	921	781229961
830	571787000	876	672221376	922	783777448
831	573856191	877	674526133	923	786330467
832	575930368	878	676836152	924	788889024
833	578009537	879	679151439	925	791453125
834	580093704	880	681472000	926	794022776
835	582182875	881	683797841	927	796597983
836	584277056	882	686128968	928	799178752
837	586376253	883	688465387	929	801765089
838	588480472	884	690807104	930	804357000
839	590589719	885	693154125	931	806954491
840	592704000	886	695506456	932	809557568
841	594823321	887	697864103	933	812166237
842	596947688	888	700227072	934	814780504
843	599077107	889	702595369	935	817400375
844	601211584	890	704969000	936	820025856

t er.	Cube.	Root or Number.	Cube.	Root or Number.	Cube.
937	822656953	959	881974079	980	941192000
938	825293672	960	884736000	981	944076141
9	827936019	961	887503681	982	946966168
	830584000	962	890277128	983	949862087
	833237621	963	893056347	984	952763904
	835896888	964	895841344	985	955671625
3	838561807	965	898632125	986	958585256
4	841232384	966	901428696	987	961504803
945	843908625	967	904231063	988	964430272
946	846590536	968	907039232	989	967361669
947	849278123	969	909853209	990	970299000
948	851971392	970	912673000	991	973242271
949	854670349	971	915498611	992	976191488
950	857375000	972	918330048	993	979146657
951	860085351	973	921167317	994	982107784
952	862801408	974	924010424	995	985074875
953	865523177	975	926859375	996	988047936
954	868250664	976	929714176	997	991026973
955	870983875	977	932574833	998	994011992
956	873722816	978	935441352	999	997002999
957	876467493	979	938313739	1000	1000000000
958	879217912				

A TABLE OF POWERS.

R. or 1ſt Power	1	2	3	4	5	6	7	8	9
2d Power	1	4	9	16	25	36	49	64	81
3d Power	1	8	27	64	125	216	343	512	729
4th Power	1	16	81	256	625	1296	2401	4096	6561
5th Power	1	32	243	1024	3125	7776	16807	32768	59040
6th Power	1	64	729	4096	15625	46656	117649	262144	531441
7th Power	1	128	2187	16384	78125	279936	823543	2097152	4782969
8th Power	1	256	6561	65536	390625	1679616	5764801	16777216	43046721
9th Power	1	512	19683	262144	1953125	10077696	40353607	134217728	387420489
10th Power	1	1024	59049	1048576	9765625	60466176	282475249	1073741824	3486784401
11th Power	1	2048	177147	4194304	48828125	362797056	1977326743	8589934592	31381059609
12th Power	1	4096	531441	16777216	244140625	2176782336	13841287201	68719476736	282429536481
13th Power	1	8192	1594323	67108864	1220703125	13060694016	96889010407	549755813888	2541865828329
14th Power	1	16384	4782969	268435456	6103515625	78364164096	678223072849	4398046511104	22876792454961
15th Power	1	32768	14348907	1073741824	30517578125	470184984576	4747561509943	35184372088832	205891132094649

Root or Number.	Cube.	Root or Number.	Cube.	Root or Number.	Cube.
937	822656953	959	881974079	980	941192000
938	825293672	960	884736000	981	944076141
939	827936019	961	887503681	982	946966168
940	830584000	962	890277128	983	949862087
941	833237621	963	893056347	984	952763904
942	835896888	964	895841344	985	955671625
943	838561807	965	898632125	986	958585256
944	841232384	966	901428696	987	961504803
945	843908625	967	904231063	988	964430272
946	846590536	968	907039232	989	967361669
947	849278123	969	909853209	990	970299000
948	851971392	970	912673000	991	973242271
949	854670349	971	915498611	992	976191488
950	857375000	972	918330048	993	979146657
951	860085351	973	921167317	994	982107784
952	862801408	974	924010424	995	985074875
953	865523177	975	926859375	996	988047936
954	868250664	976	929714176	997	991026973
955	870983875	977	932574833	998	994011992
956	873722816	978	935441352	999	997002999
957	876467493	979	938313739	1000	1000000000
958	879217912				

A TABLE OF POWERS.

R. or 1ſt Power	1	2	3	4	5	6	7	8	9
2d Power	1	4	9	16	25	36	49	64	81
3d Power	1	8	27	64	125	216	343	512	729
4th Power	1	16	81	256	625	1296	2401	4096	6561
5th Power	1	32	243	1024	3125	7776	16807	32768	59040
6th Power	1	64	729	4096	15625	46656	117649	262144	531441
7th Power	1	128	2187	16384	78125	279936	823543	2097152	4782969
8th Power	1	256	6561	65536	390625	1679616	5764801	16777216	43046721
9th Power	1	512	19683	262144	1953125	10077696	40353607	134217728	387420489
10th Power	1	1024	59049	1048576	9765625	60466176	282475249	1073741824	3486784401
11th Power	1	2048	177147	4194304	48828125	362797056	1977326743	8589934592	31381059609
12th Power	1	4096	531441	16777216	244140625	2176782336	13841287201	68719476736	282429536481
13th Power	1	8192	1594323	67108864	1220703125	13060694016	96889010407	549755813888	2541865828329
14th Power	1	16384	4782969	268435456	6103515625	78364164096	678223072849	4398046511104	22876792454961
15th Power	1	32768	14348907	1073741824	30517578125	470184984576	4747561509943	35184372088832	205891132094649

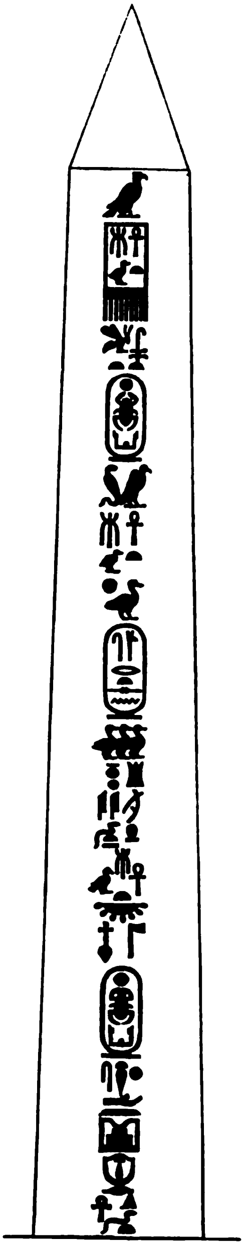

Fig. 13.
THE OBELISK AT HELIOPOLIS.

Fig. 14. represents a series of obeliscal areas, where the central or primitive triangle has the height to side of base as 1 : 2. The sectional axes being as 1, 3, 5, 7, &c., and ordinates, as 2×1, 2×2, 2×3, 2×4, &c., which equal twice the square root of the whole axis from the apex of the triangle or obelisk.

By varying the primitive triangle, a variety of designs for ceilings or panels may be formed.

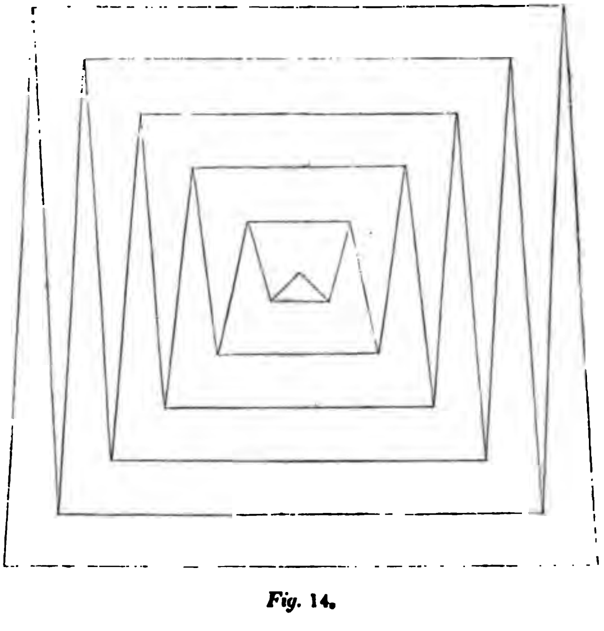

Fig. 14.

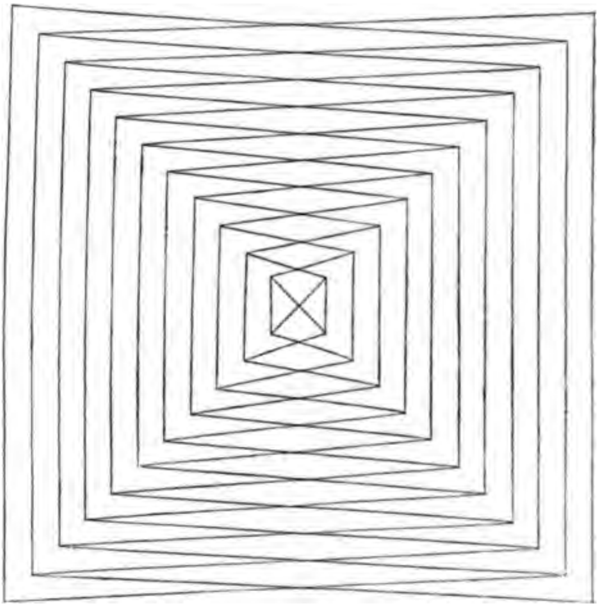

Fig. 15.

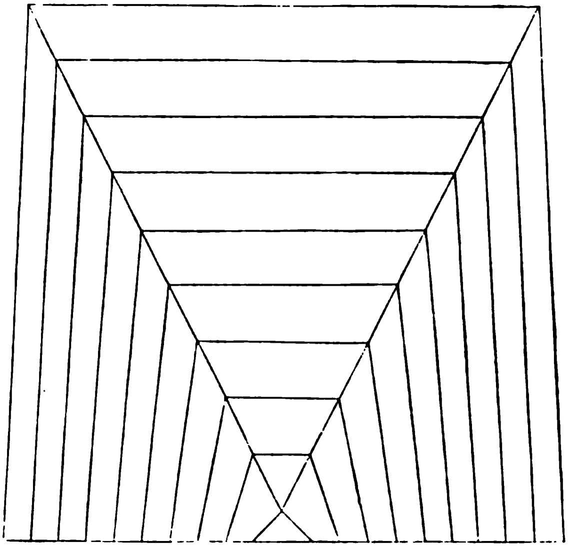

Fig. 16.

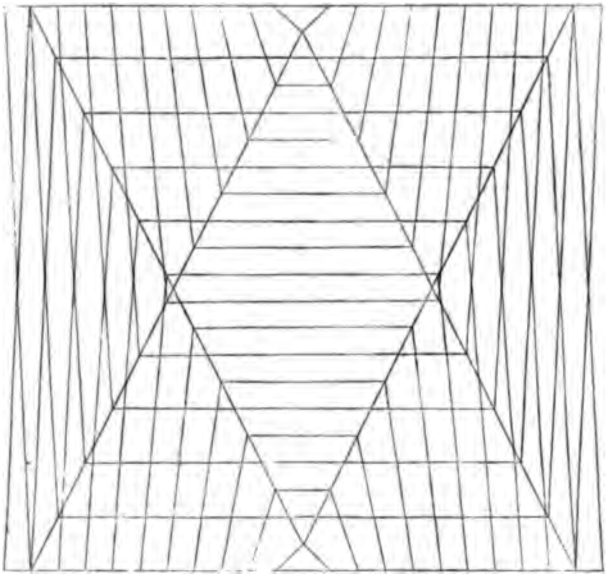

Fig. 17.

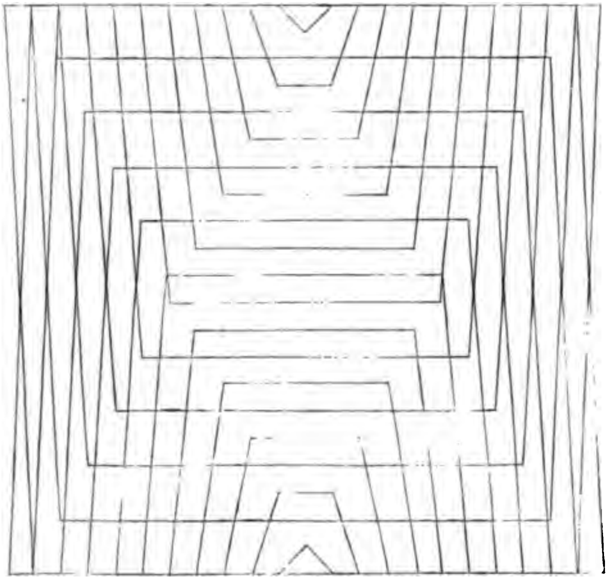

Fig. 18.

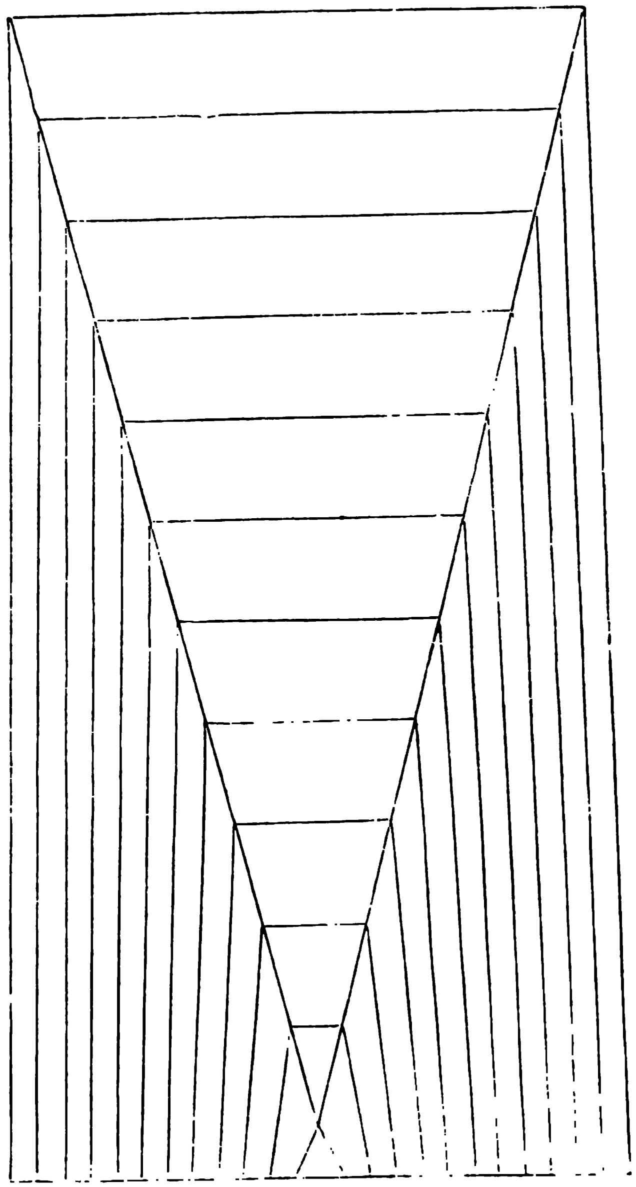

Fig. 19.

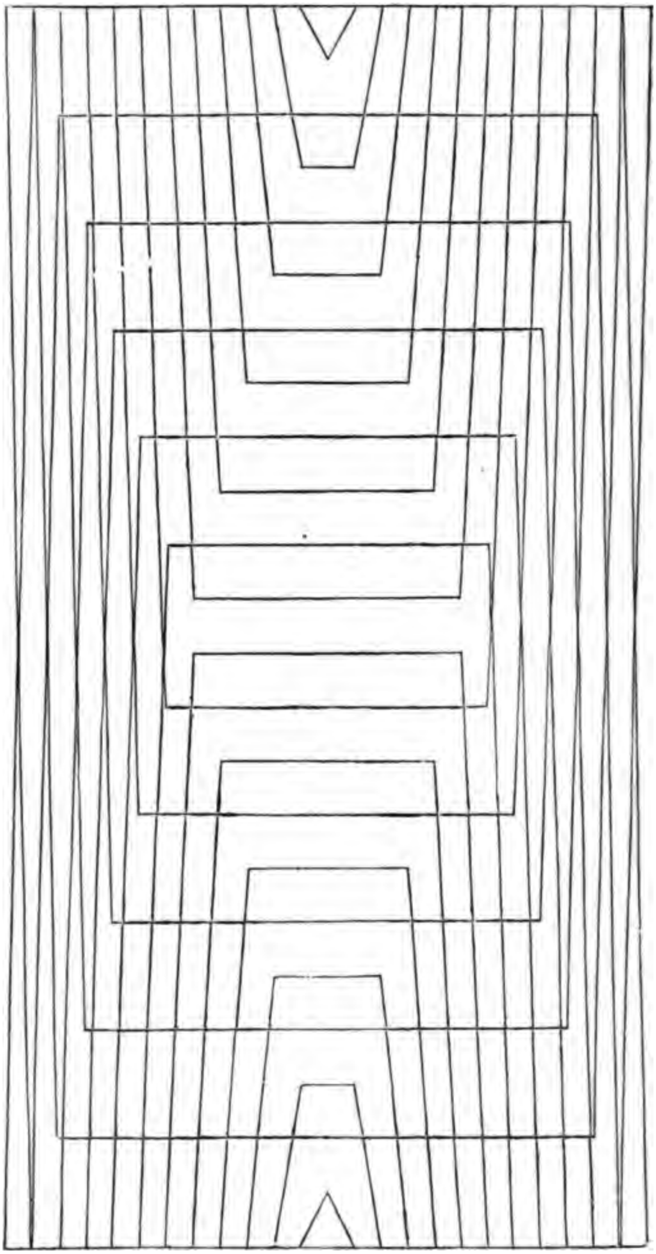

Fig. 20.

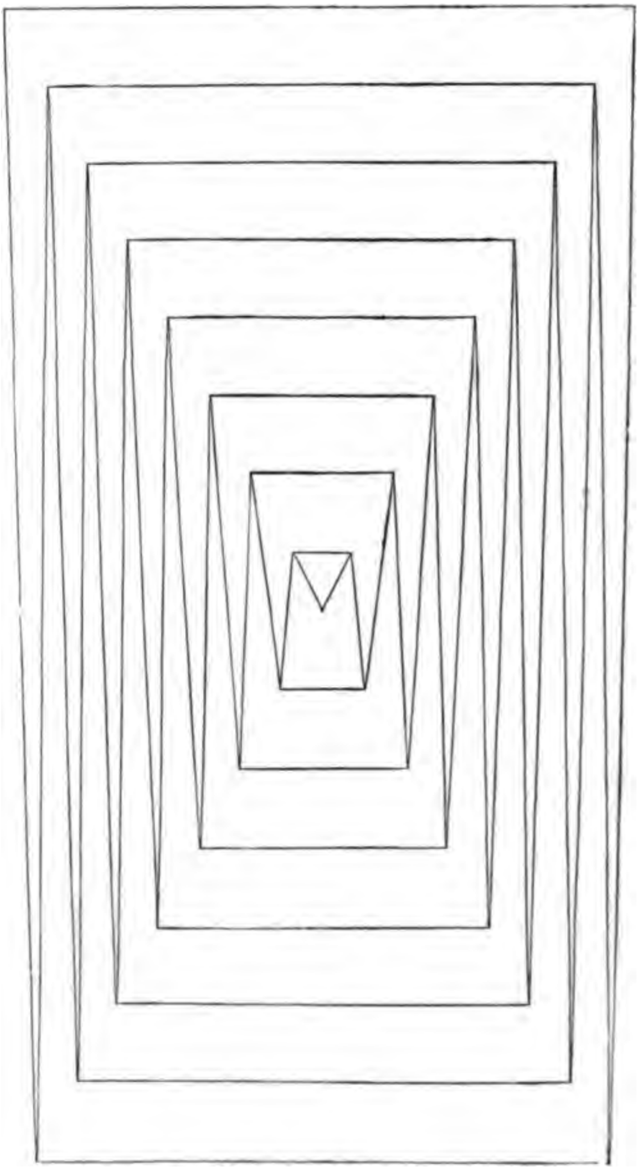

Fig. 21.

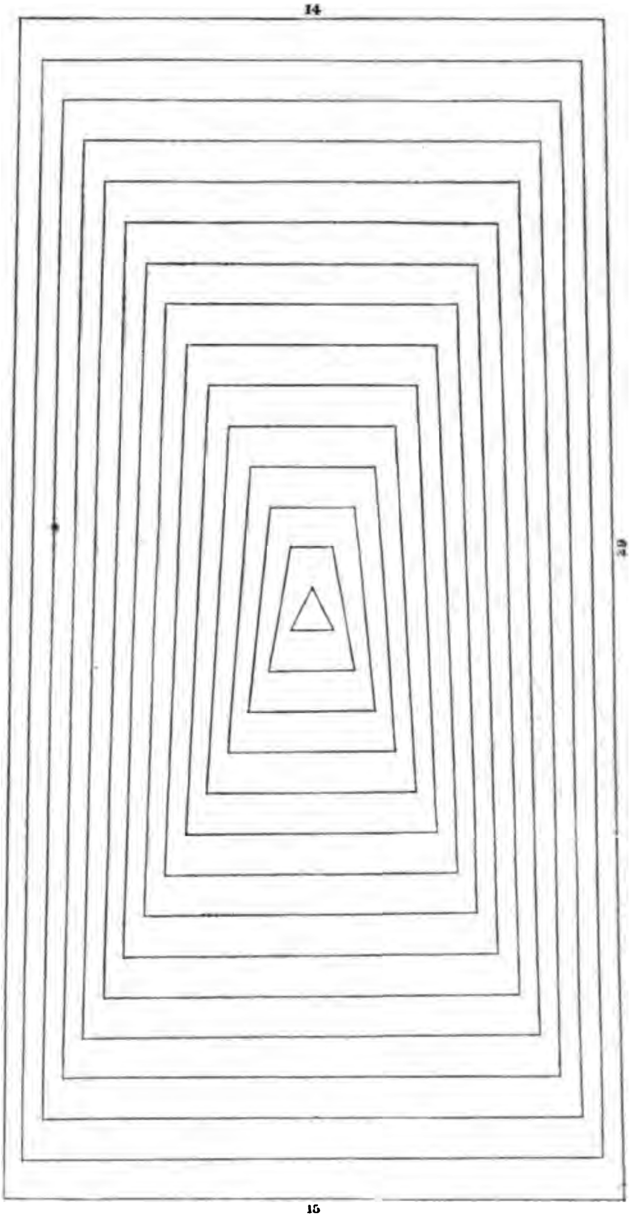

Fig. 22.

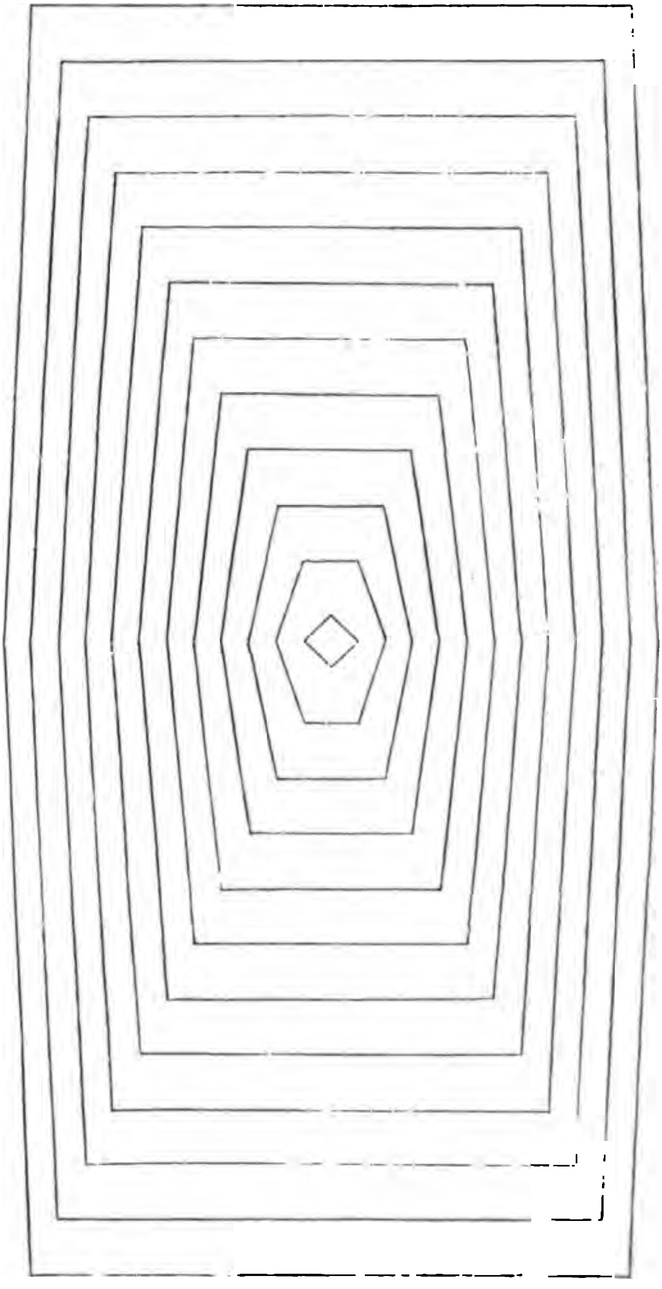

Fig. 23.

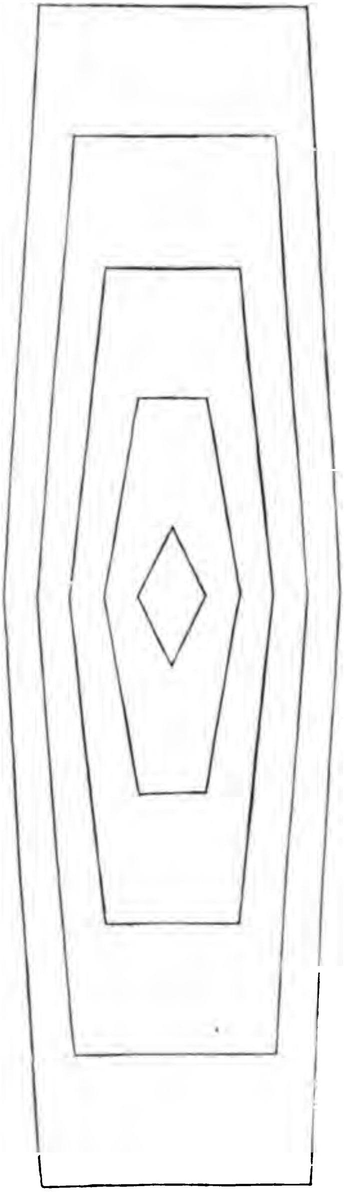

Fig. 24.

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